ON THE POSSIBILITY OF DETERMINING THE π-π SCATTERING AMPLITUDES FROM THE ANALYSIS OF THE $\gamma + p \rightarrow N + \pi + \pi$ REACTIONS NEAR THRESHOLD

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It is shown that a study of the reactions $\gamma + p \rightarrow p + \pi^+ + \pi^-$, $p + \pi^0 + \pi^0$, $n + \pi^+ + \pi^0$ near threshold should allow the determination of the $\pi-\pi$ zero energy scattering amplitudes if the $\pi-\pi$ interaction has resonance character, and the determination of a certain combination of these amplitudes if the interaction does not have resonance character.

AT present there does not exist any conclusive evidence on the interaction between π mesons. It is therefore of importance to find any parameters which are connected with this interaction. In the present paper it is shown that an experimental investigation of the photoproduction of two mesons near the threshold can yield information on the zero energy $\pi-\pi$ scattering amplitude.

In the following we shall consider two cases:

1) The interaction between π mesons does not have resonance character at small energies. In this case the energy and angular distribution of the reaction can be obtained by the method developed in references 1-3.

However, the case of photoproduction of two π mesons differs considerably from the decay of the τ meson which was treated in references 1-3. It allows generally an easier way to determine the $\pi-\pi$ scattering amplitudes.

2) The $\pi-\pi$ scattering at small energies is of resonance character. The characteristic energy dependence associated with this case has been already investigated for other reactions.[4] Since at resonance the final state interaction of the π mesons produces an effect of the order of magnitude unity this case would be particularly favorable from the experimental point of view.

1. THE CASE OF A NONRESONATING INTERACTION

In references 1-3 it has been shown that in reactions involving the creation of three particles a final state interaction leads to considerable changes in the angular and energy distributions. In particular, at energies close to threshold when terms up to the first power in kr_0 have been retained (k — momentum corresponding in order of magnitude to the energy excess above the threshold; r_0 — range of the interaction) the matrix element of the S matrix has the form

$$\langle f \mid S \mid i \rangle = (1 + ik_{12}a_{12} + ik_{13}a_{13} + ik_{23}a_{23}) \langle f_0 \mid S \mid i \rangle, \quad (1)$$

where k_{12}, k_{13}, k_{23} and a_{12}, a_{13}, a_{23} are the relative momenta and the scattering amplitudes between two particles respectively and the wave function of the final state in the matrix element of the right hand side f_0, is to be taken at zero energy of the three particles. The quantity $\langle f_0 | S | i \rangle$ nevertheless still depends on the energy through the initial state wavefunction $| i \rangle$. However, close to threshold the matrix element $\langle f_0 | S | i \rangle$ is an analytic function of the energy E and thus differs from the matrix element at threshold $\langle f_0 | S | i_0 \rangle$ by a term of the order $\sim \Delta E \sim k^2$. Therefore for an accuracy linear in k one can set $\langle f_0 | S | i \rangle = \langle f_0 | S | i_0 \rangle$. The fact that $\langle f_0 | S | i \rangle$ is analytical can be understood if one recalls that the branch points in matrix elements of the type $\langle f | S | i \rangle$ at the thresholds usually are due to the final state. This can be demonstrated using a method analogous to the method which was used by Lehmann[5] to show the analyticity of the scattering amplitude as a function of the momentum transfer.

It has been shown[2,3] for the case of τ-meson decay that, owing to the reality of the matrix elements of the type $\langle f_0 | S | i \rangle$, the terms linear in k vanish in the expressions for the probability of the process. This circumstance not only considerably decreases the magnitude of the effect but also leads to the necessity of a much more detailed investigation of the final three-particle state. For this situation in the expressions for the probability there are two types of terms which are quadratic in k. They are, first, terms which determine the correlations between the outgoing particles, and which can be expressed in terms of the scattering amplitudes between pairs of particles, and second, terms which cannot be evaluated but which do not influence the angular distributions. Thus one cannot obtain the energy distributions near the threshold. It is therefore of value only to study the angular correlations.

It will be shown below that in the case of photoproduction of two π mesons the terms linear in k are present in the expression for the probability of the process when the reaction has a real intermediate state which differs from the initial and the final state (viz., in the present case, a nucleon plus a π meson) and when the reaction can go via channels which differ in the charge of the created particles. The relations given below allow in principle to determine a certain combination of the $\pi-\pi$ scattering amplitudes without a detailed investigation of the angular correlations of the reaction.

There are three possible reactions of production of two π mesons by γ quanta:

$$\gamma + p \rightarrow p + \pi^+ + \pi^-, \quad (2.1)$$
$$\gamma + p \rightarrow p + \pi^0 + \pi^0, \quad (2.2)$$
$$\gamma + p \rightarrow n + \pi^+ + \pi^0, \quad (2.3)$$

We shall denote the respective matrix elements at zero energy by

$$\lambda_1 = \rho_1 \exp(i\varphi_1), \qquad \lambda_2 = \rho_2 \exp(i\varphi_2), \qquad \lambda_3 = \rho_3 \exp(i\varphi_3).$$

Utilizing the method which was applied in reference (2) to the decay of the τ meson one can easily obtain expressions for the squares of the matrix elements of the reactions (2) with an accuracy of terms linear in k:

$$|\langle p\pi^+\pi^- | S | p\gamma\rangle|^2 = \rho_1^2 \Big[1 + \rho_{12}\sin\varphi_{12}\cdot \tfrac{2}{3} k_{12}(a_2 - a_0) + \rho_{13}\sin\varphi_{13}\cdot \tfrac{2}{3}\sqrt{2}\, k_{23}(b_{3/2} - b_{1/2})\Big],$$
$$|\langle p\pi^0\pi^0 | S | p\gamma\rangle|^2 = \rho_2^2 \Big[1 + \rho_{21}\sin\varphi_{21}\cdot \tfrac{4}{3} k_{12}(a_2 - a_0) + \rho_{23}\sin\varphi_{23}\cdot \tfrac{2}{3}\sqrt{2}\,(k_{13} + k_{23})(b_{3/2} - b_{1/2})\Big],$$
$$|\langle n\pi^+\pi^0 | S | p\gamma\rangle|^2 = \rho_3^2 \Big[1 + \rho_{31}\sin\varphi_{31}\cdot \tfrac{2}{3}\sqrt{2}\, k_{23}(b_{3/2} - b_{1/2}) + \rho_{32}\sin\varphi_{32}\cdot \tfrac{2}{3}\sqrt{2}\, k_{13}(b_{3/2} - b_{1/2})\Big],$$
$$\rho_{ik} = \rho_k/\rho_i, \qquad \varphi_{ik} = \varphi_i - \varphi_k. \quad (3)$$

Here a_0 and a_2 are the $\pi-\pi$ mesons scattering amplitudes at zero energy for states with isotopic spin 0 and 2 (since the π mesons close to threshold are created in a relative S-state an isotopic spin 1 is impossible), while $b_{1/2}$ and $b_{3/2}$ are $\pi-\pi$ meson scattering amplitudes at zero energy for states with isotopic spin 1/2 and 3/2. In k_{12}, k_{13}, and k_{23} the indices 1 and 2 refer to mesons, the index 3 to the nucleon. Comparing with the case of the τ-meson decay, one sees that the quantities λ_1, λ_2, and λ_3 are not real and have relative phases differing from zero. They can be expressed in terms of measurable quantities. By studying all three reactions (2) we can determine the charge exchange amplitudes $(a_2 - a_0)/3$ and $(\sqrt{2}/3) \times (b_{3/2} - b_{1/2})$. However, since the amplitudes $b_{3/2}$ and $b_{1/2}$ are well known it is possible to simplify the analysis considerably by utilizing the isotopic invariance. So one can determine $(a_2 - a_0)/3$ by studying only one reaction (e.g. $\gamma + p \rightarrow p + \pi^+ + \pi^-$).

The electromagnetic interaction is the sum of an isotopic scalar and a vector. Therefore the final state isotopic spin can be 1/2 or 3/2 (but not 5/2). Each of the final states (2) can be expanded as the sum of two states. In one of them the isotopic spin of the two π mesons $T_{12} = 0$ and the total isotopic spin $T = 1/2$; in the other $T_{12} = 2$, $T = 3/2$. We can easily verify that the following relation between λ_1, λ_2 and λ_3 arises:

$$\lambda_3 = -(\lambda_1 + \lambda_2)/\sqrt{2}, \quad (4)$$

from which follows, for example

$$\rho_{12}\sin\varphi_{12} = -\sqrt{2}\,\rho_{13}\sin\varphi_{13}. \quad (5)$$

This relation allows us to determine $a_2 - a_0$ from only the first reaction if the coefficients of k_{12} and k_{23} have been found experimentally.

2. THE CASE OF A RESONATING INTERACTION

The formulae given above concern the case where the $\pi-\pi$ interaction at small energies does not have a resonance character, i.e., if the $\pi-\pi$ scattering amplitude is of the order of the range of the forces (Compton wavelength of the π meson): $a \approx r_0 \approx 1/\mu$. We now assume the opposite case, i.e., we take $a \gg r_0$ and, as before, $kr_0 \ll 1$ while we assume $ka \approx 1$. In this case the π-nucleon interaction evidently does not play such an important role as the $\pi-\pi$ interaction since the former as is well known does not have a resonance character at zero energy. Effects of this type have been already treated repeatedly.[4] We therefore give only the final results. For the three processes (2) we have in this case [instead of (3)]:

$$|\langle p\pi^+\pi^- | S | \gamma p\rangle|^2 = \rho_1^2 (1 + k_{12}^2 a_0^2)^{-1} (1 + k_{12}^2 a_2^2)^{-1} \times \Big\{\Big[1 + \tfrac{1}{3}\rho_{12}\sin\varphi_{12} k_{12}(a_2 - a_0)\Big]^2 + k_{12}^2 \Big[\tfrac{1}{3}(2a_2 + a_0) - \tfrac{1}{3}\rho_{12}\cos\varphi_{12}(a_2 - a_0)\Big]^2\Big\},$$
$$|\langle p\pi^0\pi^0 | S | \gamma p\rangle|^2 = \rho_2^2 (1 + k_{12}^2 a_0^2)^{-1} (1 + k_{12}^2 a_2^2)^{-1} \times \Big\{\Big[1 + \tfrac{2}{3}\rho_{21}\sin\varphi_{21} k_{12}(a_2 - a_0)\Big]^2 + k_{12}^2 \Big[\tfrac{1}{3}(a_2 + 2a_0) - \tfrac{2}{3}\rho_{21}\cos\varphi_{21}(a_2 - a_0)\Big]^2\Big\},$$
$$|\langle n\pi^+\pi^0 | S | \gamma p\rangle|^2 = \rho_3^2 (1 + k_{12}^2 a_2^2)^{-1}. \quad (6)$$

As can be seen from (6) in this case one can determine both amplitudes a_0 and a_2 separately.

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