ON THE MEASUREMENT OF THE MOMENTUM OF FAST PARTICLES AND THE INVESTIGATION OF NUCLEAR INTERACTIONS IN THE $10^{10}-10^{12}$ ev ENERGY RANGE

M. I. DAĬON and V. Kh. VOLYNSKIĬ

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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A new method for the direct measurement of the momentum of charged particles in a magnetic field in the $10^{10}-10^{12}$ ev energy range is proposed. The method is based on the simultaneous use of spark counters and nuclear emulsions. The possibility of applying the method to measurements of momentum spectra and to the investigation of nuclear interactions of protons with matter is discussed.

THE method consists of the following procedure: Three spark counters I, II, and III (Fig. 1), triggered by a Geiger-Müller counter system, are placed in the magnet gap. The spark counters make it possible to localize the trajectory of particles in the counter plane with an accuracy of up to $\sim 1\ \text{mm}^2$. (At present, counters of 18×80 mm have been constructed for a gap width of 2 mm. The sparks are photographed through transparent electrodes together with a coordinate-grid background.) Nuclear emulsions 1, 2, and 3 (100 – 200 μ thick) are placed under the spark counters.* An approximate localization of the trajectory of the particle is given by the spark counters. (A more accurate determination of the particle-trajectory coordinates is based on the tracks in the emulsion.)

The method, utilizing the tracks in the emulsion, makes it possible to localize the trajectory with a maximum accuracy, and thus to attain a high accuracy in the determination of the curvature of the trajectory and, consequently, of the particle momentum. It should be noted that, in certain cases where the load is small, Geiger counters of a small diameter, Conversi tubes, etc, can be used instead of spark counters. For a study of nuclear interactions, a multiplate cloud chamber, an emulsion stack, or another arrangement for observing the interactions, can be placed under the magnet. The interesting events can then be selected by a suitable triggering method.

*For greater accuracy in the determination of the trajectory and the coordinates (see below), glass plates coated with emulsion on both sides should be used. The direction of the trajectory is determined from the point of intersection of the tracks in the emulsions with the surfaces of the plate.

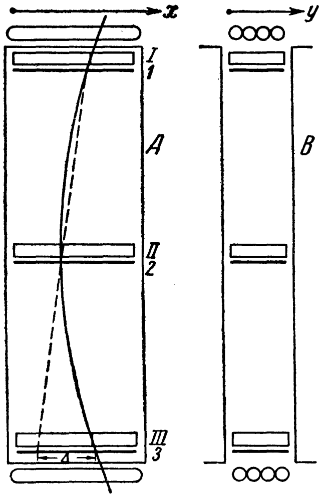

FIG. 1

ACCURACY OF MOMENTUM MEASUREMENTS

Knowing the coordinates of the trajectory x_1, x_2, x_3, we can determine the quantity Δ (Fig. 1), which is related to the particle momentum by the equation

$$p = 300\, HL^2 / 4\Delta,$$

where p is the particle momentum, L the length of the telescope, H the magnetic field intensity, and

$$\Delta = x_3 + \frac{l_2}{l_1} x_1 - \left(1 + \frac{l_2}{l_1}\right) x_2.$$

The error in the momentum is due to the following causes:

1. Errors in the determination of the coordinates x_1, x_2, and x_3, and errors in the determination of l_1 and l_2 (l_1 and l_2 are the distances between the emulsions, and $l_1 + l_2 = L$). We assume that $l_1 = l_2$; we then have $\Delta = x_3 + x_1 - 2x_2$. Let

us denote the standard error in the determination of coordinates x_1, x_2, x_3 by d_0. It can be assumed that $d_0 \approx 2.5 \times 10^{-4}$ cm. The standard error of Δ will then be* $\delta\Delta_{geom} = \sqrt{6}\, d_0 \approx 6 \times 10^{-4}$ cm.

2. The error due to the scattering in the layer of matter inside the gap. The projection of the mean-square scattering angle on the plane A is given by the equation

$$\sqrt{\overline{\theta^2}} = (2 \cdot 10^7 \sqrt{t}) / (\sqrt{2}\, p\beta) = (10^7 \sqrt{2t}) / (p\beta),$$

where t is the thickness of the scatterer in radiation units. For the large energies that we are dealing with, $\beta \approx 1$. The standard error of the deviation due to the scattering is equal to

$$\delta\Delta_{scat} = (L/2)\sqrt{\overline{\theta^2}} = (L/p)\ 10^7 \sqrt{t/2}.$$

The total error of the deviation is equal to

$$\overline{\Delta^2_{sq}} = 6d_0^2 + 10^{14}(L/p)^2 t/2.$$

Let us consider the value $k = \Delta_M / \sqrt{\overline{\Delta^2_{sq}}}$, where Δ_M is the actual deviation of the particle in the magnetic field

$$k = 300\, HL^2/4p \sqrt{6d_0^2 + 10^{14}(L/p)^2 t/2}.$$

This formula makes it possible to determine the accuracy of the method. Let us put $H = 10^4$ gauss and $t = 1/5$ radiation lengths. The values of momentum for which $k = 1$ and $k = 5$, i.e., those determined with errors of 100 and 20% respectively, are given in Table I.

TABLE I

Telescope length, L, cm	p, ev/c	
	k = 1	k = 5
100	$1.25 \cdot 10^{13}$	$2.5 \cdot 10^{12}$ ($\Delta = 30\ \mu$)
50	$3 \cdot 10^{12}$	$5.7 \cdot 10^{11}$ ($\Delta = 30\ \mu$)
25	$7.5 \cdot 10^{11}$	$8.5 \cdot 10^{10}$ ($\Delta = 55\ \mu$)

Noting that spark counters with a working area 250 × 120 mm can at present be produced, we can calculate the counting rate of protons at an altitude of 3000 m for different solid angles of the telescope. The results are given in Table II (where α is the absolute proton intensity).

From Table II it can be seen that, using small magnets, one can measure the momentum spectrum of protons up to the energy of $\sim 10^{12}$ ev during a comparatively short time. The aperture of the instrument is sufficient for the study of interactions in the high energy range, since about 50% of the protons will interact in the layer of matter, with a thickness equal to half of the nuclear mean free path, placed under the magnet.

*We neglect here the errors in l_1 and l_2. Their contribution to the value of Δ depends on the inclination of the trajectory and, for trajectories close to the vertical, is practically negligible.

TABLE II

p, ev/c	α, particles/cm²-sterad-day	Solid angle of the telescope, cm²-sterad		
		25	55	100
	Counting rate, particles/day			
$>10^{10}$	1.3	33	72	132
$>5 \cdot 10^{10}$	0.11	2.7	6.0	10.8
$>10^{11}$	0.04	1	2.2	4
$>5 \cdot 10^{11}$	0.004	0.1	0.22	0.4
$>10^{12}$	0.0013	0.03	0.073	0.13

LOADING OF THE EMULSION BY BACKGROUND PARTICLES

A question arises concerning the identification of a given trajectory among trajectories of particles passing through the given area of emulsion (of ~ 1 mm² area) during the total time of exposure.

The identification of an interesting event can be carried out by various methods, as, for instance, in the following way: From the three sparks in counters I, II, and III, we approximately determine the position of the trajectory and calculate the azimuth and the length of the projection of the track in the emulsion on a horizontal plane. These data serve as the basis for the search and identification of the trajectory. For a small intensity of background particles, e.g., in underground experiments, such a method of identification may be sufficient.

In the following, we consider the background problem arising in the detection of protons at an altitude of 3000 m above sea level in the momentum range $\sim 10^{10} - 10^{12}$ ev/c.

Let the total particle flux recorded by the telescope during the exposure time amount to N particles. [We assume a telescope size of $(25^2 \times 12^2) \times (40^2\ \text{cm}^2\text{-sterad})^{-1}$.] If S is the area of one emulsion plate, then the loading is equal to $n = N/S$. We disregard here the loading due to particles travelling outside the solid angle of the telescope since, owing to the large inclination of their trajectories, they can immediately be excluded from consideration in scanning.

We shall furthermore assume that the error in the determination of the direction of the trajectory in the emulsion in the plane A and in the plane B (Fig. 1) is equal to 0.3° (1/200 rad). (See first footnote.) For a telescope length equal to 40 cm, this gives an inaccuracy of ~ 1 mm in the determination of the corresponding coordinate in the

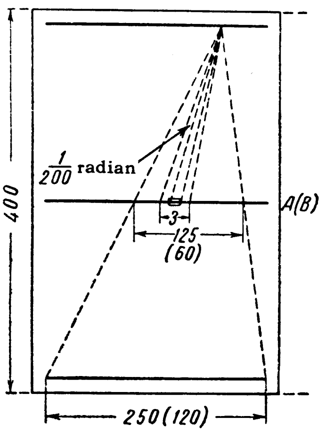

FIG. 2

adjacent row (Fig. 2). If the diameter of the spark is equal to 1 mm, then the probability of a chance coincidence of the direction of the trajectory in the emulsion with a given position of the spark is equal to 3/60 in projection B and to 3/125 in projection A. The total probability of such a chance coincidence is equal to $(3/60) \times (3/125) \approx 1/800$. For three layers of emulsion, this probability is equal to $\sim 3/800$. For a loading n, the probability of a chance coincidence between the direction of the track with the direction given by the sparks is $\sigma = 3n/800$.

Let us evaluate n. From the data on the total intensity of cosmic radiation arriving from the vertical direction and on the intensity of the proton component, we can conclude that $\sim 30{,}000$ particles will traverse the instrument during the time necessary for the detection of one proton with energy $> 10^{11}$ ev. For a proton with energy $> 5 \times 10^{11}$ ev, the number increases to $\sim 300{,}000$ particles. For an emulsion area of 250×150 mm, we obtain, respectively, loadings $n_1 = 1$ particle/mm^2 and $n_2 = 10$ particles/mm^2. It should be noted that the actual loadings will be smaller, since the emulsions are placed in a strong magnetic field.

Thus, for the detection of one proton with energy $> 5 \times 10^{11}$ ev, the loading is $n_2 = 10$ particles/mm^2, which gives $\sigma_2 = (3 \times 10)/800 = 3/80$. This means that, in reducing 80 trajectories recorded by the spark telescope, there will be three cases where, in an emulsion region with an area of 1 mm^2, two trajectories may be observed going in the direction of the sparks. It should be noted, however, that the presence of two unresolved trajectories does not represent a danger from the point of view of an error in the momentum determination, since such cases are simply excluded from consideration (3 trajectories out of 80 in this case).

From Table II, it can be seen that the detection frequency of protons with energy $> 5 \times 10^{11}$ ev is equal to ~ 2 particles in 10 days. It is sufficient to exchange the emulsion every 5 or 10 days to avoid a dangerous loading of the emulsion. For energies $> 10^{12}$ ev, the values obtained should be increased by a factor of three.

The calculation given above is tentative only, since not all the initial data have been accurately determined. For instance, one could expect that the actual localization of the particle trajectories by the spark counter may be a few times better than the value used above (1 mm^2), in which case the loading will play a still smaller role.

Finally, for a large loading, one can use the moving emulsion method for trajectory identification.

On the basis of the above, one can expect that the loading of the emulsion by background particles will not prohibit an application of the proposed method to the study of nuclear interactions of protons.

The authors express their deep gratitude to A. I. Alikhanyan for the interest shown towards our idea and for making its experimental realization possible.

Note received in proof (August 28, 1959): A spark counter telescope in a magnetic field started operation in June 1959.

[1] Gramenitskiĭ, Podgoretskiĭ, and Sharapova, JETP **30**, 277 (1956), Soviet Phys. JETP **3**, 230 (1956).

Translated by H. Kasha
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ERRATA TO VOLUME 9

On page 868, column 1, item (e) should read:

(e). Ferromagnetic weak solid solutions. By way of an example, we consider the system Fe-Me with A2 lattice, where Me = Ti, V, Cr, Mn, Co, and Ni. For these the variation of the moment m with concentration c is

$$dm/dc = (N_d)_{Me} \mp 0.642 \ \{8 \ (2.478 - R_{Me}) + 6\,|2.861 - R_{Me}| \mp [\,8\,(2.478 - R_{\mathrm{Fe}}) + 6\,(2.861 - R_{\mathrm{Fe}})]\},$$

where the signs – and + pertain respectively to ferromagnetic and paramagnetic Me when in front of the curly brackets, and to metals of class 1 and 2 when in front of the square brackets. The first term and the square brackets are considered only for ferromagnetic Me. We then have dm/dc = –3 (–3.3) for Ti, –2.6 (–2.2) for V, –2.2 (–2.2) for Cr, –2 (–2) for Mn, 0.7 (0.6) for Ni, and 1.2 (1.2) for Co; the parentheses contain the experimental values.

ERRATA TO VOLUME 10

Page	Reads	Should Read
224, Ordinate of figure	10^{23}	10^{29}
228, Column 1, line 9 from top	3.6×10^{-2} mm/min	0.36 mm/min
228, Column 1, line 16 from top	0.5 mm/sec	0.05 mm/min
329, Third line of Eq. (23a)	+(1/4 cosh r +...	+1/4(cosh r +...
413, Table II, line 2 from bottom	−0.0924±	−1.0924±
413, Table II, line 3 from bottom	+1.8730±	+0.8370±
479, Fig. 7, right, 1st line	92 hr	9.2 hr
499, Second line of Eq. (1.8)	$+\tilde{k}\sin^2\alpha/\omega_N^2 + \langle c^2\tilde{k}^2 \ldots$	$+\left(\tilde{k}/\omega_H\right)^2 \sin^2\alpha \ \langle c^2\tilde{k}^2 \ldots$
648, Column 1, line 18 from top	18 × 80 mm	180 × 80 mm
804, First line of Eq. (17)	$-{}^1/_3\left(\alpha_x^2\alpha_y^2 + \ldots\right.$	$\ldots - 3\left(\alpha_x\alpha_y^2 + \ldots\right.$
967, Column 1, line 11 from top	$\sigma(N', \pi) \approx 46\,(N', N')$	$\sigma(N', \pi) > \sigma(N', N')$
976, First line of Eq. (10)	$= \frac{e^2}{3r^2c^4}$	$= \frac{e^2}{3\hbar^2c^2}$
978, First line of Eq. (23)	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)\sin^4(\theta/2)}\right.$	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)^2\sin^4(\theta/2)}\right.$