For a large class of nuclei (in the mass number interval $150 < A < 190$ and $A > 222$) the lowest excited states of even-even nuclei are uniquely interpreted to be rotational states. Nuclei lying outside this mass number range are treated on the uniform model as spherically symmetric and their excitation attributed to quadrupole vibration. The lack of a quantitative theory of such vibrations makes a direct test of this assumption difficult. An alternative point of view was proposed in the paper of Davydov and Filippov,[6] where it was shown that the lowest excited states of even-even nuclei can be interpreted to be rotational states even if they are outside the mass number range $150 < A < 190$, $A > 222$, if one assumes that the nucleus is not axially symmetric. Then when we go from highly deformed nuclei to nuclei which are close to a filled shell the nature of the excitation does not change; only the parameters describing the deformation and the axial asymmetry change, resulting in a change in the energy of the levels and the separation of levels with the same spin. Possibly the regularities noted here in the dependence of B(E2) on energy could be explained on the basis of such a model.

[1] F. K. McGowan, Comptes Rendus du Congres International de physique nucleaire, Paris, 1959, p. 225.

[2] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

[3] Strominger, Hollander, and Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[4] P. H. Stelson and F. K. McGowan, Phys. Rev. **110**, 489 (1958).

[5] F. K. McGowan and P. H. Stelson, Phys. Rev. **109**, 901 (1958).

[6] A. S. Davydov and G. F. Filippov, JETP **35**, 440 (1958), Soviet Phys. JETP 8, 303 (1959).

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EXCITATION OF ROTATIONAL STATES OF NONAXIAL NUCLEI IN ALPHA-PARTICLE SCATTERING

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In the present paper an estimate is made of the probability of excitation of the second 2^+ level in even-even nonaxial nuclei resulting from scattering of α particles with energy $E \gtrsim E_B$ (where E_B is the height of the Coulomb barrier), for the purpose of determining the role of the competing mechanisms of excitation — direct nuclear interaction and Coulomb excitation.

Since the quasi-classical approximation is valid for the scattering of α particles with $E \gtrsim E_B$ by heavy nuclei ($kR \gg 1$), in solving our problem we can use a method which was developed in the classical theory of Coulomb excitation. In this treatment the excitation of the nucleus occurs as the result of time-dependent nuclear and electric interactions.

It is not hard to show (cf. reference 1) that the conditions under which one can treat the potential energy of interaction as a perturbation are expressible in the following form: $(kR_0)^2 M(I_i \rightarrow I_f) \ll 1$ (for the nuclear interaction) and $\eta^2 M(I_i \rightarrow I_f) \ll 1$ (for the Coulomb interaction). Here

$$M(I_i \rightarrow I_f) = \frac{1}{2I_i + 1} \sum_{m_i m_f} |\langle I_i m_i | a_\mu | I_f m_f \rangle|^2$$

is the matrix element for transition of the nucleus from the ground state to the excited state, a_μ are coordinates characterizing the deformation of the nuclear surface in a coordinate system fixed in the nucleus, and $\eta = Z_1 Z_2 e^2/\hbar v$. In the case of excitation of the second 2^+ level in a nonaxial nucleus (denoted in the sequel by 2^{+}) we have, according to Davydov and Filippov:[2]

$$M(0 \rightarrow 2^+) = \frac{\beta^2}{10}\left[1 - \frac{3 - 2\sin^2 3\gamma}{\sqrt{9 - 8\sin^2 3\gamma}}\right], \qquad \beta^2 = \sum_\mu |a_\mu|^2,$$

where γ is the parameter describing the deviation of the nucleus from axial symmetry. The quantity $M(0 \rightarrow 2^+)$ goes to zero for $\gamma \rightarrow 0$ and $\gamma \rightarrow 30°$, while it attains its maximum value $\sim 7 \times 10^{-3} \beta^2$ for $\gamma \approx 20°$. From these estimates it follows that perturbation theory is applicable to the excitation of the 2^+ level.

If we take account of the change of the orbit of the bombarding particles and the change of the electric multipole fields when the particles enter the nucleus by using the method proposed by the author,[3] then in calculating the probability of excitation of the 2^+ level we can use the perturbation formula:

$$P = \sum_{m_f} \left| \frac{1}{i\hbar} \int_{-\infty}^{\infty} \langle \psi_f \,|\, H(t) \,|\, \psi_i \rangle e^{i\omega t} dt \right|^2,$$

where $\omega = (E_f - E_i)/\hbar$, and $H(t)$ is the interaction energy. The quadrupole component of the interaction energy, which causes the transition, has the form

$$H(t) = -r \frac{\partial V(r)}{\partial r} \sum_{\mu\nu} D^{(2)}_{\mu\nu}(\Theta_i)\, a_\mu Y_{2\mu}(\Theta, \Phi) + \frac{4\pi}{5} Z_1 e \sum_{\mu} \hat{Q}_{2\mu} Y_{2\mu}(\Theta, \Phi)\, r^{-3}.$$

Here

$$V(r) = -V_0\left[1 + \exp\frac{r - R_0}{d}\right]^{-1}$$

is the potential of the interaction of the α particle with the nucleus, $d \sim 0.5 \times 10^{-13}$ cm;[4] $D^{(2)}_{\mu\nu}(\Theta_i)$ are generalized spherical functions, depending on the Euler angles, which determine the transformation from a coordinate system fixed in the nucleus to a space-fixed system; r, Θ, and Φ are the polar coordinates of the incident particle; $\hat{Q}_{2\mu}$ is the nuclear quadrupole moment operator. Choosing for the wave functions ψ_i and ψ_f the wave functions of rotational states of a nonaxial nucleus,[2] and carrying out calculations analogous to those of the classical theory of Coulomb excitation, we find

$$P = \frac{16\pi^2\eta^2 B(E2;\, 0 \to 2^+)}{125 a^4 Z_2^2 e^2} \times \sum_{\mu} \left| I_{2\mu}(\vartheta, \xi) - \frac{5kR_0V_0a^4}{6\eta E R^2 R_0 d} I_{2\mu}(\vartheta, \xi, a, R_0, d) \right|^2 \left| Y_{2\mu}\left(\frac{\pi}{2}, 0\right) \right|^2,$$

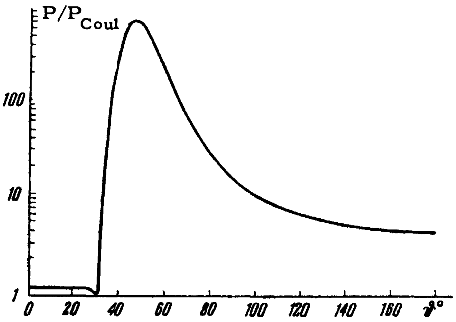
$$I_{2\mu}(\vartheta, \xi, a, R_0, d) = \int_{-\infty}^{\infty} e^{i\xi(\varepsilon \sinh w + w)} \frac{(\cosh w + \varepsilon + i\sqrt{\varepsilon^2 - 1}\, \sinh w)^{\mu}}{(\varepsilon \cosh w + 1)^{\mu - 2}} \times g(\vartheta, a, R_0, d, w)\, dw,$$

$$g(\vartheta, a, R_0, d, w) = \frac{\exp\left[\dfrac{a(\varepsilon \cosh w + 1) - R_0}{d}\right]}{\left[1 + \exp\left(\dfrac{a(\varepsilon \cosh w + 1) - R_0}{d}\right)\right]^2};$$

$$a = \frac{Z_1 Z_2 e^2}{\mu v^2}, \quad \xi = \frac{\Delta E}{2E}\eta, \quad \varepsilon = 1/\sin(\vartheta/2),$$

where ϑ is the scattering angle, μ is the reduced mass, R_0 is the radius of interaction of the α particle and the nucleus, R is the nuclear radius, $I_{2\mu}(\vartheta, \xi)$ are tabulated functions of the classical theory of Coulomb excitation,[5] and $B(E2;\, 0 \to 2^+)$ is the reduced probability for excitation of the 2^+ level.

As an example we show in the figure the angular



dependence of the ratio of the probability of excitation of the 2^+ level in Cd^{114}_{48} [$\Delta E = 1.2$ Mev, $B(E2;\, 0 \to 2^+) = 1.25 \times 10^{-50}\,\text{cm}^4$][6] to the probability of excitation by the Coulomb field of incident α particles with energy $E = 30$ Mev. As we see from the figure, in the region of angles ϑ corresponding to impact parameters $\sim R_0$, the probability of excitation of the nucleus is approximately two orders of magnitude greater than the probability of Coulomb excitation. The rapid increase in the ratio P/P_{Coul} with increasing scattering angle is associated with the fact that the maximum value of P occurs at small scattering angles ϑ. In this angle region, the strong dependence of the impact parameter on angle results in an even stronger dependence of the function $g(\vartheta, a, R_0, d, w)$ on ϑ, and this function essentially determines the change in the probability of excitation as a function of angle of scattering.

The maximum cross section for inelastic scattering is approximately an order of magnitude less than the cross section for elastic scattering of α particles by this nucleus. The large value of the cross section for excitation of the 2^+ level, which is caused by the interaction of the α particles with the nuclear surface, makes it possible to study the properties of second excited 2^+ states in even-even non-axial nuclei by recording inelastically scattered α particles.

In conclusion, I express my thanks to V. G. Neudachin for discussion of this work.

[1] L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Pergamon Press, 1958, Section 45.

[2] A. S. Davydov and G. F. Filippov, JETP **35**, 440, 703 (1958), Soviet Phys. JETP **8**, 303, 488 (1959).

[3] E. A. Romanovskiĭ, JETP **37**, 83 (1959), Soviet Phys. JETP **10**, 59 (1960).

[4] R. Woods and D. Saxon, Phys. Rev. **95**, 577 (1954).

[5] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. **28**, 432 (1956).

[6] H. Motz, Phys. Rev. **104**, 1353 (1956).

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