ON ELECTROMAGNETIC CORRECTIONS IN μ-e DECAY

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Electromagnetic corrections to the electron angular distribution have been obtained for the V-A theory of μ-e decay.

BEHRENDS, Finkelstein, and Sirlin[1] have calculated the electromagnetic corrections to the electron spectrum in the μ-e decay. The electron angular distribution in the decay of a polarized μ meson has been obtained by Kinoshita and Sirlin.[2] However, as was shown by Berman,[3] the infrared divergence was removed by the above-mentioned authors[1,2] incorrectly; Berman also gives the corrected energy spectrum of the decay electrons. In the present paper the corrected electron angular distribution is calculated.

We start from the interaction Lagrangian introduced by Feynman and Gell-Mann[4] (we use units such that $\hbar = c = 1$) with an additional term describing the interaction with the electromagnetic field:

$$L_{int} = 2G(\bar{\psi}_2\gamma_\mu \tfrac{1}{2}(1+\gamma_5)\psi_1)(\bar{\psi}_3\gamma_\mu \tfrac{1}{2}(1+\gamma_5)\psi_4) - ie(\bar{\psi}_1\hat{A}\psi_1) - ie(\bar{\psi}_2\hat{A}\psi_2) + \text{Herm. conj.} \quad (1)$$

(the subscripts 1, 2, 3, 4, refer to the μ meson, electron and the two neutrinos respectively).

The electromagnetic corrections consist of coherent radiative corrections and corrections to the μ-e decay with the emission of a photon (internal bremsstrahlung, see Fig. 1). The radiative corrections consist of corrections to the vertex part and the proper self energy part of the electron and μ meson. The contribution of the vertex part has the form

$$\begin{aligned}\Lambda_\rho = 2G\frac{\alpha}{2\pi}\Big\{&\frac{1}{2}\gamma_\rho(1+\gamma_5)A(\theta,\omega) + \frac{1}{2}\gamma_\rho(1-\gamma_5)\frac{\theta}{\sinh\theta}\\
&+\frac{1}{2}(1+\gamma_5)\frac{i}{4m_1}\Big[P_\rho\Big(\frac{\theta}{\sinh\theta} - \frac{2}{\sinh\theta}F_2(\theta,\omega)\Big)\\
&+2q_\rho\Big(\frac{\theta}{\sinh\theta} - 3\frac{F_2(\theta,\omega)}{\sinh\theta} + (\cosh\omega - \cosh\theta)^{-1}\\
&\times\Big(1 - \frac{\sinh\omega}{\sinh\theta}F_2(\theta,\omega) - F_3(\theta,\omega)\Big)\Big)\Big]\\
&+\frac{1}{2}(1-\gamma_5)\frac{i}{4m_2}\Big[P_\rho\Big(\frac{\theta}{\sinh\theta} + \frac{2}{\sinh\theta}F_2(\theta,\omega)\Big)\\
&-2q_\rho\Big(\frac{\theta}{\sinh\theta} + 3\frac{F_2(\theta,\omega)}{\sinh\theta} + (\cosh\omega - \cosh\theta)^{-1}\\
&\times\Big(1 - \frac{\sinh\omega}{\sinh\theta}F_2(\theta,\omega) - F_3(\theta,\omega)\Big)\Big)\Big]\Big\}. \quad (2)\end{aligned}$$

Here we used the convenient abbreviations introduced by Behrends et al[1]

$$\coth\theta = -(p_1p_2)/m_1m_2, \quad \omega = \ln(m_1/m_2),$$

$$\omega_> = \ln(L/m_2), \quad \omega_< = \ln(\lambda/m_2),$$

$$P = p_1 + p_2, \quad q = p_1 - p_2,$$

where m_1 and m_2 are the masses of the μ meson and electron, L is the upper cutoff on the momentum of the virtual photon and λ is the photon mass. The functions $A(\theta,\omega)$ and $F_i(\theta,\omega)$ are

$$A(\theta,\omega) = (\theta - F_1(\theta,\omega))\coth\theta + (1-\theta\coth\theta)(\omega - 2\omega_<) + F_3(\theta,\omega) + r_\Lambda,$$

where

$$r_\Lambda = -\tfrac{3}{2}\omega + \tfrac{1}{4} + 2\omega_< + \omega_>; \quad (3)$$

$$F_1(\theta,\omega) = L\left(\frac{2\sinh\theta}{e^\omega - e^{-\theta}}\right) - L\left(\frac{2\sinh\theta}{e^\theta - e^{-\omega}}\right) + (\omega-\theta)\ln\frac{\sinh\left[\frac{(\omega-\theta)}{2}\right]}{\sinh\left[\frac{(\omega+\theta)}{2}\right]},$$

$$F_2(\theta,\omega) = \frac{\omega\sinh\theta - \theta\sinh\omega}{2(\cosh\omega - \cosh\theta)},$$

$$F_3(\theta,\omega) = \frac{\omega\sinh\omega - \theta\sinh\theta}{2(\cosh\omega - \cosh\theta)},$$

$$L(x) = \int_0^x \frac{\ln(1-t)}{t}dt = -\sum_{k=1}^{\infty}\frac{x^k}{k^2}. \quad (4)$$

The terms $\sim\gamma_\rho$, 1 in (2) correspond to the vector and the terms $\sim\gamma_\rho\gamma_5$, γ_5 correspond to the axial vector interactions.

After mass renormalization the radiative corrections corresponding to the electron and μ meson self energies give a contribution equal to

$$\Sigma_1 + \Sigma_2 = (G\alpha/2\pi)\gamma_\rho(1+\gamma_5)(\tfrac{3}{2}\omega - \tfrac{9}{4} - \omega_> - 2\omega_<). \quad (5)$$

In the absence of transitions ($\theta = \omega = 0$) the complete matrix element for radiative corrections

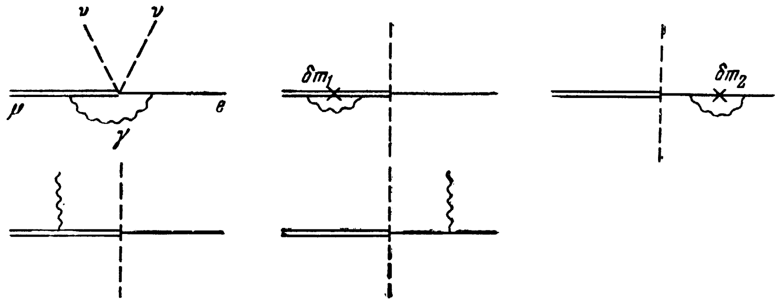

FIG. 1

should vanish. This is the case for the vector interaction. For the axial vector interaction there is a finite remainder $-(\alpha G/2\pi)\gamma_\rho\gamma_5$. It may be interpreted as a change in the axial vector coupling constant. It is of order $\alpha/2\pi$, i.e., 0.1%. Consequently second order radiative corrections leave the ratio of coupling constants in the V-A theory practically unchanged.

Second-order electromagnetic corrections, accurate to terms of order $\alpha/2\pi$, turn out to be equal for the V and A interactions in the approximation $m_2/m_1 \ll 1$. The infrared divergence in the radiative corrections is compensated by the infrared divergence in the internal bremsstrahlung. The probability per unit time for decay with the emission of a photon of a polarized μ meson (with polarization four-vector a satisfying $a\cdot p_1 = 0$) is given by

$$\frac{dw}{dt} = \frac{\alpha G^2}{3(2\pi)^5}\frac{d^3p_2}{\varepsilon_1\varepsilon_2}\frac{d^3k}{\varepsilon_\gamma}$$

$$\times\Big\{\Big[\frac{1}{2}(m_1^2-m_2^2) - Q^2\big(Q^2+\frac{1}{2}(m_1^2-m_2^2)\big)\Big]\Phi$$

$$-4Q^2 + \frac{(kQ)^2}{(p_1k)(p_2k)}(m_1^2+m_2^2-2Q^2)$$

$$-m_1\Big[(m_1^2-m_2^2+2Q^2)\Big[\Phi(ap_2) + \frac{(kQ)}{(p_1k)(p_2k)}$$

$$\times\Big[\frac{1}{2}(m_1^2+m_2^2+Q^2)\frac{(ka)}{(kp_1)} - (ap_2)\Big]$$

$$-\frac{(ka)}{(kp_1)}\frac{m_2^2(kQ)}{(kp_2)^2}\Big] - 4Q^2\frac{(ka)}{(kp_1)}\Big]\Big\}, \qquad (6)$$

where k and ϵ_γ are the four-momentum and energy of the photon and

$$\Phi = [p_{2\rho}/(p_2k) - p_{1\rho}/(p_1k)]^2, \qquad Q = p_1 - p_2 - k.$$

For $a = 0$, (6) goes over into Eq. (17) of Behrends et al.[1]

Integrating (6) over d^3k (with $k^2 = -\lambda^2$ taken into account) and combining the result with the radiative corrections we finally obtain the following expression for the spectrum and angular distribution of electrons from the decay of a polarized μ meson (with a polarization vector ξ in the μ-meson rest system for $m_2/m_1 \ll 1$):

$$dN(\varepsilon, \varphi, \xi) = \tfrac{1}{12}G^2m_1^5(2\pi)^{-3}\{3-2\varepsilon+(\alpha/2\pi)f(\varepsilon)$$

$$-\xi\cos\varphi\,[2\varepsilon-1+(\alpha/2\pi)h(\varepsilon)]$$

$$+6\zeta(m_2/m_1)(1-\varepsilon)/\varepsilon\}\,\varepsilon^2 d\varepsilon d\Omega/4\pi. \qquad (7)$$

Here φ is the angle between the direction of electron emission and of the μ-meson spin vector, $\epsilon = \epsilon_2/\epsilon_{2\,max}$;

$$\xi = \frac{2\,\mathrm{Re}\,g_V^*g_A}{|g_V|^2+|g_A|^2},$$

$$\zeta = \frac{|g_A|^2-|g_V|^2}{|g_V|^2+|g_A|^2}, \qquad |\zeta| \leqslant (1-\xi^2)^{1/2}, \qquad (8)$$

g_V and g_A are the vector and axial vector coupling constants, ζ is a measure of the relative contributions of the V and A couplings;

$$f(\varepsilon) = 2(3-2\varepsilon)u(\varepsilon) + 6(1-\varepsilon)\ln\varepsilon$$

$$+\tfrac{1}{3}\varepsilon^{-2}(1-\varepsilon)[(5+17\varepsilon-34\varepsilon^2)(\omega+\ln\varepsilon)-22\varepsilon+34\varepsilon^2],$$

$$h(\varepsilon) = 2(2\varepsilon-1)u(\varepsilon) + (6\varepsilon-2)\ln\varepsilon$$

$$+\tfrac{1}{3}\varepsilon^{-2}(1-\varepsilon)[(1+\varepsilon+34\varepsilon^2)(\omega+\ln\varepsilon)$$

$$+3-7\varepsilon-32\varepsilon^2-4\varepsilon^{-1}(1-\varepsilon)^2\ln(1-\varepsilon)],$$

$$u(\varepsilon) = 2\sum_{k=1}^{\infty}\frac{\varepsilon^k}{k^2} - \frac{\pi^2}{3} + \frac{3}{2}\omega - 2 + \ln\varepsilon\,\{3\ln(1-\varepsilon)$$

$$-2\ln\varepsilon - 2\omega + 1\} + (2\omega - 1 - \tfrac{1}{\varepsilon})\ln(1-\varepsilon). \qquad (9)$$

These expressions are a good approximation to the spectrum from $\epsilon \sim 0.1$ on. (For smaller values of ϵ the corrections become very large and it becomes necessary to consider higher order approximations in the electromagnetic field.) At the end of the spectrum (at $\epsilon = 1$) $f(\epsilon)$ and $h(\epsilon)$ diverge. This divergence may be removed by introducing an experimental energy interval $\Delta\epsilon$.[1] The function $(\alpha/2\pi)f(\epsilon)/(3-2\epsilon)$ was tabulated by Berman.[3] We give a table of values of $(\alpha/2\pi)\times h(\epsilon)$. In the absence of electromagnetic correc-

The correction function to the angular distribution ($\Delta\epsilon = 0$)

ε	$\frac{\alpha}{2\pi} h(\varepsilon)$	ε	$\frac{\alpha}{2\pi} h(\varepsilon)$
0.1	0.077	0.7	0.016
0.2	0.038	0.8	0.0033
0.3	0.031	0.9	−0.018
0.4	0.030	0.095	−0.038
0.5	0.027	0.099	−0.076
0.6	0.023		

tions the spectrum and angular distribution may be expressed as follows

$$dN(\varepsilon, \varphi, \xi) = \tfrac{1}{12} G^2 m_1^5 (2\pi)^{-3} [6(1-\varepsilon) + \tfrac{4}{3}\rho_M (4\varepsilon - 3)$$
$$-\xi \cos\varphi (2(1-\varepsilon) + \tfrac{4}{3}\delta_M (4\varepsilon - 3))$$
$$+ 6\zeta (m_2/m_1)\varepsilon^{-1}(1-\varepsilon)]\varepsilon^2 d\varepsilon d\Omega / 4\pi, \qquad (10)$$

where ρ_M is the parameter introduced by Michel and δ_M is the δ of Kinoshita and Sirlin.[2]

Conclusions may be drawn from a comparison of (7) and (10) about the effect of electromagnetic corrections on the μ-e decay. The mean μ-meson lifetime (τ) is reduced by 0.42%. The constants introduced[1] to characterize the electromagnetic corrections are equal to $\Lambda_1 = 0.13$, $\Lambda_2 = -0.022$ (for an experimental interval $\Delta\epsilon = 0$). Thus we find for the ρ-parameter $\rho = \frac{3}{4}(1 - 0.0042) = 0.747$ (defined in reference 1 as $\rho = 3\tau/4\tau_0$ where τ_0 is the mean lifetime in the absence of electromagnetic corrections). The ρ introduced in this manner depends only weakly on the magnitude of the electromagnetic corrections to the spectrum since even if the spectrum should change significantly the area under the spectral curve, which is equal to the mean lifetime, may remain unchanged as it did in the present case. Since the spectrum, including electromagnetic corrections, when divided by ϵ^2 may be approximated by a straight line for ϵ in the range $0.3 \lesssim \epsilon \lesssim 0.95$, another means of introducing ρ is made possible by relating ρ to the tangent of the angle of inclination of this line: $\tan\alpha = 6 + 16\rho/3$ (see reference 2). This yields $\rho = 0.70$. For this definition of ρ the comparison with experiment should be made only in the middle part of the spectrum, whereas with the previous definition the comparison is possible over the entire spectrum (see Rosenson[5]). For the angular part we similarly approximate the quantity $2\epsilon - 1 + (\alpha/2\pi)h(\epsilon)$ by a straight line and obtain $\delta = 0.74$ (in the absence of electromagnetic correction $\delta_M = \frac{3}{4}$).

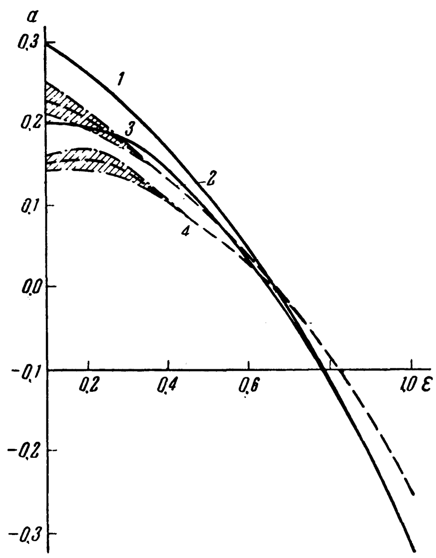

FIG. 2. Curves 1 and 2 give the asymmetry coefficient $a(\epsilon, \xi, \zeta)$ (introduced in reference 2) without electromagnetic corrections for $\xi = -1$ and $\xi = -0.76$; curves 3 and 4 – the same but including electromagnetic corrections. The shaded area indicates possible deviation from the equality $\zeta = 0$, i.e. from $|g_A| = |g_V|$ (for $\xi = -0.76$; $|\zeta| \leqslant 0.65$).

The values of the asymmetry coefficient introduced by Kinoshita and Sirlin,[2] including corrections resulting from the covariant method of elimination of the infrared divergence, are shown in Fig. 2.

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Note added in proof (August 31, 1959). The question of electromagnetic corrections is also studied in the recent paper by Kinoshita and Sirlin.[6]

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[6] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

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