# EXCITATION OF NUCLEAR VIBRATIONAL AND ROTATIONAL STATES IN THE SCATTERING OF CHARGED PARTICLES

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A semiclassical method is presented for calculating the probability of exciting nuclear vibrational and rotational states through the interaction of a scattered charged particle with the surface of the target nucleus.

THE scattering of fast protons and neutrons by nuclei having excited rotational or vibrational states has been considered in a number of papers. Drozdov[1,2] and Inopin[3] have calculated the inelastic scattering cross sections subject to the following assumptions: 1) The wavelength $1/k$ of the incident particle is much smaller than the nuclear radius R ($kR \gg 1$); 2) the nucleus absorbs all particles impinging on it (the black-nucleus model); 3) the nucleus is assumed to be at rest during the collision (the "adiabatic" approximation); 4) the energy of an incident proton is considerably higher than the Coulomb barrier of the nucleus. It is shown in those papers that for diffraction scattering the angular distribution of nucleons possessing a given energy and scattered with the excitation of a given collective level in an even-even nucleus is identical for vibrational and rotational levels. The validity of the resulting formulas is subject to the following inequalities: $\vartheta < 1$ (diffraction hypothesis) and $\vartheta kR\beta \ll 1$ or $\vartheta kR\sqrt{\Delta E/2C_2} \ll 1$, where $\vartheta$ is the scattering angle, $\beta$ is the nuclear deformation parameter, $C_2$ is the nuclear surface-tension parameter[4] and $\Delta E$ is the nuclear excitation energy.

In the present paper inelastic scattering cross sections are calculated for heavy charged particles interacting with a semitransparent nonspherical even-even nucleus (with zero nuclear spin) in the shape of an ellipsoid of revolution with small eccentricity, and with an even-even nucleus possessing vibrational states. The adiabatic approximation is used, as in the papers of Drozdov and Inopin. The inelastic scattering cross section is calculated by means of the semiclassical method which was developed by Ter-Martirosyan in a paper on Coulomb excitation of nuclei.[5] The results are valid for all scattering angles.

## 1. STATEMENT OF PROBLEM AND METHOD OF CALCULATING EXCITATION PROBABILITY

In the collision of a heavy charged particle with a nucleus the inelastic scattering that is accompanied by the excitation of low-lying levels of the target-nucleus evidently results from a direct interaction, since the observed yields considerably exceed those that would be expected if a compound nucleus were formed.[6] We shall therefore not consider the inelastic scattering of charged particles proceeding through compound-nucleus formation, and shall investigate inelastic scattering resulting from Coulomb excitation and the excitation of surface vibrations in the target-nucleus.

We introduce the coordinate system K with its Z axis perpendicular to the plane of motion of the charged particle and its X axis representing the axis of symmetry of the trajectory, and the system K′ with its Z′ axis representing the axis of symmetry of the nucleus. Nuclear orientation in the K system will be given by the Euler angles $\theta_i$.

The particle-nucleus interaction energy which results in the excitation of surface vibrations (for small deformations of the nuclear surface) may be represented by

$$V(\mathbf{r}, \theta_i) = V_0(\mathbf{r}) + V_1(\mathbf{r}, \theta_i). \qquad (1.1)$$

Here $V_0(\mathbf{r})$ is the central part of the interaction potential; $V_0(r) = -V_0 f(r)$, where $f(r)$ is the form factor of the potential. For a nucleus with a sharply defined boundary $f(r) = 1$ when $r \le R_0$ and $f(r) = 0$ when $r > R_0$ ($R_0$ is the effective radius of interaction between a nucleus and a punctiform charged particle). For a nucleus in which the thickness of the diffuse layer is $\Delta R$ we have

$$f(r)=\left[1+\exp\frac{r-R_0}{d}\right]^{-1},\qquad d=\frac{\Delta R}{4.4}.$$

The particle-nucleus interaction energy is first represented in the K′ coordinate system and then transformed to the K system. Limiting ourselves to inelastic scattering with the nucleus making a transition from its ground state to an excited state with spin $2^+$ and assuming that only $\alpha_{\lambda\mu}$ with $\lambda = 2$ differs from zero, we have

$$V_1=-V_0 r\frac{\partial f(r)}{\partial r}\sum_\mu \alpha_\mu Y_{2\mu}(\Theta,\Phi),\qquad(1.2)$$

where

$$\alpha_\mu=\sum_\nu a_\nu D^{(2)}_{\mu\nu}(\theta_i).\qquad(1.3)$$

In (1.2) and (1.3) $a_\nu$ are the coordinates of a nuclear deformation in the K′ system [$a_0 = \beta\cos\gamma$, $a_2 = a_{-2} = (\beta/\sqrt{2})\sin\gamma$, $a_1 = a_{-1} = 0$], $\alpha_\mu$ are nuclear deformation coordinates in the K system and $D^{(2)}_{\mu\nu}$ is the transformation matrix of second order spherical harmonics. Since $a_2 = a_{-2} = 0$ for an axially symmetrical nucleus, (1.2) can be rewritten as follows:

$$V_1=-V_0 r\frac{\partial f(r)}{\partial r}\beta\cos\gamma\sum_\mu\sqrt{\frac{4\pi}{5}}Y_{2\mu}(\theta,\varphi)Y_{2\mu}(\Theta,\Phi).\qquad(1.4)$$

If the quasi-classical condition $kR \gg 1$ is satisfied, particle motion in the nuclear field $Z_1Z_2e^2/r + V_0(r)$ can be described by means of classical mechanics, assuming $\mathbf{r} = \mathbf{r}(t)$. Assuming, furthermore, that the energy $\Delta E = E_f - E_i$ transferred to the nucleus in a collision is small compared with the collision energy E ($\Delta E < E$), we may neglect the change in the character of particle motion as energy is imparted to the nucleus. In other words, we shall assume that in inelastic scattering the trajectory of a charged particle is indistinguishable from that of an elastically scattered particle.

We know[7] that when $kR_0 \gg 1$ and $\eta = Z_1Z_2e^2/\hbar v \gg 1$, Coulomb scattering plays the principal role for scattering angles $\vartheta \le 2\eta/kR_0$, while diffraction scattering occurs for $\vartheta > 2\eta/kR_0$. In the scattering of $\alpha$ particles (as well as of $C^{12}$, $N^{14}$ and other ions) by heavy nuclei at angles $\vartheta > 2\eta/kR_0$ the angular distribution of elastically scattered particles does not possess the diffraction maxima and minima which appear, for example, in the scattering of $\alpha$ particles by light nuclei, but is characterized by an exponential reduction of the cross section as the scattering angle increases. This property of the angular distribution has been explained by the optical-model theory of elastic $\alpha$-particle scattering[8] as well as by a number of quasi-classical theories.[9,10] Porter[10] accounts for the non-Rutherfordian differential cross section for elastic scattering by assuming that the decreased absolute value of the elastic scattering cross section is due to the fact that $\alpha$ particles are knocked out of the beam upon traversing a nucleus. It is also assumed that the Coulomb trajectory of the particles is not changed because of $V_0(r)$. The differential cross section for elastic scattering is then given by

$$\sigma(\vartheta)_{\text{elast}}=\sigma(\vartheta)_{\text{Ruth}}\exp\left\{-2R_0\Pi_0 h\left(\frac{D}{R_0},\frac{\Delta R}{R_0}\right)\right\},$$

$$D(\vartheta)=\frac{Z_1Z_2e^2}{2E}\left(1+\frac{1}{\sin(\vartheta/2)}\right),\qquad(1.5)$$

where $\Pi_0$ is the absorption coefficient for particles passing through the center of the nucleus and $h(D/R_0, \Delta R/R_0)$ is a tabulated function.[10] If the nucleus has a sharply defined boundary we have

$$h(D/R_0,\ \Delta R/R_0)_{\Delta R=0}=\sqrt{1-(D/R_0)^2}.\qquad(1.6)$$

We shall use the collision-parameter method to determine inelastic scattering cross sections. When all conditions mentioned above are satisfied the differential cross section for inelastic scattering is

$$\sigma(\vartheta)_{\text{inelast}}=\sigma(\vartheta)_{\text{elast}}P(\vartheta),\qquad(1.7)$$

where P is the probability of nuclear transition from the ground state to the final state. Nuclear excitation results from the time-dependent action of the incident particle's electromagnetic field and from the time-dependent interaction of the particle with the noncentral part of the potential (1.1). Assuming, furthermore, that the combined effect of the fields is small and can be treated as a first approximation, we have

$$P=\sum_{M_f}|b_{if}+b^c_{if}|^2,\qquad(1.8)$$

where

$$b_{if}=\frac{1}{i\hbar}\int_{-\infty}^{\infty}e^{i\omega t}\langle f|V_1(\mathbf{r}(t),\theta_i)|i\rangle\,dt,\qquad(1.9)$$

$\omega = \Delta E/\hbar$, $M_f$ is the projection of the nuclear spin in the final state on the Z axis and $b^c_{if}$ is a transition amplitude for Coulomb excitation.[6]

In calculating nuclear transition amplitudes $b_{if}$ resulting from nuclear forces we first consider the excitation of rotational states in even-even nuclei. This becomes the problem of scattering on a nucleus with fixed orientation, that is, we determine the transition amplitude $b_{if}$ which is dependent on the orientation and then average this amplitude over all orientations.

The wave functions of the deformed nucleus in the ground and excited states are[4]

$$\Psi_i \approx Y_{00}(\theta, \varphi)\varphi_i(\beta, \gamma), \quad \Psi_f \approx Y_{IM}(\theta, \varphi)\varphi_f(\beta, \gamma), \tag{1.10}$$

where $\varphi_i(\beta, \gamma)$ and $\varphi_f(\beta, \gamma)$ represent the surface vibrations of a nucleus with axial symmetry.

Substituting (1.4) and (1.10) into (1.9), we obtain the following expression for the amplitude of a nuclear transition from state i to state f:

$$b_{if} = -(V_0\beta / i\hbar\sqrt{5})\sum_\mu S_{2\mu}, \tag{1.11}$$

where

$$S_{2\mu} = \int_{-\infty}^{\infty} e^{i\omega t} Y_{2\mu}(\Theta, \Phi)\frac{\partial f(r)}{\partial r}dt, \tag{1.12}$$

$$\bar\beta = \langle\varphi_f(\beta, \gamma)\,|\,\beta\cos\gamma\,|\,\varphi_i(\beta, \gamma)\rangle. \tag{1.13}$$

We shall now consider the excitation of vibrational states in even-even nuclei. In this case $\alpha_\mu$ can be associated with creation and annihilation operators of excitation quanta (phonons)[4] with spin $\lambda = 2$ and spin projection $\mu$ on the fixed Z axis:

$$\alpha_\mu = \sqrt{\hbar\omega / 2C_2}\,[q_\mu + (-1)^\mu q_\mu^*] \tag{1.14}$$

where $q_\mu^*$ is a phonon-creation operator and $q_\mu$ is an annihilation operator. The wave functions of the ground state and first excited state will then be $\Psi_i = \Psi_{00}^0$ (in the absence of phonons) and $\Psi_f = \Psi_{2\mu}^1$ (for one phonon with spin 2 and its projection $\mu$). Calculating the matrix elements of the operators $q_\mu$ and $q_\mu^*$, which are then substituted into (1.9), we obtain

$$b_{if} = -(V_0 / i\hbar)\sqrt{\hbar\omega/2C_2}\sum_\mu S_{2\mu}, \tag{1.15}$$

where $S_{2\mu}$ is defined by (1.12).

## 2. CALCULATION OF EXCITATION PROBABILITY

For the purpose of calculating the excitation probability we write the equation of motion of a charged particle in the parametric form

$$r = a(\varepsilon\cosh w + 1), \quad x = a(\cosh w + 1),$$

$$y = a\sqrt{\varepsilon^2 - 1}\,\sinh w, \quad t = (a / v)(\varepsilon\sinh w + w), \tag{2.1}$$

where $a = Z_1Z_2e^2/\mu v^2$ and $\epsilon$ is the orbital eccentricity. Since $\mathbf{r} = (r, \Theta, \Phi) = (r, \pi/2, \Phi)$ in the K coordinate system, we have

$$Y_{2\mu}(\Theta, \Phi) = \left(\frac{x + iy}{r}\right)^\mu Y_{2\mu}\left(\frac{\pi}{2}, 0\right). \tag{2.2}$$

After (2.1) and (2.2) are inserted in (1.12), the excitation probability of collective levels of even-even nuclei is obtained from (1.11) and (1.8):

$$P = \frac{9A}{25}\left(\frac{R}{a}\right)^4 \eta^2 \sum_\mu \left| Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \Big| I_{2\mu}(\vartheta, \xi) - \frac{5}{6}\frac{kR_0}{\eta}\left(\frac{V_0}{E}\right)\left(\frac{a}{R}\right)^2 \frac{a^2}{R_0 d} \times I_{2\mu}(\vartheta, \xi, a, R_0, d)\Big|^2, \quad \text{if } d \neq 0, \tag{2.3}$$

$$P = \frac{9A}{25}\left(\frac{R}{a}\right)^4 \eta^2 \sum_\mu \left| Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \Big| I_{2\mu}(\vartheta, \xi) - \frac{5}{6}\frac{kR_0}{\eta}\left(\frac{V_0}{E}\right)\left(\frac{a}{R}\right)^2 K_{2\mu}(\vartheta, \xi, a, R_0)\Big|^2, \quad \text{if } d = 0, \tag{2.4}$$

where $A = \bar\beta^2/5$ for the excitation of rotational states and $A = \Delta E/2C_2$ for the excitation of vibrational states; R is the nuclear radius ($R = 1.2 \times 10^{-13}$ cm); $\epsilon = 1/\sin(\vartheta/2)$; $I_{2\mu}(\vartheta, \xi)$ are the classical orbital integrals for E2 Coulomb excitation, which were tabulated in reference 6:

$$I_{2\mu}(\vartheta, \xi, a, R_0, d) = \int_{-\infty}^{\infty} e^{i\xi(\varepsilon\sinh w + w)} \frac{(\cosh w + \varepsilon + i\sqrt{\varepsilon^2 - 1}\sinh w)^\mu}{(\varepsilon\cosh w + 1)^{\mu - 2}} \times g(\vartheta, a, R_0, d, w)\,dw, \tag{2.5}$$

$$K_{2\mu}(\vartheta, \xi, a, R_0) = a\int_{-\infty}^{\infty} e^{i\xi(\varepsilon\sinh w + w)} \frac{(\cosh w + \varepsilon + i\sqrt{\varepsilon^2 - 1}\sinh w)^\mu}{(\varepsilon\cosh w + 1)^{\mu - 1}} \times \delta[a(\varepsilon\cosh w + 1) - R_0]\,dw$$
$$= \frac{e^{i\xi(\varepsilon\sinh\Lambda + \Lambda)}}{\varepsilon\sinh\Lambda}\left\{\frac{(\cosh\Lambda + \varepsilon + i\sqrt{\varepsilon^2 - 1}\sinh\Lambda)^\mu}{(\varepsilon\cosh\Lambda + 1)^{\mu - 1}} - \frac{(\cosh\Lambda + \varepsilon - i\sqrt{\varepsilon^2 - 1}\sinh\Lambda)^\mu}{(\varepsilon\cosh\Lambda + 1)^{\mu - 1}}\right\}, \tag{2.6}$$

where

$$g(\vartheta, a, R_0, d, w) = \exp\left[\frac{a(\varepsilon\cosh w + 1) - R_0}{d}\right] \times \left[1 + \exp\frac{a(\varepsilon\cosh w + 1) - R_0}{d}\right]^{-2}, \tag{2.7}$$

$$\Lambda = \ln\left[\frac{1}{\varepsilon}\left(\frac{R_0}{a} - 1\right) + \sqrt{\frac{1}{\varepsilon^2}\left(\frac{R_0}{a} - 1\right)^2 - 1}\,\right]. \tag{2.8}$$

In calculating (2.3) and (2.4) we have used

$$b_{if}^c = (3/5i)\,A\,(R/a)^2\eta Y_{2\mu}(\pi/2, 0)\,I_{2\mu}(\vartheta, \xi), \tag{2.9}$$

which is easily obtained by writing Eq. (2Г, 7) of reference 6 in our notation.

To calculate the fuctions $I_{2\mu}(\vartheta, \xi, a, R_0, d)$ in (2.3) we must calculate the following integrals:

$$I_{22}(\vartheta, \xi, a, R_0, d) = I_1(\vartheta, \xi, a, R_0, d) - I_2(\vartheta, \xi, a, R_0, d), \tag{2.10}$$

$$I_{20}(\vartheta, \xi, a, R_0, d) = I_0(\vartheta, \xi, a, R_0, d), \tag{2.11}$$

$$I_{2-2}(\vartheta, \xi, a, R_0, d) = I_1(\vartheta, \xi, a, R_0, d) + I_2(\vartheta, \xi, a, R_0, d), \tag{2.12}$$

$$I_1(\vartheta, \xi, a, R_0, d) = 2\int_0^\infty [(\cosh w + \varepsilon)^2 - (\varepsilon^2 - 1)\sinh^2 w]\, g(\vartheta, a, R_0, d, w) \times \cos[\xi(\sinh w + w)]\, dw, \qquad (2.13)$$

$$I_2(\vartheta, \xi, a, R_0, d) = 4\sqrt{\varepsilon^2 - 1}\int_0^\infty (\sinh w)(\cosh w + \varepsilon)\, g(\vartheta, a, R_0, d, w) \times \sin[\xi(\sinh w + w)]\, dw, \qquad (2.14)$$

$$I_0(\vartheta, \xi, a, R_0, d) = 2\int_0^\infty (\varepsilon \cosh w + 1)^2 g(\vartheta, a, R_0, d, w) \times \cos[\xi(\sinh w + w)]\, dw. \qquad (2.15)$$

The function $g(\vartheta, a, R_0, d, w)$ is defined by (2.7). The imaginary parts of the integrals in (2.5) vanish because the imaginary parts of the integrands are odd. Since $g(\vartheta, a, R_0, d, w)$ decreases rapidly as the parameter $w$ increases, it is usually sufficient for the calculation of $I_0$, $I_1$, and $I_2$ to perform a numerical integration from 0 to 2 — 2.5.

## 3. CONDITIONS FOR THE SEMICLASSICAL METHOD

For calculating the excitation probability of collective nuclear levels it was assumed that the combined effect of the electric and nuclear fields is small and can be regarded as a time-dependent perturbation. To determine the conditions for the validity of this hypothesis we require that the excitation probability P be smaller than unity for any scattering angle $\vartheta$. When the incident particle energy exceeds the Coulomb barrier height we have $|b_{if}| > |b_{if}^C|$, with opposite signs for the two quantities. Therefore in analyzing the validity conditions we neglect $b_{if}^C$ and obtain the following expression for the probability:

$$P \approx A(kR_0)^2\left(\frac{V_0}{E}\right)^2 \begin{cases} I(\vartheta, \xi, a, R_0, d), & d \neq 0 \\ K(\vartheta, \xi, a, R_0), & d = 0, \end{cases} \qquad (3.1)$$

where

$$I(\vartheta, \xi, a, R_0, d) = \frac{1}{4}\left(\frac{a}{d}\right)^2\left(\frac{a}{R_0}\right)^2 \sum_\mu \left|Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \times \left|I_{2\mu}(\vartheta, \xi, a, R_0, d)\right|^2, \qquad (3.2)$$

$$K(\vartheta, \xi, a, R_0) = \frac{1}{4}\sum_\mu \left|Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 |K_{2\mu}(\vartheta, \xi, a, R_0)|^2. \qquad (3.3)$$

The highest value of this probability is obtained at scattering angles $\vartheta$ determined by the relation $D(\vartheta) = a(1+\epsilon) = R_0$. Considering that for these

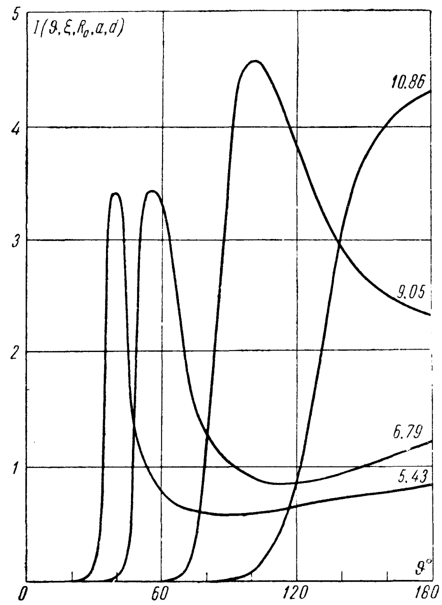


FIG. 1. $I(\vartheta, \xi, R_0, a, d)$ as a function of the scattering angle for $\xi = 0$ and $R_0/d = 22$. The ratio a/d is indicated on each curve.

scattering angles the absolute values of $I(\vartheta, \xi, a, R_0, d)$ (Fig. 1) and $K(\vartheta, \xi, a, R_0)$ are of the order of unity, we have

$$P < 1, \quad \text{if} \quad A(kR_0)^2(V_0/E)^2 < 1. \qquad (3.4)$$

Experiments on the elastic scattering of $\alpha$ particles by heavy nuclei indicate $V_0 \approx 30-40$ Mev. When the incident $\alpha$-particle energy is within this range (3.4) reduces to $\bar{\beta}^2(kR_0)^2 < 1$ or $(\Delta E/2C_2)(kR_0)^2 < 1$. We know[11,12] that the collective states of axially symmetrical even-even nuclei are of two types — excitations which are and which are not accompanied by considerable change of the nuclear quadrupole moment. Transitions of the first type occur between states with $\gamma = 0$; for these transitions we have

$$\bar{\beta} = \langle \varphi_f(\beta, \gamma) | \beta\cos\gamma | \varphi_i(\beta, \gamma)\rangle \approx \beta.$$

A transition from the ground level (with $\gamma = 0$) to an excited level with $\gamma = \pi$ occurs with a change of both the sign and absolute value of $\bar{\beta}$; $\bar{\beta} \ll \beta$ for the absolute value. Transitions of the second type occur in almost spherical nuclei and do not play an important part in markedly nonspherical nuclei when the energy change is small. Davydov and Filippov[12] have indicated a large number of even-even nuclei whose second excited states with spin $2^+$ may be assigned to this second type. The experimental data now available[13] indicate that $\bar{\beta}$ for these transitions is smaller than $\beta$ by a factor of 10 or more.

It is easily calculated that when $\alpha$ particles with 20 — 50 Mev are scattered by heavy nuclei $kR_0 \sim 20$, which is reduced to 10 for deuteron scattering. For the excitation of the first nuclear rotational levels we therefore have $\bar{\beta}(kR_0) \approx \beta kR_0 \approx (0.1 - 0.3)\,20 > 1$ and the condition for applying perturbation theory to inelastic scattering is not fulfilled. For the excitation of second levels with spin $2^+$ (excitations of the second type) we have $\bar{\beta}(kR_0) \ll \beta(kR_0) < 1$.

A similar conclusion is reached by analyzing the requirement that the product $(\Delta E/2C_2)(kR_0)^2$ be small compared with unity. For the excitation of the first levels of even-even nuclei with spin $2^+$, $C_2$ (the surface tension parameter) lies between 15 Mev and 100 Mev for most nuclei,[6] whereas $C_2 \sim 2000 - 2000$ Mev for the second levels[13]. In other words, for the excitation of first vibrational levels of even-even nuclei $P > 1$, but $P < 1$ for the excitation of second levels. The semiclassical method can therefore be used to analyze experimental data on inelastic scattering of charged particles accompanied by the excitation of second levels with spin $2^+$ in even-even nuclei.

## 4. INELASTIC SCATTERING CROSS SECTION

The expressions obtained for the excitation probability of vibrational and rotational nuclear levels enable us to put the formulas for the differential inelastic scattering cross section into final form:

$$\sigma(\vartheta) = 9\cdot 10^{-2} A\gamma_i^2 a^2 \left(\frac{R}{a}\right)^4 \sin^{-4}\frac{\vartheta}{2} \exp\left\{-2R_0\Pi_0 h\left(\frac{D}{R_0}, \frac{\Delta R}{R_0}\right)\right\}$$
$$\times \sum_\mu \left|Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \Big| I_{2\mu}(\vartheta, \xi) - \frac{5}{6}\frac{kR_0}{\eta}\left(\frac{V_0}{E}\right)\left(\frac{a}{R}\right)^2 \frac{a^2}{R_0 d} I_{2\mu}(\vartheta, \xi, a, R_0, d)\Big|^2, \qquad (4.1)$$

if $d \neq 0$, and

$$\sigma(\vartheta) = 9\cdot 10^{-2} A\gamma_i^2 a^2 \left(\frac{R}{a}\right)^4 \sin^{-4}\frac{\vartheta}{2}$$
$$\times \exp\{-2R_0\Pi_0\sqrt{1-(D/R_0)^2}\} \sum_\mu \left|Y_{2\mu}\left(\frac{\pi}{2}, 0\right)\right|^2 \Big| I_{2\mu}(\vartheta, \xi) - \frac{5}{6}\frac{kR_0}{\eta}\left(\frac{V_0}{E}\right)\left(\frac{a}{R}\right)^2 K_{2\mu}(\vartheta, \xi, a, R_0)\Big|^2, \qquad (4.2)$$

if $d = 0$.

Equations (3.10) and (4.1) have been used to calculate the excitation probability of a second level with spin $2^+$ in $Th^{232}_{90}$ ($\Delta E = 0.790$ Mev) by $\alpha$ particles with the energies 25, 30, 40, and 50 Mev and the corresponding angular and energy distributions.

Figure 2 gives $P(\vartheta)$ curves for different energies, Fig. 3 gives angular distributions for inelastically scattered $\alpha$ particles and Fig. 4 gives the total cross section as a function of energy. In calculating the differential cross sections the parameters of $T = \exp\{-2R_0\Pi_0 h(D/R_0, \Delta R/R_0)\}$, which takes into account the degree of attenuation of the $\alpha$-particle beam, were selected in agreement with Porter:[10] $R_0 = 10.5 \times 10^{-13}$ cm, $\Delta R = 2.1 \times 10^{-13}$ cm and $\Pi_0 = 3.5$. The depth $V_0$ of the potential well was 40 Mev[8] and $C_2 = 2960$ Mev.[13]

It is apparent from Figs. 3 and 4 that the angular dependence of the inelastic scattering cross section possesses a single maximum, whose half-width decreases as the incident-particle energy rises. The absolute value of the differential cross

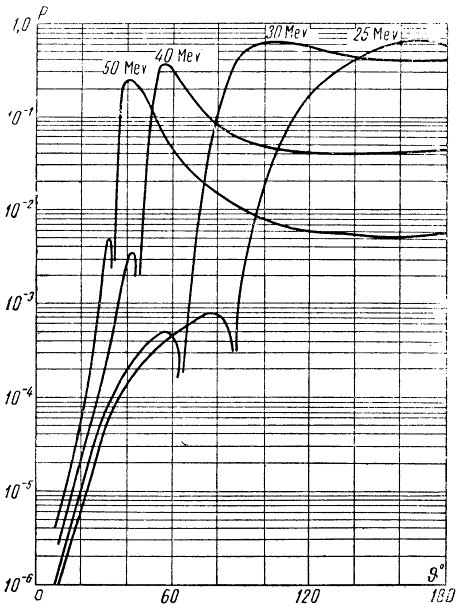


FIG. 2. Excitation probability of a second level with spin $2^+$ in $Th^{232}$ ($\Delta E = 0.790$ Mev) as a function of scattering angle for a few values of incident $\alpha$-particle energy.

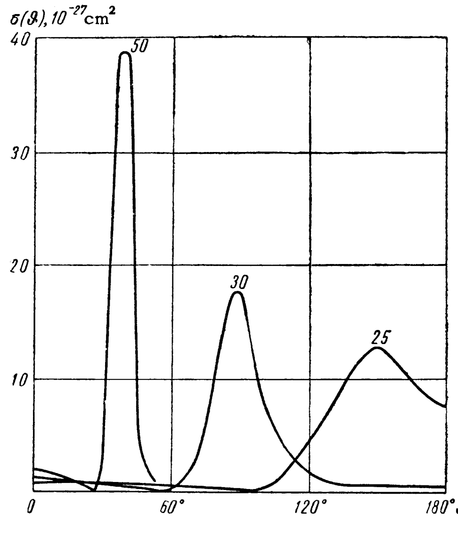


FIG. 3. Angular distributions of $\alpha$ particles scattered inelastically on $Th^{232}$ ($\Delta E = 0.790$ Mev). Numbers on the curves indicate the energies of the incident $\alpha$ particles in Mev.

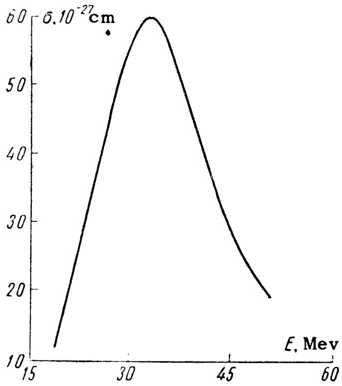


FIG. 4. Energy dependence of total cross section for inelastic scattering of α-particles on $Th^{232}$.

section maximum for inelastic scattering increases with the energy. The total cross section increases with energy up to a maximum at some $E_m$ and then decreases. With increasing energy $\alpha$ particles remain in the vicinity of a nucleus for a shorter time; also, fewer $\alpha$ particles remain unabsorbed by the nucleus and can be scattered inelastically.

When the energy $\Delta E$ transferred during collision is such that the parameter $\xi$ differs from 0 the general character of all curves is conserved. The maxima of the $\sigma(\vartheta)$ curves are shifted toward larger angles, the absolute value of the cross section is reduced and $E_m$, for which the total cross section reaches its maximum, is shifted toward higher energies.

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