We also calculated the parameters f_1 and f_2 in saturation of one of the transitions, satisfying the selection rules $\Delta M = \Delta m = \pm 1$. However, these expressions are too unwieldy to repeat here.

In conclusion, the author thanks G. R. Khutsishvili for help in this work.

[1] G. R. Khutsishvili, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1031 (1958), Soviet Phys. JETP **8**, 720 (1959); Nuovo cimento, in press.

[2] C. D. Jeffries, Phys. Rev. **106**, 164 (1957).

ON THE "LARMORON" PLASMA THEORY

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IN present-day quantum theory widespread use is made of the method introducing quasi-particles; it usually leads to a more effective and visualizable theory. It seems probable that a similar introduction of effective particles would also appreciably simplify plasma theory (albeit classical).

We shall use the term "larmoron" to define an effective particle situated at the guiding center of the Larmor motion of the real particle with a magnetic moment $\mu = mv_{\perp}^2/2H$ (where m is the particle mass, $v_{\perp}$ the component of the velocity perpendicular to magnetic field H) and an energy equal to the total energy of the real particle (we think this nomenclature more felicitous than the expression "Larmor circle" used elsewhere). The idea of larmorons has already been used often (but not always consistently) in many papers (see, for instance, references 1, 2, and others).

For a more consistent definition of a larmoron one must introduce its mean lifetime τ and the average translational velocities of its motion (u and v are the components perpendicular to the field H, and w the one along H). If we assume that the magnetic field is uniform, the velocity of the motion of the real particle under the action of an external acceleration a (along the x axis) in a field H (along the z axis) is of the form

$$v_x = v_x^{(0)} \cos \omega t - (v_y^{(0)} - a/\omega) \sin \omega t, \quad v_z = v_z^{(0)},$$

$$v_y = v_x^{(0)} \sin \omega t + (v_y^{(0)} - a/\omega) \cos \omega t + a/\omega, \quad (1)$$

where $\mathbf{v}^0$ is the velocity of the real particle at time $t = 0$; ω is the Larmor frequency. Since the probability that a larmoron will live through the interval of time t to $t + dt$ is equal to $e^{-t/\tau} dt/\tau$, we have

$$u = \int_0^\infty v_x e^{-t/\tau} \frac{dt}{\tau} = \frac{v_x^{(0)} - v_y^{(0)}\omega\tau + a\tau}{1 + (\omega\tau)^2},$$

$$v = \frac{v_y^{(0)} + v_x^{(0)}\omega\tau + a\omega\tau^2}{1 + (\omega\tau)^2}, \quad w = v_z^{(0)}. \quad (2)$$

The total larmoron energy ϵ can be expressed in terms of its velocity as follows:

$$\varepsilon = \frac{m}{2} \{v_x^{(0)\,2} + v_y^{(0)\,2} + v_z^{(0)\,2}\} = \frac{m}{2} \{[1 + (\omega\tau)^2](u^2 + v^2)$$
$$- 2a\tau [u + (\omega\tau)v] + (a\tau)^2 + w^2\}. \quad (3)$$

If we assume that the distribution function for the velocities $\mathbf{v}^0$ had a Maxwellian form the distribution function for the velocities u, v, w is of the form

$$f_0(u, v, w) = n(m/2\pi kT)^{3/2} [1 + (\omega\tau)^2]$$
$$\times \exp\left\{-\frac{m}{2kT} \left[\left(u\sqrt{1 + (\omega\tau)^2} - \frac{a\tau}{\sqrt{1 + (\omega\tau)^2}}\right)^2\right.\right.$$
$$\left.\left. + \left(v\sqrt{1 + (\omega\tau)^2} - \frac{a\omega\tau^2}{\sqrt{1 + (\omega\tau)^2}}\right)^2 + w^2\right]\right\}, \quad (4)$$

where n is the number of larmorons per unit volume. The transport equation for the larmorons of one kind under the conditions where there is a density and a temperature gradient along the x axis can be written in the form $u \,\partial f/\partial x = (\partial f/\partial t)_{\text{coll}}$, since the larmoron acceleration is equal to zero (if $\mathbf{a} \perp \mathbf{H}$). If we use the usual method of Lorentz's electron theory, i.e., put $f = f_0 + u\chi$ we get for the transport equation and its solution

$$\partial f_0/\partial x = -\chi/\tau, \quad f = f_0 - \tau u\, \partial f_0/\partial x. \quad (5)$$

Substituting (4) into (5) and then evaluating, by the usual equations, the fluxes of electric charge, number of particles, and energy, we easily obtain the well known formulae for the coefficients of electrical conductivity, diffusion, and thermal conductivity of a plasma. We must note that the coefficient of electrical conductivity is evaluated with the first term of (5), i.e., with f_0 and with $a = eE/m$.

All calculations in terms of the "larmoron" theory of a plasma are very simple and suffer not at all from the unwieldiness which is so characteristic for the transport theory of a plasma. The weak point of the larmoron theory, in the formu-

lation given here, is the definition of the mean life time τ of a larmoron. One can as a first approximation use for τ the average time of free flight of the real particle, evaluated without taking the magnetic field into account, but changing the values of the parameters in the logarithmic term (the Larmor radius or the Coulomb screening distance, depending on their relative magnitude). In this way we indeed get from (4) and (5) the well known formulae for transport phenomena in a dilute plasma. It is, however, possible in the "larmoron" theory to give also a more rigorous determination of the quantity τ, in particular, by using to this purpose the method of interpreting the collision terms for larmorons in a way similar to the one used in quantum theories of transfer.

If the magnetic field or the acceleration a are non-uniform and depend on the time, we must change (1) and (2) will as a consequence become more complicated. But the "larmoron" theory will even in that case be appreciably simpler than the usual transport theory of a plasma.

I express my gratitude to A. E. Glauberman for discussing this paper.

[1] L. Spitzer Jr., Physics of Fully Ionized Gases, Interscience, New York, 1956, Russ. Transl. IIL, M., 1957.

[2] S. T. Belyaev, Физика плазмы и проблема управляемых термоядерных реакций (Plasma Physics and the Problem of Controlled Thermonuclear Reactions) U.S.S.R. Acad. Sci. Press, M., 1958, pp. 50-66.

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A DYNAMICAL PRINCIPLE FOR SECOND-ORDER EQUATIONS

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A new method to construct a quantum theory of spinor fields based upon second order equations has recently been suggested.* The results of quantum electrodynamics are not changed in such an approach, but a number of interesting possibil-

ities arise for a more correct description of other processes involving spinor particles. The unusual commutation relations for spinor wave functions are due to a number of peculiarities of the theory.

In this note we show that the application of Schwinger's dynamical principle to systems with a Lagrangian of the second order gives unique commutation rules for fermion and boson fields. It is clear from the derivation that the same result is also valid for fields with higher derivatives: Lagrangians of odd order lead to anticommutativity of spinors and Lagrangians of even order require that they commute.

Taking the Lagrangian in the form (m is the eigen mass, the rest of the notation is the same as in reference 1):

$$\mathcal{L}(x) = (1/2m)\,\partial_\mu\chi(x)\,\alpha_\mu\alpha_\nu\partial_\nu\chi(x) - \mathcal{H}(\chi(x)), \tag{1}$$

we get, after taking the variation, the equation of motion

$$\alpha_\mu\alpha_\nu\partial^2_{\mu\nu}\chi + m\delta_l\mathcal{H}/\delta\chi = \partial^2_{\mu\nu}\chi\alpha_\mu\alpha_\nu + m\delta_r\mathcal{H}/\delta\chi = 0. \tag{2}$$

We first of all determine the commutation rule for χ on the hypersurface σ with $d\sigma_\mu \to d\sigma_0 = d\sigma$, using the shift operator G_χ

$$G_\chi = \frac{1}{2m}\int_\sigma d\sigma_\mu\,(\delta\chi\alpha_\mu\alpha_\nu\partial_\nu\chi + \partial_\nu\chi\alpha_\nu\alpha_\mu\delta\chi)$$

$$= \frac{1}{m}\int_\sigma d\sigma_\mu\delta\chi\alpha_\mu\alpha_\nu\partial_\nu\chi = \frac{1}{m}\int_\sigma d\sigma_\mu\partial_\nu\chi\alpha_\nu\alpha_\mu\delta\chi. \tag{3}$$

As $d\sigma_\mu \to d\sigma_0$ we get

$$G_\chi = \frac{1}{m}\int_\sigma d\sigma\partial_\nu\chi\alpha_\nu\alpha_0\delta\chi = \frac{1}{m}\int_\sigma d\sigma\,(\alpha_0\alpha_\nu)^T\delta\chi\partial_\nu\chi,$$

$$[\alpha_0\chi, G_\chi] = i\alpha_0\delta\chi.$$

The matrices α_μ can be of two kinds: a) $\alpha_\mu^T = -\alpha_\mu$, Dirac algebra; b) $\alpha_\mu^T = +\alpha_\mu$, Kemmer algebra. In both cases $\alpha_\mu\alpha_\nu = (\alpha_\nu\alpha_\mu)^T$ so that always

$$[\delta\chi, \partial_\mu\chi] = 0. \tag{4}$$

The commutation relations are the same in both cases:

$$(1/m)\,[\alpha_0\chi(x), \partial_\nu'\chi(x')\,\alpha_\nu\alpha_0] = i\alpha_0\delta_\sigma(x - x'), \tag{5}$$

Using Green's theorem

$$\chi_\alpha(x) = \frac{1}{m}\int_\sigma d\sigma_\mu\,(\Delta(x - x')\,(\alpha_\mu\alpha_\nu\partial_\nu'\chi(x'))_\alpha$$

$$- \partial_\nu'\Delta(x - x')\,(\alpha_\nu\alpha_\mu\chi(x'))_\alpha) \tag{6}$$

we get for arbitrary points x and x':

$$[\chi_\alpha(x), \chi_\beta(x')] = i\delta_{\alpha\beta}\Delta(x - x'). \tag{7}$$

It is easy to generalize this result to charged