# *STABILITY OF A HOLLOW GAS CONDUCTOR IN A MAGNETIC FIELD*

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Stability conditions are obtained for a hollow gaseous conductor located in a longitudinal magnetic field.

THE author has shown earlier[1] that if a conductor carrying a return current is placed on the axis of a discharge chamber then, under certain conditions, the discharge will be localized within a narrow region inside the chamber and will form a hollow thin walled cylinder separated from the walls. It seems that in this case the conditions required to stabilize the discharge ought to be less rigid. We therefore investigate in the present paper the question of the stability of such a plasma layer with respect to small oscillations.

We consider a hollow plasma cylinder of internal radius $r_2$ and external radius $r_3$ situated in a region bounded by two coaxial perfectly conducting cylinders of radii $r_1$ $(r_1 < r_2)$ and $r_4$ $(r_4 > r_3)$. The initial system of equations consists of the magnetohydrodynamic equations for a perfectly conducting fluid, Maxwell's equations for the field, and boundary conditions on the surfaces of discontinuity (see, for example, reference 2).

We assume that in equilibrium the plasma pressure $p_0$ is balanced by magnetic forces set up by currents flowing in the surface of the plasma, that the velocity is equal to zero, that the plasma is uniform with respect to $\varphi$ and z, and that the field components $H_\varphi^0$ and $H_z^0$ differ from zero. We shall assume that within the plasma $H_\varphi^0 = 0$, while $H_z^0$ is everywhere uniform, i.e.,

$$H_z^0 = \begin{cases} H_{z1}^0 \\ H_{z2}^0, \\ H_{z3}^0 \end{cases} \qquad H_\varphi^0 = \begin{cases} 2I_1/cr & \text{for } r_1 < r < r_2 \\ 0 & \text{for } r_2 < r < r_3 \\ 2I_3/cr & \text{for } r_3 < r < r_4 \end{cases} \tag{1}$$

with

$$H_{z1}^{02} + H_{\varphi1}^{02} = H_{z3}^{02} + H_{\varphi3}^{02} = H_{z2}^{02} + 8\pi p_0, \tag{2}$$

where

$$H_{\varphi1}^0 = 2I_1/cr_2, \qquad H_{\varphi3}^0 = 2I_3/cr_3.$$

By assuming that the deviations from the equilibrium state are small and by solving the equations of motion by the standard method (see, for example, reference 2), we obtain a dispersion relation for the frequency spectrum of the system:*

$$\begin{aligned}\Delta(\Omega^2, k) = &-\alpha_{11}\{\alpha_{11}[I_m(\zeta r_3)K_m(\zeta r_2) - I_m(\zeta r_2)K_m(\zeta r_3)] \\ &+ (\alpha_{12}/\zeta r_2)[I_m'(\zeta r_2)K_m(\zeta r_3) - I_m(\zeta r_3)K_m(\zeta r_2)]\} \\ &+ (\alpha_{22}/\zeta r_3)\{\alpha_{11}[I_m'(\zeta r_3)K_m(\zeta r_2) - I_m(\zeta r_2)K_m'(\zeta r_3)] \\ &+ (\alpha_{12}/\zeta r_2)[I_m'(\zeta r_2)K_m'(\zeta r_3) \\ &- I_m'(\zeta r_3)K_m'(\zeta r_2)]\} = 0,\end{aligned} \tag{3}$$

where

$$\alpha_{12} = h_{\varphi1}^2 + (\varepsilon_{1m}/kr_2)[mh_{\varphi1} + kr_2h_{z1}]^2,$$

$$\alpha_{22} = h_{\varphi3}^2 - (\varepsilon_{2m}/kr_3)[mh_{\varphi3} + kr_3h_{z3}]^2,$$

$$\varepsilon_{1m} = \frac{I_m(kr_2)K_m'(kr_1) - I_m'(kr_1)K_m(kr_2)}{I_m'(kr_2)K_m'(kr_1) - I_m'(kr_1)K_m'(kr_2)},$$

$$\varepsilon_{2m} = \frac{I_m(kr_3)K_m'(kr_4) - I_m'(kr_4)K_m(kr_3)}{I_m'(kr_4)K_m'(kr_3) - I_m'(kr_3)K_m'(kr_4)},$$

$$\alpha_{11} = 1 - \frac{4\pi\gamma p_0}{H_{z2}^{02}}\frac{\Omega^2}{k^2 - \Omega^2}, \qquad \zeta^2 = \frac{(\Omega^2 - k^2)(\Omega^2 - q^2k^2)}{q^2k^2 - (1+q^2)\Omega^2},$$

$$q^2 = H_{z2}^{02}/4\pi\gamma p_0, \qquad h_{\varphi,z} = H_{\varphi,z}^0/H_{z2}^0,$$

$$\Omega^2 = \rho_0\omega^2/\gamma p_0. \tag{3'}$$

Primes indicate differentiation with respect to the argument; $I_m$ and $K_m$ the modified Bessel functions of order m.

We obtain the stability criterion from the con dition that the dispersion relation have no solutions, $\Omega^2 < 0$. It has the form

$$\begin{aligned}\alpha_{22}(k) < kr_3\{&[I_m(kr_3)K_m(kr_2) - I_m(kr_2)K_m(kr_3)] \\ &+ (\alpha_{12}/kr_2)[I_m'(kr_2)K_m(kr_3) \\ &- I_m(kr_3)K_m'(kr_2)]\}\{[I_m'(kr_3)K_m(kr_2) \\ &- I_m(kr_2)K_m'(kr_3)] + (\alpha_{12}/kr_2)[I_m(kr_2)K_m'(kr_3) \\ &- I_m'(kr_3)K_m'(kr_2)]\}^{-1}.\end{aligned} \tag{4}$$

Indeed, in this case, as $\Omega^2$ varies from 0 to

*We assume, as usual, that all the quantities are proportional to exp (iωt + ikz + imφ).

$-\infty$, $\Delta(\Omega^2, k)$ is always less than zero, and, consequently, Eq. (3) does not have any roots $\Omega^2 < 0$.

It is easily seen that when $H^0_{Z2} = 0$ the inequality (4) agrees exactly with the stability criterion obtained for a solid cylindrical conductor,[3,4] i.e., the stability criteria are not relaxed without a "frozen-in" longitudinal field. We also note that when $h^2_{\varphi 3} = 0$ the condition (4) is fulfilled for all m and k, i.e., absolute stability exists.

Without going into a detailed investigation of the stability criterion obtained above, we consider only the stability conditions for the two most dangerous forms of perturbation, with m = 0 and m = 1. In carrying this out we assume that the outer metal wall is absent; i.e., we shall set $r_4 = \infty$ in the expression for $\epsilon_{2m}$.

1) m = 0. As k varies from 0 to $\infty$ the right-hand side of the inequality (4) increases, whereas the left-hand side decreases. Consequently equilibrium will be stable when

$$H^{02}_{z2} > \frac{1}{2} H^{02}_{\varphi 3} (\delta_2^2 - 1) \frac{H^{02}_{\varphi 1} + 2\delta_1^2 H^{02}_{z1} / (\delta_1^2 - 1)}{\delta_2^2 [H^{02}_{\varphi 1} + 2\delta_1^2 H^{02}_{z1} / (\delta_1^2 - 1)] - H^{02}_{\varphi 3}}, \quad (5)$$

where $\delta_1 = r_2/r_1$; $\delta_2 = r_3/r_2$. When $H^0_{Z1} = H^0_{Z3} = 0$ relation (5) coincides with the stability criterion obtained for a solid cylinder.

2) m = 1. In this case the stability condition depends on the magnitude of $h_{Z3}$. In the case $kr_3 \ll 1$ it has the form

$$H^{02}_{z2} > \frac{\delta_2^2 - 1}{\delta_2^2 + 1} H^{02}_{\varphi 3} \ln \frac{1}{kr_3}, \quad (6)$$

if $h_{Z3} = 0$, and

$$|H^0_{z2}| > \frac{|H^0_{\varphi 3}|}{kr_3} \frac{2|h_{z3}|(\delta_2^2 - 1)}{h^2_{z3}(\delta_2^2 - 1) + (\delta_2^2 + 1)}, \quad (6')$$

if $h_{Z3} \gg kr_3$.

It follows from the foregoing that when $\delta_2 \to 1$ the stabilizing field in the case of a hollow cylindrical conductor is less than the field required for the stabilization of a solid conductor by a factor $\sim (\delta_2^2 - 1)^{-1/2}$ (when $h_{Z3} = 0$) and by a factor $\sim (\delta_2^2 - 1)^{-1}$ (when $h_{Z3} \neq 0$).

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