REMARKS ON THE OPTICAL MODEL OF THE NUCLEUS

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1. In the scattering of particles from a center of force with the potential $U(\mathbf{r})$, the scattering amplitude, $f(\vartheta)$, has the form

$$f(\vartheta) = -\frac{1}{4\pi}\int e^{-ik_0\mathbf{n}\mathbf{r}}\,U(\mathbf{r})\,\psi(\mathbf{r})\,d\tau, \qquad (1)$$

where k_0 is the wave number far away from the scatterer, and $\mathbf{n}$ is the unit vector in the direction of the scattering. In some cases the wave function is almost undistorted near the center of force, i.e., $\psi(\mathbf{r}) \sim e^{ik_0z}$. The substitution of this expression in (1) leads to the solution of the problem in the so-called Born approximation. In the interaction of fast particles with nuclei, $\psi(\mathbf{r})$ can often be written down in the approximation of geometrical optics: $\psi \sim e^{ikz}$, where k is the wave number inside the nucleus, which is connected with the potential U by the relation $k_0^2 - k^2 = U$. Substituting this expression in the basic formula (1), we obtain the usual optical-model solution. Both methods are therefore equivalent. However, for the investigation of any special problem, one may be more convenient than the other.

2. Let us assume that the scatterer consists of a number of identical and independent centers. If k is sufficiently close to k_0, we then obtain, after some simple transformations,

$$f(\vartheta) = f_0(\vartheta)\sum\psi(\Delta_m)\exp(-ik_0\mathbf{n}\Delta_m), \qquad (2)$$

where $f_0(\vartheta)$ is the scattering amplitude for a free single center, and Δ_m is the radius vector of the m-th center. Physically, formula (2) expresses the interference of the secondary waves from the various elementary scatterers.

We now introduce the density of elementary scatterers, $\rho(\Delta)$, and obtain

$$f(\vartheta) = f_0(\vartheta)\,F(\vartheta), \qquad (3)$$

where

$$F(\vartheta) = \int\rho(\Delta)\,\psi(\Delta)\exp(-ik_0\mathbf{n}\Delta)\,d\tau \qquad (4)$$

is interpreted as a generalized form factor. In Born approximation $\psi(\Delta) \sim e^{ik_0z}$, and $F(\vartheta)$ goes over into the usual form factor.[1,2]

3. It can be shown that the scattering amplitude obtained by the usual optical model considerations is

$$f_{\mathrm{opt}} = f_0(0)\int\exp(-ik_0\mathbf{n}\Delta)\rho(\Delta)\psi(\Delta)d\tau = f_0(0)\,F(\vartheta), \qquad (5)$$

this result is different from the more exact expression (3). The comparison of (3) and (5) gives

$$f(\vartheta) = f_{\mathrm{opt}}(\vartheta)\,f_0(\vartheta)\,/\,f_0(0). \qquad (6)$$

The differential scattering cross section is

$$\sigma(\vartheta) = \sigma_{\mathrm{opt}}(\vartheta)\,\sigma_0(\vartheta)\,/\,\sigma_0(0). \qquad (7)$$

Since $f(0) = f_{\mathrm{opt}}(0)$, the total interaction cross section is the same in both cases, according to the optical theorem. The inelastic scattering cross section is therefore

$$\sigma_{in} = \sigma_{in\,\mathrm{opt}} + \int\sigma_{\mathrm{opt}}(\vartheta)\{1 - \sigma_0(\vartheta)\,/\,\sigma_0(0)\}\,d\Omega. \qquad (8)$$

The relations (6) to (8) have the same form in the laboratory system as in the center of mass system. It is evident from the physical meaning of these results, that they are also valid in the relativistic case.

4. The available experimental data on the scattering of electrons from nuclei[3] were investigated with the help of the formulas (3) and (4), in which $f_0(\vartheta)$ was replaced by the amplitude for Coulomb scattering from a point charge. The quantity $\rho_e(\Delta)$ obtained in this way gives the distribution of the charge in the nucleus. For a comparison with the data on the scattering of π mesons and nucleons from nuclei it would be more accurate to consider not the charge distribution, but the density distribution, of the nucleons, $\rho_n(\Delta)$. For this purpose one must replace $f_0(\vartheta)$ in (3) by the scattering amplitude obtained from the experiments on the scattering of electrons from protons. The corresponding correction to the quantity $\overline{r^2}$ amounts to a few percent (5.5% for C_6^{12}). The experiments on the scattering of π mesons and nucleons,[4-7] on the other hand, are usually studied with the help of formula (5). The transition to the more exact formulas (6) and (7) also involves corrections of a few percent.

For this reason, the scattering of particles of different types[4] should be compared along the lines of the above mentioned considerations.

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ANGULAR DISTRIBUTION OF TRITONS FROM THE REACTION Li^7 *(α, t)* Be^8

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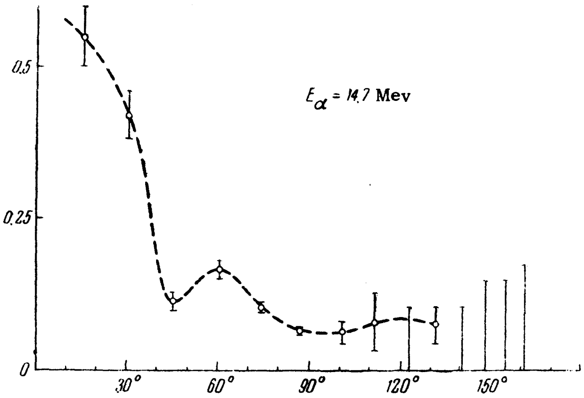
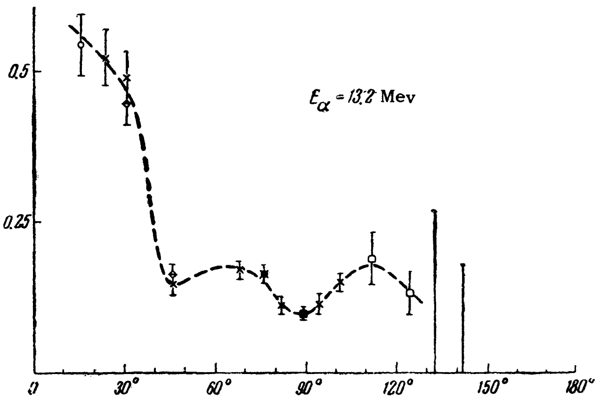
FOR the purpose of studying the reaction mechanism we used the nuclear emulsion method to investigate the angular distributions of tritons from the reaction $Li^7(\alpha, t)Be^8$ $(Q = -2.56$ Mev) with α particles accelerated in the cyclotron to energies of 8.34, 10.15, 11.5, 13.2, and 14.7 Mev. At all energies we obtained similar angular distributions. The curves in the figures show the dependence of the differential cross section (in relative units) on angle in the center of mass system with E_α = 13.2 and 14.7 Mev. The differently designated points were obtained in different experiments. At the larger angles we only evaluated the upper limit of the cross section.

The form of the angular distributions and its weak dependence on the energy of bombarding α particles show the important role of the direct interaction mechanism. Comparison with the Butler theory[1] showed that we can obtain satisfactory correspondence between theoretical and experimental curves with the angular momentum transferred to the nucleus at the time of collision $l = 1$ (the only possible value in line with the known values of nuclear spin[2] and the law of conservation of parity).

If, as is customary, we relate the isotropic part of the angular distributions, to compound nuclear processes, then we can see from the graph that the contribution of this process is large for $E_\alpha = 13.2$ Mev. We should note that in this case the full energy of motion in the c.m. system, if we include the energy spread of the α-particle beam and the energy loss in the target, corresponds to an energy ~ 16.9 Mev for the level of the compound nucleus B^{11}.





The absolute values of the differential cross section for a 16° angle (c.m.) are equal to $9.2^{+3.7}_{-1.85}$ mbn/sterad for E_α = 13.2 Mev and $9.4^{+4.0}_{-2.0}$ for E_α = 14.7 Mev.

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