ON THE POLARIZATION OF THE ELECTRONS IN BREMSSTRAHLUNG PROCESSES

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Submitted to JETP editor June 5, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 872-873 (September, 1959)

THE polarization effects in the bremsstrahlung of electrons have been investigated in detail in a number of papers.[1-3] However, the question of the change of polarization of the electron beam due to the bremsstrahlung process has received almost no attention.

To investigate this question, we use the method developed by Lipps and Tolhoek[4] and Rozentsveĭg and the author.[5] We shall describe the state of polarization of the incoming electrons with momentum $\mathbf{p}_1$ by the density matrix

$$\rho(\zeta_1, \mathbf{p}_1) = \eta^{(+)}(\mathbf{p}_1)\tfrac{1}{2}(1 + \zeta\Sigma\gamma_4)\eta^{(+)}(\mathbf{p}_1),$$

where

$$\eta^{(+)}(\mathbf{p}) = (m - i\hat{p})\gamma_4/2\varepsilon, \quad \Sigma = i\gamma\gamma_4\gamma_5.$$

The vector $\boldsymbol{\zeta}$, which characterizes the polarization of the electron in the laboratory system, is connected with the polarization vector $\boldsymbol{\zeta}^0$ in the rest system of the electron by the relation

$$\zeta^0 = \zeta - \mathbf{p}(\zeta\mathbf{p})/\varepsilon(\varepsilon + m),$$

where ϵ is the energy of the electron.

The vector $\boldsymbol{\zeta}_2$, which describes the state of polarization of the electrons emitting the bremsstrahlung quantum, is given by

$$\zeta_2 = \mathrm{Sp}\,[\Sigma\eta^{(+)}(\mathbf{p}_2)S\eta^{(+)}(\mathbf{p}_1)\rho(\zeta_1, \mathbf{p}_1)\eta^{(+)}(\mathbf{p}_1)S^+\eta^{(+)}(\mathbf{p}_2)]/Q,$$
$$Q = \mathrm{Sp}\,[\eta^{(+)}(\mathbf{p}_2)S\eta^{(+)}(\mathbf{p}_1)\rho(\zeta_1, \mathbf{p}_1)\eta^{(+)}(\mathbf{p}_1)S^+],$$

where S is the scattering matrix element for the bremsstrahlung process. Here $\mathbf{k}$ and ω are the wave vector and the energy of the emitted photon; the indices 1 and 2 refer to the initial and final states of the electron, respectively. As a result of the calculations we obtain

$$\begin{aligned}Q\zeta_2 = {} & Q\zeta_1 - (p_2k)^{-2}\{-\varepsilon_1(p_1k)(p_2k)\zeta_1 + m^2\varepsilon_1(\mathbf{k} - \mathbf{p}_1)(\zeta_1, \mathbf{k} + \mathbf{p}_1 + \mathbf{p}_2) + \varepsilon_1[(p_2k) - (\overset{*}{p}_1p_2)]\,\mathbf{p}_2(\zeta_1\mathbf{k})\\
& + [m^2\varepsilon_2 - \omega(p_1p_2) + \omega(p_2k) + \varepsilon_2(p_1k)](\zeta_1\mathbf{p}_1)\mathbf{p}_2 + \varepsilon_1[m^2 + (\overset{*}{p}_1k)](\zeta_1\mathbf{p}_2)\mathbf{p}_2\} - (p_1k)^{-2}\{-\varepsilon_1(p_1k)(p_2k)\zeta_1\\
& + m^2\varepsilon_1(\zeta_1, \mathbf{k} + \mathbf{p}_2)(\mathbf{k} - \mathbf{p}_1) + m^2(\varepsilon_2 - \omega)(\zeta_1\mathbf{p}_1)(\mathbf{k} - \mathbf{p}_1) + \varepsilon_1[(\overset{*}{p}_2k) - (\overset{*}{p}_1p_2)](\zeta_1\mathbf{k})\mathbf{p}_2\\
& + [m^2\varepsilon_2 + \varepsilon_2(p_1k) + \omega(\overset{*}{p}_1p_2) - \omega(\overset{*}{p}_2k)](\zeta_1\mathbf{p}_1)\mathbf{p}_2 + \varepsilon_1[m^2 + (p_1k)](\zeta_1\mathbf{p}_2)\mathbf{p}_2\} - [2/(p_1k)(p_2k)]\\
& \times\{\varepsilon_1[(p_1k)(p_2k) - \omega^2(p_1p_2) - \omega^2m^2]\zeta_1 + \varepsilon_1(p_1p_2)(\zeta_1\mathbf{k})\mathbf{k} - [m^2\varepsilon_1 + \omega(p_1p_2) - \varepsilon_2(p_1k)](\zeta_1\mathbf{p}_1)\mathbf{k} - \varepsilon_1[m^2 + (p_1k)](\zeta_1\mathbf{p}_2)\mathbf{k}\\
& + \varepsilon_1[m^2 - p_2k](\zeta_1\mathbf{k})\mathbf{p}_1 + [\varepsilon_1(p_2k) - \varepsilon_2(p_1k) - \varepsilon_2(p_1p_2)](\zeta_1\mathbf{p}_1)\mathbf{p}_1 + \varepsilon_1[(p_2k) - (p_1k) - (p_1p_2)]\,\mathbf{p}_1(\zeta_1\mathbf{p}_2)\\
& + \varepsilon_1[(\overset{*}{p}_1p_2) - \varepsilon_2\omega](\zeta_1\mathbf{k})\mathbf{p}_2 + [\varepsilon_2(p_1p_2) - \varepsilon_1(p_2k) - \varepsilon_1\varepsilon_2\omega](\zeta_1\mathbf{p}_1)\mathbf{p}_2 + \varepsilon_1[(p_1p_2) - (p_2k) - \varepsilon_2\omega](\zeta_1\mathbf{p}_2)\mathbf{p}_2\},\end{aligned} \tag{1}$$

$$\begin{aligned}Q = {} & \frac{\varepsilon_1}{(p_2k)^2}[-m^4 - m^2(p_1^*p_2) + m^2(p_2k) - m^2(p_1^*k) - (p_1k)(p_2k)] + \frac{\varepsilon_1}{(p_1k)^2}[-m^4 - m^2(p_1^*p_2) + m^2(p_2^*k) - m^2(p_1k) - (p_1k)(p_2^*k)]\\
& + \frac{2\varepsilon_1}{(p_1k)(p_2k)}[-m^2\omega^2 - m^2(p_1p_2) - (p_1p_2)(\overset{*}{p}_1p_2) + (p_2k)(\overset{*}{p}_1p_2) - (p_1k)(\overset{*}{p}_1p_2) - \omega^2(p_1p_2) + \varepsilon_1\omega(p_2k) + \varepsilon_2\omega(p_1k)],\end{aligned} \tag{2}$$

where

$$(p_1k) = \mathbf{p}_1\mathbf{k} - \varepsilon_1\omega, \quad (\overset{*}{p}_1k) = \mathbf{p}_1\mathbf{k} + \varepsilon_1\omega \quad \text{etc.}$$

The expression for Q coincides, up to a factor, with the bremsstrahlung cross section computed from the Bethe-Heitler formula.

In some special cases the expression (1) can be considerably simplified.

In the nonrelativistic limit ($\epsilon_1, \epsilon_2 \sim m$) we obtain $\boldsymbol{\zeta}_2^0 = \boldsymbol{\zeta}_1^0$, i.e., the polarization of the beam does not change.

In the limiting case of extremely soft bremsstrahlung quanta ($\omega \to 0$, i.e., $\epsilon_2 \approx \epsilon_1 = \epsilon$) we have

$$\zeta_2 = \zeta_1 + \frac{(\zeta_1, \mathbf{p}_1 + \mathbf{p}_2)}{m^2 + \varepsilon^2 + \mathbf{p}_1\mathbf{p}_2}(\mathbf{p}_2 - \mathbf{p}_1). \tag{3}$$

Going over to the vector $\boldsymbol{\zeta}_2^0$, we see that this vector, while preserving its absolute magnitude, is rotated around the normal to the $\mathbf{p}_1$, $\mathbf{p}_2$ plane through the angle φ, given by

$$\tan\varphi = \frac{(\gamma - 1)[(\gamma + 1) + (\gamma - 1)\cos\vartheta]}{[2\gamma + (\gamma^2 - 1)\cos\vartheta + (\gamma - 1)^2\cos^2\vartheta]}\sin\vartheta, \tag{4}$$

where $\gamma = \epsilon/m$. For $\gamma \to 1$ we have $\varphi \to 0$, and for $\gamma \to \infty$: $\varphi \to \vartheta$.

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Translated by R. Lipperheide
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THE $\mu \rightarrow e + \gamma$ *AND* $\mu \rightarrow e + \nu + \bar{\nu} + \gamma$ *DECAYS*

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Submitted to JETP editor June 9, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 873-875 (September, 1959)

THE main characteristics of universal A-V interaction can be explained by the hypothesis of the existence of an intermediate boson of large mass.[1] One of the consequences of this hypothesis is the possibility of μ-meson decay according to the scheme $\mu \rightarrow e + \gamma$. For a point A-V interaction such a decay is forbidden. Feinberg[2] calculated the probability of the decay $\mu \rightarrow e + \gamma$ and showed that the ratio $\rho_1 = R(\mu \rightarrow e + \gamma)/R(\mu \rightarrow e + \nu + \bar{\nu})$ depends on the cutoff parameter Λ. If $\Lambda = M$, where M is the boson mass, then $\rho_1 \cong 10^{-4}$. For $\Lambda < M$, ρ_1 can be arbitrarily small.

In the present work, a search for the decay $\mu \rightarrow e + \gamma$ was made, using a 17-liter freon bubble chamber, sensitive to γ rays.[3] The chamber was placed in the external π^+ beam of the synchrocyclotron of the Joint Institute for Nuclear Research. π^+ mesons of energy approximately 200 Mev were slowed by a graphite filter, and stopped inside the chamber. About 20,000 stereophotographs were obtained and scanned twice.

In the scanning, a search was made for cases in which the π-μ-e decay was accompanied by an electron-positron pair directed along and following the end of a π- or μ-meson track. The direction of the pair corresponded to the direction of flight of the γ ray to within an accuracy of $\sim 5-7°$, and the length of the track of the μ meson from the π-meson decay was 0.16 cm. Therefore, the case in which the π-μ-e decay was accompanied by a directed pair could relate to one of three processes:

$$\mu \rightarrow e + \nu + \bar{\nu} + \gamma, \quad (1)$$

$$\pi \rightarrow \mu + \nu + \gamma, \quad (2)$$

$$\mu \rightarrow e + \gamma. \quad (3)$$

In the decay (3) the angle between electron and γ ray is 180°. In 91,000 π-μ-e decays, not a single such case was found. From this, knowing the efficiency of the chamber for detecting γ rays, it is possible to determine an upper limit for the quantity ρ_1. The efficiency was calculated on an electronic computer for various γ-ray energies by the Monte-Carlo method, taking into account the distribution of the number of π mesons stopping inside the chamber. The efficiency turned out to be equal to 0.253 for the decay (3). From this, $\rho_1 \leq 4.3 \times 10^{-5}$. This quantity is in agreement with results of experiments in which the decay (3) was studied using counter methods.[4-6]

In scanning the photographs, cases of directed pairs with various angles $(e, \gamma) < 180°$ were found. One of these cases is shown in Fig. 1. We interpret

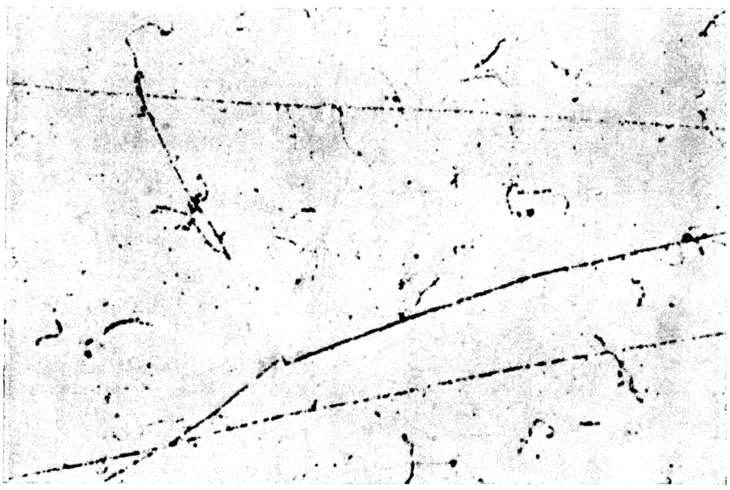

FIG. 1. Radiative decay of a μ meson by scheme (1).