# EFFECT OF MULTIPLE SCATTERING ON PAIR PRODUCTION BY HIGH-ENERGY PARTICLES IN A MEDIUM

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Submitted to JETP editor April 11, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1010-1016 (October, 1959)

The effect of multiple scattering on the process of pair production by a fast particle going through a medium is considered. Calculations are carried out using a method developed previously by Migdal.

## 1. INTRODUCTION

LANDAU and Pomeranchuk[1,2] have shown that radiation processes in media are cut down considerably at high energies because of multiple scattering. Taking this effect into account, Migdal[3-5] obtained cross sections for bremsstrahlung and pair production by $\gamma$ rays.

In this work, the influence of the medium on pair production by charged particles is considered. With this in mind, following Migdal,[5] we establish the connection between the transition probability and the density matrix, and then use the equations for the density matrix, averaged over the coordinates of the scattering atoms.

Expressions (16) — (19) obtained for the cross section go over at low energies into those from the theory neglecting the influence of the medium, and at high energies give a substantially reduced probability for the process considered.

## 2. EFFECT OF MULTIPLE SCATTERING ON PAIR PRODUCTION BY CHARGED PARTICLES IN THE MEDIUM

We denote systems of electron and positron solutions to the Dirac equation in the medium by $\Psi_s$ and $\Phi_s$, respectively, defining them by

$$\Psi_s(\mathbf{r}, t) = e^{-iHt} u_{\mathbf{p}_s}^{\lambda_s} e^{i\mathbf{p}_s\mathbf{r}},$$

$$\Phi_s(\mathbf{r}, t) = e^{-iHt} v_{\mathbf{p}_s}^{\lambda_s} e^{-i\mathbf{p}_s\mathbf{r}},$$

where $v_{\mathbf{p}}^{\lambda}$ and $u_{\mathbf{p}}^{\lambda}$ are unit spin amplitudes,

$$H = H_0 + \sum_n V(\mathbf{r} - \mathbf{r}_n)$$

is the Dirac Hamiltonian in the external field of the scatterers.*

Taking into account the fact that at high energies the scattering leaves the spin state and absolute value of the momentum almost unchanged, we obtain

$$\Psi_s(\mathbf{r}, t) \approx \sum_{p=p_s} u_{\mathbf{p}}^{\lambda_s} e^{i\mathbf{p}\mathbf{r}} (e^{-iHt})^{+}_{\mathbf{p}, \mathbf{p}_s},$$

$$\Phi_s(\mathbf{r}, t) \approx \sum_{p=p_s} v_{\mathbf{p}}^{\lambda_s} e^{-i\mathbf{p}\mathbf{r}} (e^{-iHt})^{-}_{-\mathbf{p}, -\mathbf{p}_s}, \qquad (1)$$

where the signs + and − remind us that the matrix element does not depend on the orientation of the spin, but does depend on the sign of the energy.

Processes of first and second order (see reference 6) contribute to the effect considered. The largest term in the integral cross section comes from the second-order process in which the effects of the external field on the wave functions of the resulting electron and positron are taken into account, but the initial particle is viewed as free. The corresponding matrix element has the form*

$$M = \frac{1}{2} e^2 \int I_\mu(x) (\overline{\Psi}(x') \gamma_\mu \Phi(x')) D(x' - x)\, dx\, dx', \qquad (2)$$

where

$$I_\mu = P_\mu \exp\{i(\mathbf{p} - \mathbf{p}')\mathbf{r} - i(E - E')t\}$$

is the transition current of initial particles. If the latter have zero spin, then

$$P_\mu = (p + p')_\mu / 2EE',$$

and for spin $\frac{1}{2}$

$$P_\mu = (\overline{u}_{\mathbf{p}'}^{\lambda} \gamma_\mu u_{\mathbf{p}}^{\lambda_0}).$$

Using Eq. (1), the matrix element (2) takes the form:

$$M = \frac{e^2 P_\mu}{\omega^2 - k^2} \int_0^{t_0} dt\, e^{-i\omega t} \sum_{\mathbf{p}_1} (\overline{u}_{\mathbf{p}_1}^{\lambda_1} \gamma_\mu v_{\mathbf{k}-\mathbf{p}_1}^{\lambda_2}) (e^{iHt})^{+}_{\mathbf{p}_-, \mathbf{p}_1} (e^{-iHt})^{-}_{\mathbf{p}_1 - \mathbf{k}, -\mathbf{p}_+}.$$

Here $t_0$ is the time of motion in the medium,

*We employ the system of units in which $\hbar = c = m_e = 1$.

*The notation is that used in the book of Akhiezer and Berestetskiĭ.[7]

$\mathbf{k} = \mathbf{p} - \mathbf{p}'$, $\omega = E - E'$. The quantity $\langle |M^2| \rangle$, where $\langle \ldots \rangle$ denotes an average over the coordinates of the scatterers, enters into the transition probability. In the future it will be convenient to integrate the quantity $\langle |M|^2 \rangle$ over the angle of emission of the positron, since, as is easily seen from Eq. (1), this is practically equivalent to integration over $d^3\mathbf{p}_+$. We obtain

$$\int \langle |M|^2 \rangle d^3\mathbf{p}_+ = \mathrm{Re} \frac{2e^4 P_\mu P_\nu^*}{(\omega^2 - k^2)^2} \int_0^{t_0} dt \int_0^{t_0} dt' e^{i\omega(t'-t)}$$

$$\times \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} (\bar{u}^{\lambda_1}_{p_1} \gamma_\mu v^{\lambda_2}_{\mathbf{k}-\mathbf{p}_1}) (\bar{v}^{\lambda_2}_{\mathbf{k}/2-\mathbf{p}_2} \bar{\gamma}_\nu u^{\lambda_1}_{\mathbf{k}/2+\mathbf{p}_2})$$

$$\times \langle (e^{iHt})^+_{p_-, \mathbf{p}_1} [e^{iH(t'-t)}]^-_{\mathbf{p}_1-\mathbf{k}, \mathbf{p}_2-\mathbf{k}/2} (e^{-iHt'})^+_{p_2+\mathbf{k}/2, \mathbf{p}_-} \rangle. \quad (3)$$

Employing the equality

$$(e^{-iHt'})^+_{p_2+\mathbf{k}/2, \mathbf{p}_-} = \sum_{\mathbf{p}'} [e^{-iH(t'-t)}]^+_{p_2+\mathbf{k}/2, \mathbf{p}'} (e^{-iHt})^+_{\mathbf{p}', \mathbf{p}_-},$$

denoting $t' - t = \tau$ and using the fact that one can average independently over the coordinates of the scatterers which enter through the factors $e^{\pm iHt}$ and $e^{\pm iH\tau}$, we can put (3) into the form

$$\int \langle |M|^2 \rangle d^3\mathbf{p}_+ = \frac{2e^4}{(\mathbf{k}^2 - \omega^2)^2} \mathrm{Re}\, P_\mu P_\nu^* \int_0^{t_0} dt \int_0^{t_0-t} d\tau\, e^{i\omega\tau}$$

$$\times \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} (\bar{u}^{\lambda_1}_{\mathbf{p}_1} \gamma_\mu v^{\lambda_2}_{\mathbf{k}-\mathbf{p}_1}) (\bar{v}^{\lambda_2}_{\mathbf{k}/2-\mathbf{p}_2} \bar{\gamma}_\nu u^{\lambda_1}_{\mathbf{k}/2+\mathbf{p}_2})$$

$$\times f_0^{++}(\mathbf{p}_-, \mathbf{p}_1, t)\, f_{\mathbf{k}}^{+-}(\mathbf{p}_1, \mathbf{p}_2, \tau),$$

$$\langle (e^{iHt})^+_{\mathbf{p}_-, \mathbf{p}_1} (e^{-iHt})^+_{\mathbf{p}', \mathbf{p}_-} \rangle = \delta_{p_1, p'} f_0^{++}(\mathbf{p}_-, \mathbf{p}_1, t)$$

$$\langle (e^{-iH\tau})^+_{p_2+\mathbf{k}/2, \mathbf{p}_1} (e^{iH\tau})^-_{p_1-\mathbf{k}, p_2-\mathbf{k}/2} \rangle = f_{\mathbf{k}}^{+-}(\mathbf{p}_1, \mathbf{p}_2, \tau). \quad (4)$$

The functions $f_0$ and $f_{\mathbf{k}}$ were introduced by Migdal.[5] They obey the same equation as the averaged density matrix (see reference 4).

We now sum the expression (4) over the spins of the final state and average over those of the initial state, obtaining

$$I = \frac{1}{2} \sum_{\lambda_1, \lambda_2} \int \langle |M|^2 \rangle d^3\mathbf{p}_+ = e^4 \mathrm{Re} \Lambda_{\mu\nu}(\mathbf{p}, \mathbf{p}') \int_0^{t_0} dt \int_0^{t_0-t} d\tau\, e^{i\omega\tau}$$

$$\times \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} \frac{G_{\mu\nu}(\mathbf{p}_1, \mathbf{p}_2)}{(k^2 - \omega^2)^2} f_0^{++}(\mathbf{p}_-, \mathbf{p}_1, t)\, f_{\mathbf{k}}^{+-}(\mathbf{p}_1, \mathbf{p}_2, \tau), \quad (5)$$

where

$$G_{\mu\nu}(\mathbf{p}_1, \mathbf{p}_2) = \sum_{\lambda_1, \lambda_2} (\bar{u}^{\lambda_1}_{\mathbf{p}_1} \gamma_\mu v^{\lambda_2}_{\mathbf{k}-\mathbf{p}_1}) (\bar{v}^{\lambda_2}_{\mathbf{k}/2-\mathbf{p}_2} \bar{\gamma}_\nu u^{\lambda_1}_{\mathbf{k}/2+\mathbf{p}_2}),$$

$$\Lambda_{\mu\nu}(\mathbf{p}, \mathbf{p}') = \begin{cases} (p + p')_\mu (p + p')^*_\nu / 2EE' & \text{for spin } 0, \\ \frac{1}{4EE'} \mathrm{Sp}\, \gamma_\mu (i\hat{p} - m)\, \bar{\gamma}_\nu (i\hat{p}' - m) & \text{for spin } \frac{1}{2}. \end{cases} \quad (6)$$

In so far as we are interested in only the integral cross sections, we can make the following replacements (compare reference 6)

$$G_{\mu\nu}\Lambda_{\mu\nu} \to \tfrac{1}{2} G_\perp \Lambda_\perp$$

$$+ (1 - k^{-2}\omega_1\omega)(1 - k^{-2}\omega_2\omega) G_{44}\Lambda_{44}, \quad (7)$$

where

$$G_\perp = G_{11} + G_{22}, \qquad \Lambda_\perp = \Lambda_{11} + \Lambda_{22}.$$

The further transformation of (5) and calculation of quantities entering into (7) is carried out just as in the work of Migdal.[5] Introducing the notation

$$\mathbf{p} = \mathbf{n}p(1 - \theta^2/2) + p\boldsymbol{\theta}, \qquad \mathbf{p}' = \mathbf{n}(p - k - p\theta^2/2) + p\boldsymbol{\theta},$$

$$\mathbf{p}_1 \approx \mathbf{n}q + g\boldsymbol{\eta}_0, \quad \mathbf{p}_2 \approx \mathbf{n}g + g\boldsymbol{\eta},$$

$$q = |\mathbf{p}_-|, \quad g = |q - k/2|, \quad (8)$$

we have

$$G_{\mu\nu}\Lambda_{\mu\nu} \to \frac{1}{2}\Lambda_\perp \left\{ \frac{k^2}{q^2(k-q)^2} + \frac{[q^2 + (k-q)^2] g^2 \eta\eta_0}{q^2(k-q)^2} \right\}$$

$$+ \frac{1}{2}\Lambda_{44} \left[ \frac{1 + g^2\eta^2}{q(k-q)} - \frac{m^2 + p^2\theta^2}{p(p-k)} \right] \left[ \frac{1 + g^2\eta_0^2}{q(k-q)} - \frac{m^2 + p^2\theta^2}{p(p-k)} \right]. \quad (9)$$

Here, for scalar particles

$$\tfrac{1}{2}\Lambda_\perp = p^2\theta^2 / p(p-k), \qquad \tfrac{1}{2}\Lambda_{44} = (p - k/2)^2 / p(p-k),$$

and for spin-$\frac{1}{2}$ particles

$$\frac{1}{2}\Lambda_\perp = \frac{m^2k^2 + [p^2 + (p-k)^2] p^2\theta^2}{2p^2(p-k)^2}, \qquad \frac{1}{2}\Lambda_{44} = 1.$$

Further we find

$$I = e^4 \mathrm{Re} \int_0^{t_0} dt \int_0^{t_0-t} d\tau e^{i\omega\tau}$$

$$\times \int d\boldsymbol{\vartheta}\, d\boldsymbol{\eta}\, v_0(\boldsymbol{\vartheta}, t)\, v(\boldsymbol{\eta}, \tau) \frac{\Lambda_{\mu\nu} G_{\mu\nu}}{(k^2 - \omega^2)^2}, \quad (10)$$

where we have set

$$\boldsymbol{\vartheta} = \mathbf{p}_-/p_- - \mathbf{p}_1/p_1,$$

$$f_0^{++}(\mathbf{p}_-, \mathbf{p}_1, t)\, d^3p_1/(2\pi)^3 \approx \delta(p_- - p_1)\, dp_1 v_0(\boldsymbol{\vartheta}, t)\, d\boldsymbol{\vartheta},$$

$$f_{\mathbf{k}}^{+-}(\mathbf{p}_1, \mathbf{p}_2, \tau)\, d^3p_2/(2\pi)^3 \approx \delta(p_2 - g)\, dp_2 v(\boldsymbol{\eta}, \tau)\, d\boldsymbol{\eta}.$$

Here the function $v_0(\boldsymbol{\vartheta}, t)$ is normalized by

$$\int v_0(\boldsymbol{\vartheta}, t)\, d\boldsymbol{\vartheta} = 1. \quad (11)$$

In addition, we use $\boldsymbol{\xi}$ to denote the vector angle between $\mathbf{k}$ and $\mathbf{p}_-$. Then

$$\boldsymbol{\vartheta} = \mathbf{p}_-/p_- - \mathbf{n} + \mathbf{n} - \mathbf{p}_1/p_1 = \boldsymbol{\xi} - \boldsymbol{\eta}_0 g/q. \quad (12)$$

The probability of pair production with summed energy between $k$ and $k + dk$ and electron energy between q and q + dq, per unit distance in the medium, can now be written in the form

$$w(p, k, q)\,dk\,dq = \frac{k^2\,dk q^2\,dq}{(2\pi)^6}\int \frac{dI}{dl_0}\,d\boldsymbol{\theta}\,d\boldsymbol{\xi}.$$

Using (11) and (12) and the equality $d\boldsymbol{\xi} d\boldsymbol{\theta} = (g/q)^2 d\boldsymbol{\xi} d\boldsymbol{\eta}_0$, we obtain

$$w(p, k, q) = \frac{k^2 g^2 e^4}{(2\pi)^6}\,\mathrm{Re}\int_0^{t_0} d\tau e^{i\omega\tau}\int d\boldsymbol{\theta}\,d\boldsymbol{\eta}\,d\boldsymbol{\eta}_0\frac{\Lambda_{\mu\nu}G_{\mu\nu}}{(k^2-\omega^2)^2}v(\boldsymbol{\eta},\tau), \tag{13}$$

where

$$k^2 - \omega^2 = k^2(m^2 + p^2\theta^2)/p(p-k),$$

and $\Lambda_{\mu\nu}G_{\mu\nu}$ is given by (9).

The integrals in (13) have the same form as those in the work of Migdal. The only difference consists in the fact that in our case $k \neq \omega$, in so far as we are dealing with virtual quanta. This can be taken into account by simply redefining the parameter $\alpha$ which enters into the equation for the function $e^{i\omega\tau}v(\boldsymbol{\eta}, \tau)$ (see reference 5). We have

$$\alpha = \frac{k(M^2 + p^2\theta^2)}{4Qp(p-k)}, \qquad \beta = \frac{kg^2}{4Qq(k-q)},$$

$$M^2 = m^2 + \frac{p(p-k)}{q(k-q)}.$$

The meaning of the parameters $\alpha$ and $\beta$ is clear from the expansion

$$[\varepsilon_{p_2+k/2} - \varepsilon_{p_2-k/2} - \omega]/2Q = \alpha + \beta\eta^2/2. \tag{14}$$

In our notation

$$Q = Z^2 e^4 nL/8\pi g^2,$$

where $L = \ln(\theta_{max}/\theta_{min})$. In order of magnitude, Q corresponds to the root mean square scattering angle per unit track length. We discuss the choice of angles $\theta_{max}$ and $\theta_{min}$ below.

As shown in reference 5, the effect of multiple scattering is determined by the size of the parameter

$$\tilde{s} = \frac{\alpha}{2\sqrt{2\beta}} = \frac{q(k-q)(M^2+p^2\theta^2)}{8p(p-k)}\sqrt{\frac{k}{q(k-q)Qg^2}}.$$

For $\tilde{s} > 1$, the effect of multiple scattering vanishes. We note now that, in so far as the time is concerned, times $\tau \lesssim t_k = s/\alpha Q(1+s)$ are essential, so that for $t_0 \gg t_k$, the integration over $d\tau$ can be extended to infinity. Then one obtains

$$\mathrm{Re}\int_0^\infty e^{i\omega\tau}d\tau\int d\boldsymbol{\eta}\,d\boldsymbol{\eta}_0\,v(\boldsymbol{\eta},\tau) = \frac{\pi}{6Q\alpha^2}G(\tilde{s}),$$

$$\mathrm{Re}\int_0^\infty e^{i\omega\tau}d\tau\int d\boldsymbol{\eta}\,d\boldsymbol{\eta}_0(\boldsymbol{\eta}\boldsymbol{\eta}_0)\,v(\boldsymbol{\eta},\tau) = \frac{2\pi}{3\alpha\beta Q}\Phi(\tilde{s}),$$

$$\mathrm{Re}\int_0^\infty e^{i\omega\tau}d\tau\int \eta^2\,d\boldsymbol{\eta}\,d\boldsymbol{\eta}_0\,v(\boldsymbol{\eta},\tau)$$

$$= \mathrm{Re}\int_0^\infty e^{i\omega\tau}d\tau\int \eta_0^2\,d\eta\,d\eta_0\,v(\eta,\tau) = -\frac{\pi G(\tilde{s})}{3\alpha\beta Q},$$

$$\mathrm{Re}\int_0^\infty e^{i\omega\tau}d\tau\int \eta^2\eta_0^2\,d\eta\,d\eta_0\,v(\eta,\tau) = \frac{2\pi}{3Q\beta^2}G(\tilde{s}). \tag{15}$$

The first of two of these equations were obtained by Migdal. The remaining are derived analogously.

Now, employing (13) – (15), we finally obtain for spin-½ particles, after integration over $d\boldsymbol{\theta}$,

$$w_{1/2}(p, k, q) = \frac{2r_0^2 nZ^2}{3\pi(137)^2 k^2}$$

$$\times L\left\{\frac{p^2+(p-k)^2}{p^2}\left[A(s,x) + 2\frac{q^2+(k-q)^2}{k^2}B(s,x)\right]\right.$$

$$+\frac{k^2}{p^2}\left[C(s,x) + 2\frac{q^2+(k-q)^2}{k^2}D(s,x)\right]$$

$$\left.+\frac{8q(k-q)}{k^2}\frac{p-k}{p}E(s,x)\right\}, \tag{16}$$

and for particles with zero spin

$$w_0(p, k, q) = \frac{4r_0^2 nZ^2}{3\pi(137)^2 k^2}$$

$$\times L\left\{\frac{p-k}{p}\left[A(s,x) + 2\frac{q^2+(k-q)^2}{k^2}B(s,x)\right]\right.$$

$$\left.+\frac{4q(k-q)}{k^2}\frac{(p-k/2)^2}{p^2}E(s,x)\right\}. \tag{17}$$

Here

$$A(s,x) = \int_{1+x}^\infty \frac{(z-x-1)\,G(sz)\,dz}{z^2(z-1)^2},$$

$$B(s,x) = \int_{1+x}^\infty \frac{(z-x-1)\,\Phi(sz)\,dz}{z(z-1)^2}, \quad C(s,x) = x\int_{1+x}^\infty \frac{G(sz)\,dz}{z^2(z-1)^2},$$

$$D(s,x) = x\int_{1+x}^\infty \frac{\Phi(sz)\,dz}{z(z-1)^2}, \tag{18}$$

$$E(s,x) = \int_{1+x}^\infty \frac{G(sz)\,dz}{z^2} \qquad s = \tfrac{1}{8}[k/q(k-q)Qg^2]^{1/2},$$

$$x = m^2 q(k-q)/p(p-k), \tag{19}$$

and $\Phi$ and G are functions introduced by Migdal.[5] For s > 1

$$\Phi(s) \approx G(s) \approx 1,$$

and for $s \ll 1$:

$$\Phi(s) \approx 6s, \qquad G(s) \approx 12\pi s^2 \approx (6s)^2.$$

In the intermediate region $0.1 < s < 1$, these functions can be approximated, to within a reasonable accuracy, by the following simple expressions

$$\Phi(s) \approx 6s/(6s+1), \quad G(s) \approx (6s)^2/[(6s)^2+1]. \quad (20)$$

The error from using these expressions oscillates from 10% at the edges of the interval to 20% in the middle of it.

The integrals (18) are easy to find by using expressions (20). We will not give the very lengthy formulae which result here, but consider only limiting cases.

1. $s \gg 1/(1+x)$. Then, in complete agreement with the results of the theory[6] neglecting effects of the medium, we obtain

$$A(s, x) = (1+2x)\ln\left(1+\frac{1}{x}\right) - 2,$$

$$B(s, x) = (1+x)\ln\left(1+\frac{1}{x}\right) - 1,$$

$$C(s, x) = \frac{1+2x}{1+x} - 2x\ln\left(1+\frac{1}{x}\right),$$

$$D(s, x) = 1 - x\ln\left(1+\frac{1}{x}\right),$$

$$E(s, x) = 1/(1+x). \quad (21)$$

2. $s \ll 1/(1+x)$. Then

$$A(s, x) \approx 36s^2 \ln\frac{1}{6sx}, \quad B(s, x) \approx 6s\ln\frac{1}{6sx},$$

$$C(s, x) \approx 36s^2, \quad D(s, x) \approx 6s, \quad E(s, x) \approx 3\pi s. \quad (22)$$

We turn now to understanding the magnitudes of the angles $\theta_{max}$ and $\theta_{min}$, which enter into L. The quantity $\theta_{min} = q_{min}/g = Z^{1/3}/137g$ is determined by the minimum momentum transfer; $\theta_{max}$ is of the same order of magnitude as the mean angle of particle emission. For $s > 1$, the latter is determined by Eq. (14), giving

$$\theta_{max} \sim \sqrt{\overline{\eta^2}} \sim \sqrt{\alpha/\beta} \sim \sqrt{1+x}/g \equiv \theta_0.$$

The multiple scattering has the effect of broadening the angular distribution. As shown in reference 5, for $s \ll 1$ we obtain $\eta^2 \sim \theta_0^2/s$, so that

$$\theta_{max} \sim \theta_0/\sqrt{s} = \sqrt{(1+x)/sg^2}.$$

At high energies, the quantity $\theta_0/\sqrt{s}$ can exceed the angle of diffraction on the nucleus, $\theta_1 = 1/Rg$, where $R = 0.5\, r_0 Z^{1/3}$. Then one should take $\theta_{max} = \theta_1$. Thus, we find

$$L - \begin{cases} \ln(190\, Z^{-1/3}\sqrt{1+x}) & \text{for } s>1,\ \sqrt{1+x} < 190\, Z^{-1/3} \\ 2\ln(190\, Z^{-1/3}) & \text{for } s>1,\ \sqrt{1+x} > 190\, Z^{-1/3} \\ \ln(190\, Z^{-1/3}\sqrt{(1+x)/s}) & \text{for } s<1,\ \sqrt{(1+x)/s} < 190\, Z^{-1/3} \\ 2\ln(190\, Z^{-1/3}) & \text{for } s<1,\ \sqrt{(1+x)/s} > 190\, Z^{-1/3} \end{cases} \quad (23)$$

One knows the numerical factor in the argument of the logarithm only to within order of magnitude. The number 190 was only a convenient choice.

Through the use of Eqs. (22) and (23) for $k \ll p/m$, $x \ll 1$, $s \ll 1$, Eqs. (16) and (17) assume the same form

$$w(p, k, q) \approx \frac{16 r_0^2 n Z^2}{\pi (137)^2 k^2} L \frac{q^2 + (k-q)^2}{k^2} s \ln\frac{1}{6sx}. \quad (24)$$

The condition $s \ll 1$, according to Eq. (19), means

$$s \approx 1400\,[kt/q(k-q)]^{1/2} \ll 1, \quad (25)$$

where t is the radiation length in centimeters, and k and q are in units of $m_e c^2$.

Multiple scattering will have a substantial effect on the integral cross section if Eq. (25) is fulfilled for values of q and $k - q \ll p/m$, i.e., for energies of the initial particle $p \gg 2 \cdot 10^6 t \mu c^2$.

The theory described here is applicable to electrons when the energies of the pair particles are much less than the energy of the electron.

## 3. DISCUSSION OF RESULTS

We considered the effect of multiple scattering, only on second-order processes. If the initial particle is an electron, the effect of the medium will also substantially decrease the contribution of the first-order processes, so that, as in the usual theory, they can be completely neglected in the region $k \ll p$. If $m \gg 1$, the medium has little effect on the first-order processes, and their contribution is given by formulae obtained previously.[6]

We note, further, that at high energies, when the multiple scattering (or the effect of polarization of the medium) cuts down substantially the probability of bremsstrahlung with emission of soft quanta, direct production of pairs may become the main source of low-energy particles appearing in showers. In fact, according to Eq. (25), the effect of the medium on pair production is appreciable only for $k > 10^{12}$ ev, whereas its effect on bremsstrahlung becomes important already for $p^2/k > 10^{12}$ ev (see reference 5). Take, for example, $p = 5 \times 10^{11}$ ev. Then, already for $p/k > 200$, the production of pairs is more important than emission of bremsstrahlung. This situation should be taken into account in analysis of shower processes, especially when the showers have penetrated only small depths.

In conclusion, I would like to express my deep gratitude to Prof. A. B. Migdal for his interest in the work and valuable discussions.

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Translated by G. E. Brown
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