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ELECTRICAL, OPTICAL AND ELASTIC PROPERTIES OF DIAMOND TYPE CRYSTALS
IV. INTERACTION BETWEEN CONDUCTION ELECTRONS AND LATTICE VIBRATIONS

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We consider the interaction produced between conduction electrons and lattice vibrations by the polarization due to these vibrations. The possibility of existence of polaron states is investigated. The electron mobility is evaluated.

THE interaction of an electron with the lattice vibrations of an ionic crystal can be considered by taking into account the polarization caused by the vibrations even in the approximation of point-like ions. In the case of homeopolar crystals, there is no polarization in this approximation. This forces us to use either the Bloch potential, which does not completely represent the interaction,[1] or, as is commonly done lately, to use the deformation potential.[2 3] But, as shown in reference 4 (henceforth referred to as II,) consideration of the dipole moment of the atoms leads to a polarization due to lattice vibrations. This polarization should cause an interaction between the electron and the lattice vibrations. In this paper we develop on this basis a quantitative theory of the interactions. This theory is used to investigate the existence of polarons in the homeopolar crystals and to evaluate the mobility of the conduction electrons.

To simplify the calculation we do not take into the consideration the complexity of the isoenergetic surfaces in the conduction band,[5] but adopt a simple model of spherically symmetric surfaces with the center at $\mathbf{k} = 0$. This simplification is not a principal one, and can be got rid of by known methods.[6]

1. INERTIAL POLARIZATION OF THE CRYSTAL

We consider forced lattice vibrations at zero frequency, i.e., atomic displacement and dipole moments due to a constant external field. Let $\mathbf{D}'(\mathbf{r})$ be the intensity of the external electric field, $\mathbf{D}'_s(\mathbf{K})$ and $\mathbf{D}'(\mathbf{K})$ the Fourier coefficients in the expansion of $\mathbf{D}'(\mathbf{r}_s^l)$ in terms of reduced and free wave vectors:

$$\mathbf{D}'(\mathbf{r}_s^l) = \sum_{\mathbf{K}} \mathbf{D}'_s(\mathbf{K}) \exp(i\mathbf{K}\mathbf{r}_s^l), \tag{1}$$

$$\mathbf{D}'_s(\mathbf{K}) = \sum_{\mathbf{b}} \mathbf{D}'(\mathbf{K} + 2\pi\mathbf{b}) \exp(i2\pi\mathbf{b}\mathbf{r}_s), \tag{2}$$

where $\mathbf{r}_s^l$ are the coordinates of the s-th atom in the l-th cell, b the vector of the reciprocal lattice. Introducing the dimensionless Fourier coefficients

$$\mathbf{D}_s = (d^2/e)\mathbf{D}'_s, \quad \mathbf{D} = (d^2/e)\mathbf{D}'$$

(d is the lattice constant and e is the electron charge) we get the equations of vibrations, which we are interested in, from (4,II), in which we use a frequency $\Omega = 0$ and subtract $\mathbf{D}_1$ from the left hand side of the second equation:

$$L(\mathbf{v}_1 - \mathbf{v}_2) + F(\mathbf{w}_1 - \mathbf{w}_2) = i\mu\varkappa[(D/2)(\mathbf{s} \overset{+}{\times} \mathbf{v}_2) + (C/2)(\mathbf{s} \overset{+}{\times} \mathbf{w}_2)] - \mu^2\varkappa^2\varphi(\mathbf{v}_2; \mathbf{w}_2);$$

$$A\mathbf{w}_1 + F(\mathbf{v}_1 - \mathbf{v}_2) - (16\pi/3)\mathbf{w} + 16\pi(\mathbf{w}, \mathbf{s})\mathbf{s} - \mathbf{D}_1 = i\mu\varkappa \times [(C/2)(\mathbf{s} \overset{+}{\times} \mathbf{v}_2) + S(\mathbf{s} \overset{+}{\times} \mathbf{w}_2)] + \mu^2\varkappa^2\mathbf{f}(\mathbf{v}_2; \mathbf{w}_1; \mathbf{w}_2), \tag{3}$$

where $\mathbf{v}_S$, $\mathbf{w}_S$ are the dimensionless amplitudes of the displacements and of the dipole moments; L, F, D, C, and A are the parameters of the theory; $\mu \sim 10^{-5}$ is the small parameter of the expansion; κ is the modulus of the dimensionless wave vector and **s** is the unit vector in its direction; φ and **f** linear functions of the amplitudes; $\mathbf{w} = \mathbf{w}_1 + \mathbf{w}_2$; S = 20 and 11. We can get two other equations by permuting the indices 1, 2 and by interchanging i and −i. The symbol $\overset{+}{\times}$ denotes as in the previous articles of this series,

$$(\mathbf{s} \overset{+}{\times} \mathbf{v}) = \mathbf{i}(s_y v_z + s_z v_y) + \ldots.$$

To investigate the polaron states we must (see references 7 and 8) find the inertial part of the crystal polarization. To find the amplitudes $\mathbf{w}_S^{(i)}$ of the dipole moments of the inertial polarization, we must solve Eq. (3) for $\mathbf{v}_S$, and holding these values constant, we put $\mathbf{D}_S = 0$. This gives us equations for $\mathbf{w}_S^{(i)}$, from which we can find

$$\mathbf{w}^{(i)}\,\mathbf{s} = 2\mu^2\varkappa^2 (A + 64\pi/3)^{-1} [\gamma_1 (\mathbf{v}_1{}^0 \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s}$$
$$+(\gamma_2 - S^2/A)(\mathbf{w}_1{}^0 \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \mathbf{f}(\mathbf{v}_1{}^0; 0; 0)]\,\mathbf{s},$$
$$\gamma_3 (\mathbf{v}_1{}^0 \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \gamma_4 (\mathbf{w}_1{}^0 \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \boldsymbol{\varphi}(\mathbf{v}_1{}^0; \mathbf{w}_1{}^0) = 0,$$
$$\mathbf{w}_1^0 = \{\mathbf{D}_1 + \mathbf{D}_2 - [32\pi/(A + 64\pi/3)](\mathbf{D}_1 + \mathbf{D}_2, \mathbf{s})\,\mathbf{s}\}$$
$$\times [2(A - 32\pi/3)]^{-1}, \tag{4}$$

with the γ_i ($i = 1, 2, 3, 4$) expressed in terms of the parameters of the theory.

2. THE QUESTION OF THE EXISTENCE OF THE POLARON OF LARGE RADIUS

To investigate the polaron states we must vary the functional (see references 7 and 8)

$$J[\psi] = (\hbar^2/2m^*) \int |\nabla\psi|^2 d\tau - \tfrac{1}{2} \sum_{ls} \mathbf{P}_s^l \mathbf{D}'(\mathbf{r}_s^l) = T + U,$$
$$\mathbf{P}_s^l = ed \sum_{\mathbf{K}} \mathbf{w}_s^{(i)} \exp(i\mathbf{K}\mathbf{r}_s^l), \tag{5}$$

where m^* is the effective mass of the electron, and $\mathbf{D}'(\mathbf{r})$ is the induction produced by the electron in the state with wave function ψ.

Inserting (1) into (5) and summing over l, we get,

$$U = -\tfrac{1}{2} Ned \sum_{s\mathbf{K}} \mathbf{w}_s^{(i)}(\mathbf{K})\, \mathbf{D}'_s(-\mathbf{K}),$$

where N is the number of cells in the basic domain. In the case of states of large radius, $\mathbf{D}'(\mathbf{r})$ is a smooth function of the coordinates. In this case the Fourier coefficients are small for large values of the wave vector and we can discard in Eq. (2) the terms with $\mathbf{b} \neq 0$, thus getting

$$U = -\tfrac{1}{2} Ned \sum_{\mathbf{K}} \mathbf{w}^{(i)}(\mathbf{K})\, \mathbf{D}'(-\mathbf{K}).$$

We set, following Pekar,[7]

$$\psi(\mathbf{r}) = \gamma(1 + \alpha r)\, e^{-\alpha r}.$$

This leads to

$$J[\psi] = T_0\alpha^2 - U_0\alpha^3, \qquad T_0 = 3\hbar^2/14\,m^*,$$
$$U_0 = \frac{437\, e^2 d^2}{(1792\,\pi)^2} \left(\frac{n_0^2 - 1}{n_0^2}\right)^2 \int [\gamma_1 (\mathbf{v} \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s}$$
$$+ \mathbf{f}(\mathbf{v}; 0; 0) + (\gamma_2 - S^2/A)(\mathbf{s} \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s}]\, \mathbf{s}\, d\Omega, \tag{6}$$

where $d\Omega$ is the element of solid angle in $\mathbf{K}$-space, $\mathbf{v}$ is determined from the equation

$$\gamma_3 (\mathbf{v} \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \gamma_4 (\mathbf{s} \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \boldsymbol{\varphi}(\mathbf{v}; \mathbf{s}) = 0,$$

and n_0 is the index of refraction for waves of the largest wavelengths.

The results from above fit qualitatively with the results of Deĭgen and Pekar[3] and this indicates stability (or metastability) of the band state of the electron and also shows the absence of a polaron of large radius: $J[\psi]$ has a minimum at $\alpha = 0$.

3. INTERACTION OF THE ELECTRON WITH LATTICE VIBRATIONS AS A PERTURBATION

The mobility of the electrons, in the crystal under investigation, is large. This shows that the interaction of the electrons with the lattice vibrations is rather weak. This fact together with the absence of polarons permits one to consider the band electrons as the carriers and to consider their interaction with the lattice vibrations by perturbation-theory methods. The operator of this interaction is

$$\hat{V} = -\sum_{ls} \mathbf{P}_s^l\, \mathbf{D}'(\mathbf{r}_s^l), \tag{7}$$

where $\mathbf{D}'$ is the intensity of the electric field of the electron. We introduce normal coordinates $q^\alpha(\mathbf{K})$:

$$\mathbf{P}_s^l = ed \sum_{\alpha\mathbf{K}} q^\alpha(\mathbf{K})\, \mathbf{w}_{s\alpha}(\mathbf{K}) \exp(i\mathbf{K}\mathbf{r}_s^l), \tag{8}$$

where α is the number of the branch of the spectrum of vibration. Inserting (8) and (1) into (7) and summing over l, we get

$$\hat{V} = -edN \sum_{s\alpha\mathbf{K}} q^\alpha(\mathbf{K})\, \mathbf{w}_{s\alpha}(\mathbf{K})\, \mathbf{D}'_s(-\mathbf{K}), \tag{9}$$

$$\mathbf{D}'_s(\mathbf{K}) = \frac{4\pi i e}{V} \exp(-i\mathbf{K}\mathbf{r}) \sum_{\mathbf{b}} \frac{\mathbf{K} + 2\pi\mathbf{b}}{|\mathbf{K} + 2\pi\mathbf{b}|^2}$$
$$\times \exp[i\,2\pi\mathbf{b}(\mathbf{r}_s - \mathbf{r})], \tag{10}$$

where $\mathbf{r}$ are the coordinates of the electron and V is the volume of the basic domain.

The probability of transition per unit time due to perturbation (9) is

$$P_{\alpha'\mathbf{K}'}[\mathbf{p}, n_\alpha(\mathbf{K})]$$
$$= \frac{2\pi}{\hbar} |(\mathbf{p}, n_\alpha(\mathbf{K}) | \hat{V} | \mathbf{p}^0, n^0{}_\alpha(\mathbf{K}))|^2\, \delta(E - E^0), \tag{11}$$

where $\mathbf{p}^0$ and $\mathbf{p}$ are the quasi-momenta of the electron in the initial and final state; while $n_\alpha^0(\mathbf{K})$, $n_\alpha(\mathbf{K})$ are the corresponding numbers of phonons, with

$$n_\alpha(\mathbf{K}) = n_\alpha^0(\mathbf{K}) \pm \delta_{\alpha\mathbf{K},\,\alpha'\mathbf{K}'}\,;$$

E^0 and E are the energies of the initial and final state. The reciprocal of the lifetime of the initial state is

$$\frac{1}{\tau_0} = \sum_{\mathbf{p},\, n_\alpha,\, \alpha',\, \mathrm{K}'} P_{\alpha'\mathrm{K}'}\,[\mathbf{p}n_\alpha(\mathbf{K})]$$

$$= \frac{2\pi}{\hbar} \sum_{\mathbf{p}\alpha\mathbf{K}}{}' \sum_{ijl}^{1,2} \left| F_\alpha((-1)^i \mathbf{K}, \mathbf{p}, \mathbf{p}^0)\right|^2 \,|\,(n^0{}_\alpha((-1)^j \mathbf{K})$$

$$+ (-1)^l |\, q^\alpha((-1)^j \mathbf{K})|\, n^0_\alpha((-1)^j \mathbf{K}))\,|^2\, \delta(E - E^0), \quad (12)$$

where the prime means that the summation is to be taken over the half-space **K**, and

$$F_\alpha(\mathrm{K}, \mathbf{p}, \mathbf{p}^0) = -ed \sum_s \mathbf{w}_{s\alpha}(\mathrm{K})$$

$$\times \int \mathbf{D}'(-\mathrm{K}) \exp\left(i\,\frac{\mathbf{p}^0 - \mathbf{p}}{\hbar}\,\mathbf{r}\right) u^*_{\mathbf{p}} u_{\mathbf{p}^0}\, d\tau, \quad (13)$$

and $u_\mathbf{p}$ is the Bloch function normalized to unity for the elementary cell Δ.

The integral in (13) differs from zero only if

$$\mathrm{K} + (\mathbf{p}^0 - \mathbf{p})/\hbar = 2\pi\mathbf{b}.$$

The transitions for which $\mathbf{b} \neq 0$ are the Peierls "Umklapp" processes. In the simple band model,[5] in the case of semiconductors with not very high temperatures, these processes can be ignored. In this case, inserting (10) into (13), we get

$$F_\alpha(\mathrm{K}, \mathbf{p}, \mathbf{p}^0) = \frac{4\pi i e^2 d}{\Delta} \sum_s \mathbf{w}_{s\alpha}(\mathrm{K}) \sum_b \frac{\mathrm{K} + 2\pi\mathbf{b}}{|\mathrm{K} + 2\pi\mathbf{b}|^2} \exp(-i2\pi\mathbf{b}\mathbf{r}_s)$$

$$\times\, \delta_{\mathrm{K},(\mathbf{p}-\mathbf{p}^0)/\hbar} \int_{(\Delta)} \exp(i\,2\pi\mathbf{b}\mathbf{r})\, u^*_{\mathbf{p}} u_{\mathbf{p}^0}\, d\tau.$$

For most electrons the difference $\mathbf{p} - \mathbf{p}^0$ is small, therefore we can expand $u_\mathbf{p}$ in powers of this difference and retain only the linear factor. For $\mathbf{b} \neq 0$ the integral can be ignored, as it contains a rapidly oscillating term. When $\mathbf{b} = 0$ the term linear in $\mathbf{p} - \mathbf{p}^0$ vanishes because of the central symmetry of the crystals under investigation,[7] therefore the integral equals unity. So,

$$F_\alpha(\mathrm{K}, \mathbf{p}, \mathbf{p}^0) = \frac{4\pi i e^2 d}{\Delta} \frac{\mathbf{w}_\alpha(\mathrm{K})\,\mathbf{s}}{K} \delta_{\mathrm{K},(\mathbf{p}-\mathbf{p}^0)/\hbar}. \quad (14)$$

If we normalize the amplitude according to $I_{\alpha\alpha} = md^2$, (see reference 9, henceforth denoted by III), we get for the matrix elements in (12)

$$|\,(n_\alpha\,|\,q^\alpha\,|\,n_\alpha + 1)\,| = |\,(n_\alpha + 1\,|\,q^\alpha\,|\,n_\alpha)\,|$$

$$= \tfrac{1}{2}\,[\hbar(n_\alpha + 1)/Nmd^2\omega_\alpha]^{1/2}, \quad (15)$$

where m is the atomic mass and ω_α is the frequency of vibration of the α branch. Inserting (15) and (14) into (12), summing up over **K**, and passing from summation over **p** to integration, we get

$$\frac{1}{\tau^0} = \frac{e^4}{m\Delta\hbar^3} \sum_\alpha \int \frac{|\,\mathbf{w}_\alpha(\mathrm{K})\,\mathbf{s}\,|^2}{K^2\,\omega_\alpha(\mathrm{K})} \sum_{i=1,2} [n^0_\alpha(\mathrm{K}) + n^0_\alpha(-\mathrm{K}) + 2\delta_{i1}]$$

$$\times\, \delta\,[E(\mathbf{p}) - E(\mathbf{p}^0) - \hbar\omega_\alpha(\mathrm{K})(1 - 2\delta_{i1})]\, p^2 dp\, d\Omega,$$

$$\mathrm{K} = (\mathbf{p} - \mathbf{p}^0)/\hbar,\; E(\mathbf{p}) = p^2/2m^*. \quad (16)$$

4. MOBILITY OF ELECTRON

According to II, the frequency of light oscillations is a rapidly growing function of the wave vector, therefore the scattering of electrons by these vibrations is practically insignificant.

In the case of acoustic vibrations the phonon energy can be ignored over a wide range of temperatures.[10] We can also ignore unity as compared with n^0_α and put

$$n^0_\alpha(\mathrm{K}) = kT/\hbar\omega_\alpha(\mathrm{K}).$$

Using all these simplifications and integrating in (16) over p, we get for the reciprocal of the time of free path in the state $\mathbf{p}_0$, due to the scattering by the acoustical vibrations,

$$\frac{1}{\tau^0_{\mathrm{ac}}} = \frac{4e^4\,kTm^*\,p^0}{m\Delta\hbar^4} \sum_{\alpha\,(\mathrm{ac})} \int \frac{|\,\mathbf{w}_\alpha(\mathrm{K})\,\mathbf{s}\,|^2}{K^2\omega^2_\alpha(\mathrm{K})}\, d\Omega,$$

$$\mathrm{K} = (\mathbf{p} - \mathbf{p}^0)/\hbar, \qquad p = p^0. \quad (17)$$

From II, we get for the acoustical vibrations

$$\omega_\alpha(\mathrm{K}) = \Gamma\mu\varkappa\zeta_\alpha(\mathbf{s}), \qquad \Gamma = em^{-1/2}d^{-3/2},$$

where $\zeta_\alpha \equiv \Omega_2/\kappa$ is the solution of the characteristic equation of the system (12,II),

$$\mathbf{w}_\alpha(\mathrm{K})\,\mathbf{s} = \mu^2\varkappa^2\Psi(\mathbf{s};\ \mathbf{v}^0_{1\alpha}),$$

$$\Psi(\mathbf{s};\mathbf{v}) = 2\,[\gamma_1(\mathbf{v} \overset{+}{\times} \mathbf{s}) \overset{+}{\times} \mathbf{s} + \mathbf{f}(\mathbf{v};0;0)]\,\mathbf{s}/(A + 64\pi/3).$$

Inserting these results into (17) and averaging over the initial states of the electron, wè get

$$\frac{1}{\tau_{\mathrm{ac}}} = \frac{\sqrt{3}\,e^2\,d^2\,(m^*\,k)^{3/2}\,T^{3/2}}{\pi\hbar^4}$$

$$\times \sum_{\alpha\,(\mathrm{ac})} \int \zeta_\alpha^{-2}(\mathbf{s})\,|\,\Psi(\mathbf{s};\mathbf{v}^0_{1\alpha})\,|^2\, d\Omega\, d\Omega^0. \quad (18)$$

The integration is performed here over all directions $\mathbf{p}^0$ (element of solid angle $d\Omega^0$) and over all directions **p**.

Since the limiting frequency of the optical vibrations ω_{lim} in the crystals under investigation is large, we can consider only the range $T < \hbar\omega_{\mathrm{lim}}/k$ in calculating the scattering of the electrons on the optical vibrations. In this case the creation of phonons is practically impossible and the second term in the brackets of Eq. (16) vanishes. For all three optical branches we can put

$$\omega_\alpha(\mathrm{K}) = \omega_{\mathrm{lim}}, \qquad n^0_\alpha(\mathrm{K}) = [\exp(\hbar\omega_{\mathrm{lim}}/kT) - 1]^{-1}.$$

Inserting this into (16) and integrating over **p**, we get

$$\frac{1}{\tau^0_{\mathrm{opt}}} = \frac{2e^4m^*\,[(p^0)^2 + 2m^*\,\hbar\omega_{\mathrm{lim}}]^{1/2}}{m\Delta\hbar^3\omega_{\mathrm{lim}}[\exp(\hbar\omega_{\mathrm{lim}}/kT) - 1]} \sum_{\alpha\,(\mathrm{opt})} \int \frac{|\,\mathbf{w}_\alpha(\mathrm{K})\,\mathbf{s}\,|^2}{K^2}\, d\Omega,$$

$$\mathrm{K} = (\mathbf{p} - \mathbf{p}^0)/\hbar, \qquad p^2 = (p^0)^2 + 2m^*\hbar\omega_{\mathrm{lim}}. \quad (19)$$

From II we get for optical vibrations

$$\mathbf{w}_\alpha(\mathbf{K})\,\mathbf{s} = i\mu\varkappa\,[R_1/\varkappa_0^3(n_0^2-1)]\,(\mathbf{v}_{1\alpha}^0 \overset{+}{\times} \mathbf{s})\,\mathbf{s}, \qquad \varkappa_0 = n_0\omega_{\mathrm{lim}}/\Gamma,$$

where R_1 is a combination of theory parameters, which appears in (30a,II). Inserting this result into (19) and averaging over $\mathbf{p}^0$, we get approximately

$$\frac{1}{\tau_{\mathrm{opt}}} = \frac{e^4 r\,(m^*)^{3/2}\sqrt{2\hbar\omega_{\mathrm{lim}} + 3kT}}{16\pi^2\varkappa_0^4 n_0^2\, md\hbar^3\omega_{\mathrm{lim}}[\exp(\hbar\omega_{\mathrm{lim}}/kT) - 1]} \times \sum_{\alpha\,(\mathrm{opt})} \int |(\mathbf{v}_{1\alpha}^0 \overset{+}{\times} \mathbf{s})\,\mathbf{s}|^2\, d\Omega\, d\Omega^0, \tag{20}$$

where r is expressed in terms of the parameters of the theory according to (16,III).

The mobility of the electron is

$$u = \tau e/m^*, \qquad 1/\tau = 1/\tau_{\mathrm{ac}} + 1/\tau_{\mathrm{opt}}. \tag{21}$$

According to (18), if we consider the scattering by acoustic vibrations only, we get for the temperature dependence of mobility the well known $^3/_2$ law. But the scattering by optical vibration gives, according to (20), a deviation from this law.

5. NUMERICAL RESULTS. COMPARISON OF THEORY WITH EXPERIMENT.

We represent u from (21), (18), and (20) in the form

$$u = (m^0/m^*)^{5/2}\,(e/N_1 m^0)\,[T^{3/2} + N_2 f(T)]^{-1}, \tag{22}$$

where m^0 is the electron mass,

$$f(T) = (2\hbar\omega_{\mathrm{lim}} + 3kT)^{1/2}\,[\exp(\hbar\omega_{\mathrm{lim}}/kT) - 1]^{-1},$$

and the coefficients N_1 and N_2 are expressed in terms of a sum of integrals which appear in (18) and (20). These sums are calculated approximately in reference 11; the numerical values of the other parameters which enter in N_1 and N_2 are found in III. The numerical values of N_1 and N_2 are given in Table I.

TABLE I

	Diamond	Si	Ge
N_1	$0.5\cdot 10^9$	$0.5\cdot 10^9$	$0.5\cdot 10^9$
N_2	$3.6\cdot 10^{10}$	$1.3\cdot 10^{10}$	$0.93\cdot 10^{10}$

The presence of the term $N_2 f(T)$ in (22) causes a deviation from the $^3/_2$ law. For room temperatures and below, this deviation is most remarkable for germanium, because although N_2 increases in the direction from germanium to diamond, ω_{lim} also increases in this direction, and the latter is much more important. In the case of diamond and silicon, there was no deviation in the experiment from the $^3/_2$ law.[12,13] According to (22) these deviations should be insignificant for temperatures below 300° K. For germanium, according to the experimental data,[14] we can consider $u \sim T^{-1.65}$ in the temperature range 20 to 300° K. Table II lists the values of the ratio $T^{1.65}/F(T)$ [where $F(T) = T^{3/2} + N_2 f(T)$] for germanium in the range of 25 to 300° K. This ratio remains practically constant in the range 75 to 300° K (the experimental data are accurate to two significant figures; at this accuracy the ratio in the foregoing range is 1.9). For $T \lesssim 50$° K the ratio slightly decreases with decreasing temperature. This is caused by the fact that for such low temperatures the optical vibrations play no role. For comparison, Table II lists also the values of the ratio $T^{3/2}/F(T)$, [which reduce to $T^{1.65}/F(T)$ at 150° K]. It can be seen that $T^{3/2}/F(T)$ changes in the temperature range under investigation even in the second significant figure.

In Table III the experimental values[12,14,15] of the mobility at 300° K are given together with the values of m^*/m^0 calculated from the values of mobility, using (22). The results lie in the range of values which, according to the experimental data, m^*/m^0 takes on for different directions in the crystal.

TABLE III. Computed values of effective mass

	Diamond	Si	Ge
u, (cm²/v sec)	1800	1900	4000
m^*/m^0	0.69	0.65	0.44

Therefore, the value of the mobility, calculated from the polarization that accompanies the lattice vibrations, is quite satisfactory. This opposes Shockley's point of view,[10] according to which the electric fields connected with the lattice vibrations are insignificant, as far as the interaction of the electron with lattice vibration is concerned. According to the results obtained above, the effect caused by these fields (which is the same as that caused by polarization) is almost the dominating one.

TABLE II. Temperature dependence of mobility in germanium

T, °K	300	250	200	150	100	75	50	25
$T^{1.65}/F(T)$	1.92	1.93	1.94	1.94	1.94	1.89	1.80	1.62
$T^{3/2}/F(T)$	2.16	2.10	2.04	1.94	1.84	1.76	1.65	1.49

5. CONCLUSIONS

1. The interaction between the conduction electrons and lattice vibrations of a homeopolar crystal can be investigated on the basis of the polarization due to these vibrations.

2. In homeopolar crystals the band state of electrons is stable (or metastable), and polarons of large radius should be missing.

3. The method developed in this paper to calculate the interaction gives correct results for the values of mobility and its temperature dependence.

This gives grounds for believing that the interaction between the electron and the electric fields due to the lattice vibrations plays a significant role.

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