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RADIATIVE CAPTURE OF NEUTRONS

D. F. ZARETSKIĬ

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An expression for the partial width of the radiative capture of neutrons in states described by shell model wave functions is derived. It is shown that in the case of s neutrons this width is proportional to the reduced neutron width for these neutrons. The relative probabilities for transitions to the $2p_{1/2}$ and $2p_{3/2}$ shell model states are calculated for a number of nuclei. The intensity ratios thus obtained are in good agreement with experiment.

THE spectrum of the γ quanta resulting from the capture of thermal neutrons by nuclei has some characteristic features, which were noticed by Groshev and collaborators.[1,2] For a number of light nuclei and nuclei of intermediate atomic weight intensive single lines are observed, which are mainly due to electric dipole transitions into the shell model states $2p_{1/2}$ and $2p_{3/2}$. The above-mentioned authors also measured the intensity ratios for the transitions into these states.

We shall give below an interpretation of these transitions in terms of the model of Lane, Thomas, and Wigner.[3] The wave function of the compound nucleus for excitation energies of the order of the neutron binding energy can be written in the form (for A nucleons)

$$\psi_\lambda = \sum_{c,\,k} C^\lambda_{c,\,k} \varphi_c u_k (\mathbf{r}_A), \qquad (1)$$

where $\varphi_c(\mathbf{r}_1, \ldots, \mathbf{r}_{A-1})$ is a complete system of residual nucleus wave functions, and $u_k(\mathbf{r}_A)$ is a complete system of wave functions for a single nucleon in the self-consistent field of the nucleus.

We further assume that the wave function of the final state of the nucleus can be written in the form

$$\psi_0 = \varphi_0 u_p (\mathbf{r}_A), \qquad (2)$$

where $\varphi_0(\mathbf{r}_1, \ldots, \mathbf{r}_{A-1})$ is the wave function corresponding to the unexcited residual nucleus, and $u_p(\mathbf{r}_A)$ is the wave function of the nucleon in the final state.

The dipole transition operator for a system of A nucleons with account of the effective charge of the neutrons and protons, arising as a result of the separation of the motion of the center of mass of the system, has the form[4]

$$Q_{1m} = \sum_{n=1}^{A} Q^{(n)}_{1m}, \qquad Q^{(n)}_{1m} = e_{eff} r_n Y^*_{1m}(\theta_n, \varphi_n), \qquad (3)$$

where $Q^{(n)}_{1m}$ is the dipole transition operator for the n-th nucleon.

Let us now calculate the matrix element for the transition from the initial into the final state, $M_{1m} = \langle\lambda\,|\,Q_{1m}\,|\,0\rangle$. For this purpose we separate out of the expansion (1) the wave function corresponding to the incoming channel,

$$\psi_\lambda = C^\lambda_{0s}\varphi_0 u_s(\mathbf{r}) + \sum_{c,k}{}' C^\lambda_{ck}\varphi_c u_k(\mathbf{r}), \qquad (4)$$

where the prime on the summation sign denotes the exclusion of the state corresponding to the incoming channel. It is then easily seen that

$$M_{1m} = \sum_{c,k}{}' C^\lambda_{ck} \langle \varphi_c u_k \left| \sum_{n=1}^{A-1} Q^{(n)}_{1m} \right| \varphi_0 u_p \rangle + C^\lambda_{0s}\langle u_s\,|\,Q^{(A)}_{1m}\,|\,u_p\rangle + \sum_{c,k}{}' C^\lambda_{ck}\langle \varphi_c u_k\,|\,Q^{(A)}_{1m}\,|\,\varphi_0 u_p\rangle. \qquad (5)$$

The level width of the k-th single nucleon state, W_k, resulting from the transition of the whole system into a compound nucleus state, can be determined from the formula

$$W^2_k = \sum_\lambda (E_\lambda - E_{ck})^2 |C^\lambda_{ck}|^2 = \langle c, k\,|\,v^2\,|\,c, k\rangle, \qquad (6)$$

where v is the operator of the potential energy of the interaction of a given nucleon with all the others. Let us consider the case

$$\overline{D}_\lambda \ll W_k \ll \Delta E_k, \qquad (7)$$

where $\overline{D}_\lambda$ is the average distance between the compound nucleus levels, and ΔE_k is the distance between the single particle levels in the self-consistent field of the nucleus. It is shown in the paper of Lane, Thomas, and Wigner that in this case

$$W^2_k = B^2/\zeta^2_k, \qquad B^2 = \int (E_\lambda - E_{ck})^2 S^{(k)}_\lambda dE_\lambda,$$

$$S^{(k)}_\lambda = \gamma^2_{\lambda,k}/\overline{D}_\lambda, \qquad \zeta^2_k = (a\hbar^2/2M)\,R^2_k(a), \qquad (8)$$

where $S^{(k)}_\lambda$ is the strength function, $\gamma^2_{\lambda,k}$ is the reduced neutron width, ζ^2_k is the reduced single particle width, a is the radius of the nucleus,

$R_k(a)$ is the value of the radial nucleon wave function on the surface of the nucleus, and M is the mass of the nucleon.

For the nuclei of interest to us we have $\zeta^2 \sim 1$ Mev. The energy width of the strength function for s neutrons is, according to experiment, of the order of several Mev. The width of interest to us, W_s, is therefore also of the order of a few Mev. With the help of these estimates we can now draw conclusions as to which of the terms in expression (5) plays the most important role.

It should be noted, first of all, that we can neglect the contribution to the sum (5) of all single particle states with an energy separation $\Delta E_k \gg W_k$. If the wave function $u_p(\mathbf{r})$ corresponds to a p state, then we should keep only s states in the last term.* The energy separation between s levels with different principal quantum numbers is of the order ~ 15 Mev. We can therefore neglect the contribution of s states with principal quantum numbers different from that of the incoming channel.

Only the terms with index k = p should be kept in the first sum in (5). However, if the spacing between adjacent s and p single-nucleon levels is $\Delta E_{sp} \gg W_s$, the whole first sum can be neglected. If all the above-mentioned conditions are fulfilled, we get the result

$$|M_{1m}|^2 = |C^{\lambda}_{0s}|^2 |\langle u_s | Q^{(A)}_{1m} | u_p \rangle|^2. \qquad (9)$$

The coefficient $|C^{\lambda}_{0s}|^2$ is simply related to the reduced neutron width, which can be determined from experiment:

$$\gamma^2_{\lambda,s} = (a\hbar^2/2M)|C^{\lambda}_{0s}|^2 R^2_s(a) = |C^{\lambda}_{0s}|^2 \zeta^2_s. \qquad (10)$$

Let us estimate the order of magnitude of the matrix element (9). For this purpose we consider the normalization condition

$$\sum_{\lambda} |C^{\lambda}_{0s}|^2 = 1. \qquad (11)$$

The terms of the summation are significantly different from zero in the interval $\Delta E_{\lambda} \sim W_s$. Therefore

$$\overline{|C^{\lambda}_{0s}|^2} \approx 1/\rho_0 W_s,$$

where ρ_0 is the level density of the compound nucleus with a given value for the nuclear moment, corresponding to the capture of a thermal neutron. As a result we find for the partial width for the transition under consideration

$$\Gamma_p \approx \Gamma_{\text{single}}/\rho_0 W_s, \qquad (12)$$

*d states do not give a contribution, since they can, in our case, combine only with excited states of the residual nucleus.

where Γ_{single} is the width for a single-particle transition in the self-consistent nuclear field with neglect of compound nucleus formation. In those cases where Γ_p is known (Si^{29}, S^{33}), relation (12) is confirmed by the experimental data. At the same time we find that $W_s \sim 1$ Mev.

Let us now calculate the ratio of the transition probabilities to the states $2p_{1/2}$ and $2p_{3/2}$. The wave function of the initial state can in this case be written in the form

$$u_s = R_s(r)\chi_{1/2,m_s}/\sqrt{4\pi}, \qquad (13)$$

where $R_s(r)$ satisfies the boundary condition[4]

$$a(\partial R_s/\partial r)/R_s|_{r=a} = ik_s a, \qquad (14)$$

where k_s is the wave number of the neutron outside the nucleus, corresponding to the energy at which the maximum of the strength function S_{λ} is observed; $\chi_{1/2,m_s}$ is the spin function.

The wave function of the final state has the form

$$u_p(\mathbf{r}) = R^j_p(r) Y_{j,m_j},$$

$$Y_{j,m_j} = \sum_{m_s m} (1/2\, 1 m_s m | j m_j 1/2\, 1) Y_{1m}(\theta,\varphi)\chi_{1/2,m_s}, \qquad (15)$$

where R^j_p is the radial wave function of the p state of the nucleon with total angular momentum $j = 1/2$ and $j = 3/2$. Using the wave functions (13) and (15), we obtain the ratio of the partial widths for transitions to states with different values j in the form

$$\frac{\Gamma^{(3/2)}_{\gamma}}{\Gamma^{(1/2)}_{\gamma}} = 2\left(\frac{\omega_{3/2}}{\omega_{1/2}}\right)^3 \left|\int_0^{\infty} R^{(3/2)}_p R_s r^3 dr\right|^2 \Big/ \left|\int_0^{\infty} R^{(1/2)}_p R_s r^3 dr\right|^2, \qquad (16)$$

where $\omega_{3/2}$ and $\omega_{1/2}$ are the frequencies for the corresponding transitions.

The matrix elements in (14) can be calculated with the help of the known matrix relations

$$\langle\alpha|\ddot{r}|\beta\rangle = -\omega^2_{\alpha,\beta}\langle\alpha|r|\beta\rangle = -\frac{1}{M}\langle\alpha\left|\frac{\partial U}{\partial r}\right|\beta\rangle, \qquad (17)$$

where $U(r)$ is the self-consistent nuclear potential; $\partial U/\partial r$ is different from zero only near the surface of the nucleus. Let us estimate the magnitude of the logarithmic derivative $f^j_p = (\partial R^j_p/\partial r)/R^j_p|_{r=a}$. If $\kappa_j a \gg 1$, we have

$$f^j_p = \varkappa_j \equiv \sqrt{2ME_j}/\hbar, \qquad (18)$$

where E_j is the binding energy for the given level. If the neutron is captured by a nucleus for which S_{λ} is close to the maximum, we have $k_s \ll \kappa_j$. In this case

$$\frac{\Gamma^{(3/2)}_{\gamma}}{\Gamma^{(1/2)}_{\gamma}} = 2\left(\frac{\omega_{1/2}}{\omega_{3/2}}\right)\left|\int_0^{\infty} R^{(3/2)}_p \frac{\partial U}{\partial r} r^2 dr\right|^2 \Big/ \left|\int_0^{\infty} R^{(1/2)}_p \frac{\partial U}{\partial r} r^2 dr\right|^2. \qquad (19)$$

To calculate the value of the ratio (19) we used a potential U(r) of the form

$$U(r) = V(0)/[1 + e^{\alpha(r-r_0)}], \quad \alpha = \sqrt{2MV(0)}/\hbar. \quad (20)$$

The wave functions for the single-particle $2p_{1/2}$ and $2p_{3/2}$ states and the parameters V(0) and r_0 were taken from Nemirovskiĭ:[5]

$$V(0) = 50\,\text{MeV}, \quad \alpha r_0 = 1.98\,A^{1/3}\,[1 - 0.5\,(0.5 - Z/A)]. \quad (21)$$

In the table we compare the results of our calculations with the measured ratios (19) for those nuclei in which the $2p_{1/2}$ and $2p_{3/2}$ levels are identified. For the nuclei listed in the table, the single-particle 3s resonance occurs near the neutron binding energy.

Value of $\Gamma_\gamma^{(3/2)}/\Gamma_\gamma^{(1/2)}$

Nucleus	Experiment	By formula (19)	By formula (22)
Si^{29}	3	4.0	10
S^{33}	3.5	3.7	12
Ca^{41}	2	3.5	6
Ni^{59}	2.7	3.0	4.6

In the last column of the table we give the ratios of the widths as computed by the Weisskopf formula[4]

$$\Gamma_\gamma^{(3/2)}/\Gamma_\gamma^{(1/2)} = 2\,(\omega_{3/2}/\omega_{1/2})^3. \quad (22)$$

It is seen from the table that our estimate is in satisfactory agreement with experiment, while formula (22) gives too high values for the ratio of the corresponding intensities.

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