THREE-PHOTON ANNIHILATION OF POSITRONIUM IN THE P STATE

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A relativistically invariant expression for the probability amplitude for three-photon annihilation of positronium is obtained by the summation of an infinite number of diagrams of a definite class. The probability of three-photon annihilation of positronium in the P state is calculated in the nonrelativistic limit, and the selection rules for this process are found.

DEPENDING on the charge parity of a state of positronium, it can decay into two or three photons (a larger number of photons is less probable). Two-photon annihilation of positronium in the S state has been dealt with in a calculation by Pomeranchuk,[1] and three-photon annihilation of positronium in the S state has been considered in a paper by Ore and Powell.[2] The methods given in these papers, however, cannot be applied to the calculation of the probability of annihilation of positronium in excited states. It has been suggested by the writer[3] that the amplitude for annihilation of positronium be found by summation of an infinite number of Feynman diagrams; this makes it possible to calculate the probability for annihilation of positronium in any excited state. In the paper referred to the probability for two-photon annihilation of positronium in the P state was calculated. In the present communication the technique of summation of diagrams is applied to the problem of the three-photon annihilation of positronium.

1. THE PROBABILITY AMPLITUDE FOR THREE-PHOTON ANNIHILATION OF POSITRONIUM

The diagram that describes three-photon annihilation of free particles is shown in Fig. 1.* To get the probability amplitude for three-photon annihilation of bound particles, we adjoin to the irreducible diagram (Fig. 1) all the reducible diagrams of the "ladder type" (Fig. 2), which describe the interaction of the particles before the annihilation. We get as the result the amplitude for three-photon annihilation of an interacting electron and positron (which can also be in a bound state) in the following form (cf. reference 3):

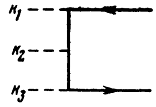

FIG. 1

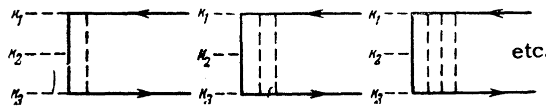

FIG. 2

$$A = -ie^3 \int \Phi_{\nu_1\nu_2\nu_3}(x_2 x x_1)\,(C\gamma_{\nu_1} K(x_2 - x)\,\gamma_{\nu_2} K(x - x_1)\,\gamma_{\nu_3})_{\rho_2\rho_1} \times \Psi_{\rho_1\rho_2}(x_1 x_2)\, d^4x\, d^4x_1\, d^4x_2. \tag{1}$$

Here $\Psi(x_1 x_2)$ is the wave function of the interacting electron and positron, which satisfies the Bethe-Salpeter equation,[4] and

$$\Phi_{\nu_1\nu_2\nu_3}(x_2 x x_1) = 2\pi \sqrt{2\pi / \omega_1\omega_2\omega_3} \times [l_{1\nu_1} l_{2\nu_2} l_{3\nu_3} \exp i (k_1 x_2 + k_2 x + k_3 x_1) + \ldots] \tag{2}$$

is the symmetrized function of the photons with frequencies $\omega_1, \omega_2, \omega_3$, momenta $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ and polarizations $\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3$. In the expression (2), and also in the subsequent calculations, the series of dots corresponds to the similar terms with all possible permutations of the photons. Furthermore $C = \alpha_2$, $\gamma_0 = \beta$, $\gamma_{1,2,3} = \beta\alpha_{1,2,3}$, and $K(x)$ is the electron Green's function.[5] Throughout we have set $\hbar = c = 1$, and have adopted the following rule of summation over vector indices:

$$ab = a_\nu b_\nu = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3.$$

The expression (1) gives the first nonvanishing contribution for the process in question. If we take into account radiative corrections, then every matrix γ in Eq. (1) is replaced by a vertex operator Γ, and every Green's function K is replaced by

*In Figs. 1 and 2 the similar diagrams that differ from each other and that shown by permutations of the three photons k_1, k_2, k_3 are omitted.

G, where Γ and G include the radiative corrections.[6] In dealing with the radiative corrections to the amplitude (1) we must add in on an equal basis with diagrams like Fig. 2 diagrams that describe the interaction of the electron and positron associated with their virtual annihilation.[3] Then the wave function $\Psi(x_1, x_2)$ involved in Eq. (1) will be the solution of a Bethe-Salpeter type equation in which along with the ordinary interaction there is also the specific exchange interaction associated with the virtual annihilation of the electron and positron.[7,6]

2. THE NONRELATIVISTIC APPROXIMATION FOR THE AMPLITUDE

In relative-momentum variables p the amplitude (1) can be rewritten in the form

$$A = \frac{ie^3\,(2\pi)^{3/2}}{\sqrt{\omega_1\omega_2\omega_3}} \int \left(\frac{C\hat{l}_3\,[\hat{p}+{}^1/_2\,(\hat{k}_3-\hat{k}_2-\hat{k}_1)+m]\hat{l}_2\,[\hat{p}+{}^1/_2(\hat{k}_3+\hat{k}_2-\hat{k}_1)+m]\hat{l}_1}{[(p+{}^1/_2(k_3-k_2-k_1))^2-m^2][(p+{}^1/_2(k_3+k_2-k_1))^2-m^2]} + \cdots \right)_{\rho_2\rho_1} \psi_{\rho_1\rho_2}(p)\, d^4p\,\delta\,(K-k_1-k_2-k_3), \qquad (3)$$

where $\psi(p)$ is the positronium wave function in the relative-momentum space, K is the total momentum of the positronium atom, m is the mass of the electron, and for any vector a we write $\hat{a} = a_\nu\gamma_\nu$.

In the calculation of the amplitude (3) we make use of the smallness of the velocity v of the relative motion of the particles in the positronium atom (v is of the order e^2) and shall hereafter neglect all terms of order v^2 and higher. For convenience we go over in Eq. (3) from the matrices γ to the two-rowed (2×2) Pauli matrices $\boldsymbol{\sigma}$, and regard as nonvanishing only those of the small components of the wave function $\psi(p)$ that are of the order of magnitude v. Then in the coordinate system of the center of mass of the positronium atom ($K_0 = 2m - E$, $\mathbf{K} = 0$) we get

$$A = \frac{ie^3\,(2\pi)^{3/2}}{\sqrt{\omega_1\omega_2\omega_3}} \Big\{ \int \Big[\Big(p_0 + \omega_3 + \frac{1}{2}E - m\Big)^2 - (\mathbf{p}+\mathbf{k}_3)^2 - m^2 \Big]^{-1}$$
$$\times \Big[\Big(p_0 - \omega_1 - \frac{1}{2}E + m\Big)^2 - (\mathbf{p}-\mathbf{k}_1)^2 - m^2 \Big]^{-1}$$
$$\times \{ \mathrm{Sp}\,\sigma_2\,[(p_0^2 + p_0(\omega_3-\omega_1) - \omega_3\omega_1)\,\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1$$
$$+ \hat{\mathbf{l}}_3(\hat{\mathbf{p}}+\hat{\mathbf{k}}_3)\,\hat{\mathbf{l}}_2\,(\hat{\mathbf{p}}-\hat{\mathbf{k}}_1)\,\hat{\mathbf{l}}_1]\,\psi^L(p) + m\,\mathrm{Sp}\,\sigma_2\,[\hat{\mathbf{l}}_3(\hat{\mathbf{p}}+\hat{\mathbf{k}}_3)\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1$$
$$+ \hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\,(\hat{\mathbf{p}}-\hat{\mathbf{k}}_1)\,\hat{\mathbf{l}}_1]\,(\psi^{M_1}(p) + \psi^{M_2}(p))$$
$$+ \mathrm{Sp}\,\sigma_2\,[(p_0 - \omega_1 + m)\,\hat{\mathbf{l}}_3\,(\hat{\mathbf{p}}+\hat{\mathbf{k}}_3)\,\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1$$
$$+ (p_0 + \omega_3 - m)\,\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\,(\hat{\mathbf{p}}-\hat{\mathbf{k}}_1)\,\hat{\mathbf{l}}_1]$$
$$\times (\psi^{M_1}(p) - \psi^{M_2}(p))\}\, d^4p + \ldots \Big\}\,\delta\,(K-k_1-k_2-k_3). \qquad (4)$$

Here $\psi^L(p)$ is the large two-rowed component of the wave function $\psi(p)$, and $\psi^{M_1}(p)$ and $\psi^{M_2}(p)$ are the small (order of magnitude v) two-rowed components of the wave function $\psi(p)$, which in the mixed representation for $t > 0$ has the following form:[8]

$$\psi(\mathbf{p},t) = \begin{pmatrix} \psi^L(\mathbf{p},t) & \psi^{M_2}(\mathbf{p},t) \\ \psi^{M_1}(\mathbf{p},t) & 0 \end{pmatrix}$$
$$= \begin{pmatrix} \varphi(\mathbf{p}) & -\varphi(\mathbf{p})\,\hat{\mathbf{p}}^T/2m \\ \hat{\mathbf{p}}\varphi(\mathbf{p})/2m & 0 \end{pmatrix} \exp\Big[-i\Big(\frac{E}{2}+\frac{\mathbf{p}^2}{2m}\Big)t\Big], \qquad (5)$$

where the index T means transposition. For $t < 0$ the sign of the exponent in Eq. (5) is reversed. E is the binding energy of the particles in the positronium atom, and for any three-dimensional vector $\mathbf{q}$ we have used throughout the notation $\hat{\mathbf{q}} = (\mathbf{q}\cdot\boldsymbol{\sigma}) = q_m\sigma_m$. The nonrelativistic two-rowed function $\varphi(\mathbf{p})$ in Eq. (5) satisfies the Schrödinger equation written for the electron and positron, and also is an eigenfunction of the operators for the total angular momentum of the system and for an angular-momentum component.

It is easy to see that in the case of the P state of positronium the functions $\psi^L(p)$, $\psi^{M_1}(p)$, and $\psi^{M_2}(p)$ make contributions of the same order of magnitude to the amplitude (4), whereas in the case of the S state the functions $\psi^{M_1}(p)$ and $\psi^{M_2}(p)$ can be neglected. If in calculating the annihilation of positronium in the S and P states we confine ourselves to the first nonvanishing contributions, then in integrating over the variable p_0 we can set $p_0 = 0$ in all the coefficients of $\psi^L(p)$, $\psi^{M_1}(p)$, and $\psi^{M_2}(p)$ in the integral (4) (cf., e.g., reference 8). Then the remaining integral over the relative momentum $\mathbf{p}$ will contain the nonrelativistic two-rowed positronium wave function $\varphi(\mathbf{p})$. Since $\varphi(\mathbf{p})$ has appreciable nonvanishing values in the region of small momenta, $|\mathbf{p}/m| = v \ll 1$, the terms in Eq. (4) that contain $\mathbf{p}^2\psi^L$, $\hat{\mathbf{p}}\psi^{M_1}$, and $\hat{\mathbf{p}}\psi^{M_2}$ are beyond the limits chosen for the accuracy of the calculation. Thus we get for the amplitude (4)

$$A = -\frac{ie^3\,(2\pi)^{5/2}}{\sqrt{\omega_1\omega_2\omega_3}}\frac{1}{2m} \Big\{ \int [2m\omega_3 + 2(\mathbf{pk}_3) + \mathbf{p}^2 - mE]^{-1}$$
$$\times [2m\omega_1 - 2(\mathbf{pk}_1) + \mathbf{p}^2 + mE]^{-1}\,[2m\,\mathrm{Sp}\,\sigma_2\,(\omega_3\omega_1\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1$$
$$+ \hat{\mathbf{l}}_3\hat{\mathbf{k}}_3\hat{\mathbf{l}}_2\hat{\mathbf{k}}_1\hat{\mathbf{l}}_1)\,\varphi(\mathbf{p}) + 2m\,\mathrm{Sp}\,\sigma_2\,(\hat{\mathbf{l}}_3\hat{\mathbf{p}}\hat{\mathbf{l}}_2\hat{\mathbf{k}}_1\hat{\mathbf{l}}_1 - \hat{\mathbf{l}}_3\hat{\mathbf{k}}_2\hat{\mathbf{l}}_2\hat{\mathbf{p}}\hat{\mathbf{l}}_1)\,\varphi(\mathbf{p})$$
$$+ m\,\mathrm{Sp}\,\sigma_2\,(\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{k}}_1\hat{\mathbf{l}}_1 - \hat{\mathbf{l}}_3\hat{\mathbf{k}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1)\,(\hat{\mathbf{p}}\varphi(\mathbf{p}) - \varphi(\mathbf{p})\,\hat{\mathbf{p}}^T)$$
$$+ \mathrm{Sp}\,\sigma_2\,(\hat{\mathbf{l}}_3\hat{\mathbf{k}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1\,(\omega_1 - m) - (\omega_3 - m)\,\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{k}}_1\hat{\mathbf{l}}_1)\,(\hat{\mathbf{p}}\varphi(\mathbf{p})$$
$$+ \varphi(\mathbf{p})\,\hat{\mathbf{p}}^T)]\,d^3p + \ldots \Big\}\,\delta(K - k_1 - k_2 - k_3). \qquad (6)$$

In the integral (6) we have kept the small quantity E in the denominator to avoid later divergences in the integration over the photon energies. The further calculations are considerably simplified if we take the photons to be circularly polarized. Then for the polarization vector $\boldsymbol{l}$ of each photon $\mathbf{k} = \mathbf{n}\omega$ we have

$$\mathbf{l} = (\boldsymbol{\tau} + i\lambda[\mathbf{n}\times\boldsymbol{\tau}])/\sqrt{2}, \qquad \lambda = \pm 1,$$

$$\boldsymbol{\tau}\times[\mathbf{n}\times\boldsymbol{\tau}] = \mathbf{n}, \qquad \hat{\mathbf{n}}\hat{\mathbf{l}} = \lambda\hat{\mathbf{l}}. \tag{7}$$

Setting $\varphi(\mathbf{x}) = \Phi_{s_z}\psi_{\text{nonrel}}(\mathbf{x})$, where Φ_{s_z} is the spin function and $\psi_{\text{nonrel}}(\mathbf{x})$ is the coordinate-space nonrelativistic wave function of the positronium atom, we get for the amplitude for annihilation of positronium, for example in the S state:

$$A = -\frac{ie^3(2\pi)^{11/2}}{2m^2\sqrt{\omega_1\omega_2\omega_3}}\{(1-\lambda_3\lambda_1)[(\mathbf{l}_3\mathbf{l}_2)\,l_{1m}$$

$$+(\mathbf{l}_2\mathbf{l}_1)\,l_{3m}-(\mathbf{l}_3\mathbf{l}_1)\,l_{2m}]+\text{c.p.}\}$$

$$\times(\mathrm{Sp}\,\sigma_2\sigma_m\Phi_{s_z})\,\psi_{\mathbf{nonrel}}(0)\,\delta(K-k_1-k_2-k_3), \tag{8}$$

where the notation c.p. in the curly brackets means the similar terms obtained by cyclic permutations of the indices 1, 2, 3. The expression (8) leads to the well known value for the probability W of three-photon annihilation of orthopositronium in the S state:[2]

$$W = \frac{2(\pi^2-9)}{9\pi n^3}(e^2)^6 m = \frac{0.72}{n^3}\cdot 10^7\ \text{sec}^{-1}, \tag{9}$$

where n is the principal quantum number fixing the energy state of the positronium atom. For parapositronium in the S state the probability of three-photon annihilation is zero according to Eq. (8), in agreement with the law of conservation of charge parity. In the writing of formulas (9) and (13) for the probability of three-photon annihilation of positronium an additional factor of $1/6$ is introduced to allow for the identity of states of the system that differ only by permutation of the momenta of the photons.

3. ANNIHILATION IN THE P STATE

In the case of the P state the wave function of the positronium atom is odd, $\varphi(-\mathbf{p}) = -\varphi(\mathbf{p})$, and by Eqs. (6) and (7) the amplitude for three-photon annihilation takes the form

$$A = -\frac{ie^3(2\pi)^{5/2}}{\sqrt{\omega_1\omega_2\omega_3}}\frac{1}{2m}\Big\{\int\frac{(1-\lambda_3\lambda_1)\,[(\mathbf{n}_1-\mathbf{n}_3)\,\mathbf{p}]\,\mathrm{Sp}\,\sigma_2\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1\varphi(\mathbf{p})}{2m^2}\,d^3p+\int(2m\omega_3+mE+\mathbf{p}^2)^{-1}(2m\omega_1$$

$$+mE+\mathbf{p}^2)^{-1}[2m\,\mathrm{Sp}\,\sigma_2\,(\omega_1\lambda_1\hat{\mathbf{l}}_3\hat{\mathbf{p}}\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1+\omega_3\lambda_3\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{p}}\hat{\mathbf{l}}_1)\,\varphi(\mathbf{p})+m\,(\omega_1\lambda_1+\omega_3\lambda_3)\,\mathrm{Sp}\,\sigma_2\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1\,(\hat{\mathbf{p}}\varphi(\mathbf{p})-\varphi(\mathbf{p})\,\hat{\mathbf{p}}^T)$$

$$+(\omega_1\omega_3(\lambda_1-\lambda_3)-m\,(\omega_1\lambda_1-\omega_3\lambda_3))\,\mathrm{Sp}\,\sigma_2\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1\,(\hat{\mathbf{p}}\varphi(\mathbf{p})+\varphi(\mathbf{p})\,\hat{\mathbf{p}}^T)]\,d^3p+\ldots\Big\}\,\delta(K-k_1-k_2-k_3). \tag{10}$$

The terms in the amplitude (10) can be divided into three groups, each of which involves two photons symmetrically and differs from the other two groups by cyclic permutation of the three photons (cf. Figs. 1 and 2). If we examine all the terms of one such group it is easily seen that the function $\varphi(\mathbf{p})$ occurring in the amplitude (10) must be antisymmetric in the spin indices. Consequently, the functions $\hat{\mathbf{p}}\varphi(\mathbf{p})$ and $-\varphi(\mathbf{p})\hat{\mathbf{p}}^T$ (the small components of the wave function (5)) make the same contribution to the amplitude (10); for example,

$$\mathrm{Sp}\,\sigma_2(\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1+\hat{\mathbf{l}}_1\hat{\mathbf{l}}_2\hat{\mathbf{l}}_3)(-\varphi\hat{\mathbf{p}}^T) = \mathrm{Sp}\,\sigma_2(\hat{\mathbf{l}}_3\hat{\mathbf{l}}_2\hat{\mathbf{l}}_1+\hat{\mathbf{l}}_1\hat{\mathbf{l}}_2\hat{\mathbf{l}}_3)\,\hat{\mathbf{p}}\varphi. \tag{11}$$

This means that in calculating the value of the expression (10) we can replace the expression $\hat{\mathbf{p}}\varphi(\mathbf{p}) - \varphi(\mathbf{p})\mathbf{p}^T$ by the factor $2\hat{\mathbf{p}}\varphi(\mathbf{p})$ or the factor $-2\varphi(\mathbf{p})\hat{\mathbf{p}}^T$, and the expression $\hat{\mathbf{p}}\varphi(\mathbf{p}) + \varphi(\mathbf{p})\hat{\mathbf{p}}^T$ is to be set equal to zero. We get

$$A = -\frac{ie^3(2\pi)^{5/2}}{\sqrt{\omega_1\omega_2\omega_3}}\frac{\mathrm{Sp}\,\sigma_2\Phi}{m}\int\left(\frac{(\lambda_1+\lambda_2)\,[1-(\mathbf{n}_1\mathbf{n}_2)]\,(\mathbf{l}_3\mathbf{p})}{2m\omega_3+mE+\mathbf{p}^2}+\text{c.p.}\right)$$

$$\times\,\psi_{\mathbf{nonrel}}(\mathbf{p})\,d^3p\,\delta(K-k_1-k_2-k_3). \tag{12}$$

Here $\varphi(\mathbf{p}) = \Phi\psi_{\text{nonrel}}(\mathbf{p})$, where Φ is the spin function and $\psi_{\text{nonrel}}(\mathbf{p})$ is the nonrelativistic coordinate wave function for positronium in the P state. In Eq. (12) terms not containing ω in the denominator have been dropped, since they make only a small contribution (see below). In Eq. (12) account has also been taken of the fact that if the energy of one of the photons goes to zero, $\omega \to 0$, then for each of the other two photons $\omega \to m$.

The spin function Φ appearing in Eqs. (12) and (10) is antisymmetric. Therefore the amplitude for three-photon annihilation of positronium in the P state is different from zero only for states with total spin $s = 0$ (parapositronium). On the other hand, for total spin $s = 1$ (orthopositronium) the probability of three-photon annihilation in the P state is zero. This selection rule is an expression of the law of conservation of charge parity.

According to Eq. (12) the probability W of three-photon annihilation of parapositronium in the P state can be written in the form

$$W = \frac{e^6}{3m^2(2\pi)^8}\int \sum_{\lambda_1\lambda_2\lambda_3} \sum_{M}\left|\int \frac{(\lambda_1+\lambda_2)\,[1-(\mathbf{n}_1\mathbf{n}_2)]\,(\mathbf{l}_3\mathbf{p})}{2m\omega_3+mE+\mathbf{p}^2}\,\psi_{\text{nonrel}}(\mathbf{p})\,d^3p\right|^2$$

$$\times\,\delta(K-k_1-k_2-k_3)\,\frac{d^3k_1d^3k_2d^3k_3}{\omega_1\omega_2\omega_3} = \frac{me^{16}(n^2-1)}{72\pi^3n^5}$$

$$\times\int \frac{m^2[1-(\mathbf{n}_1\mathbf{n}_2)]^2\omega_1\omega_2\delta(2m-E-\omega_1-\omega_2-\omega_3)}{(\sqrt{2m\omega_3+mE}+\sqrt{mE})^4\omega_3}\,d\Omega_1 d\Omega_2 d\omega_1 d\omega_2, \qquad (13)$$

where the summation over the index M comes from the averaging over the three states of parapositronium with the orbital angular momentum $l = 1$. In the expression (13) we have kept only terms in which the large factor $\ln(2m/E) \gg 1$ appears on integration over the energies of the photons. For the terms dropped in Eq. (13) the integral over the photon energies is not of an order larger than unity.

In Eq. (13) it is convenient to integrate first over the angles, using the fact that

$$d\Omega_1 d\Omega_2 = -8\pi^2 d(\cos(\mathbf{n}_1\mathbf{n}_2)). \qquad (14)$$

The integration over the angles removes the δ function, and the expression (13) takes the form

$$W = \frac{4me^{16}(n^2-1)}{9\pi n^5}\times\int_0^{m-E/2} d\omega_1$$

$$\times\int_{m-\omega_1-E/2}^{m-E/2} d\omega_2\,\frac{m^4(\omega_1+\omega_2-m+E/2)^2}{\omega_1^2\omega_2^2(\sqrt{2m(2m-\omega_1-\omega_2-E)+mE}+\sqrt{mE})^4}. \qquad (15)$$

The region of variation of the independent variables of integration ω_1 and ω_2 in Eq. (15) is limited by the laws of conservation of energy and momentum. Since two quanta can simultaneously have the energy $m - E/2$, the upper limit of the integration over ω_1 and ω_2 is $m - E/2$. If $\omega_1 < m - E/2$, the smallest value for ω_2 is given by the condition $\omega_3 = m - E/2$, i.e., it is $\omega_2 = m - \omega_1 - E/2$. The integral over ω_1 and ω_2 in Eq. (15) is equal to $\ln(2m/E) \gg 1$, if we drop a term of value about unity.

Thus the probability W of three-photon annihilation of parapositronium in the P state is

$$W = \frac{n^2-1}{9\pi n^5}\left[\ln\left(\frac{8n^2}{e^4}\right)\right](e^2)^8 m. \qquad (16)$$

In particular, if the principal quantum number n for parapositronium in the P state takes its smallest value n = 2, we have

$$W = \frac{1}{3\pi 2^5}\left[\ln\left(\frac{2^5}{e^4}\right)\right](e^2)^8 m = 0.3\cdot 10^3\ \text{sec}^{-1}$$

It follows from Eqs. (12) and (15) that in three-photon annihilation of parapositronium in the P state there is a preference for the production of two large quanta with combined energy almost equal to m, while the energy of the third quantum is of the order of E.

For parapositronium in the P state the most favored process is optical transition to the S state with probability of the order of 10^8 sec^{-1}, and subsequent two-photon annihilation.

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[1] I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **60**, 213 (1948).

[2] A. Ore and J. L. Powell, Phys. Rev. **75**, 1696 (1949).

[3] A. I. Alekseev, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1195 (1958), Soviet Phys. JETP **7**, 826 (1958).

[4] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).

[5] R. P. Feynman, Phys. Rev. **76**, 749 (1949).

[6] A. I. Alekseev, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 852 (1957), Soviet Phys. JETP **5**, 696 (1957).

[7] R. Karplus and A. Klein, Phys. Rev. **87**, 848 (1952).

[8] A. I. Alekseev, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1435 (1959), Soviet Phys. JETP **9**, 1020 (1959).

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