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AZIMUTHAL SYMMETRIES IN CASCADE REACTIONS AND PARITY CONSERVATION

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Some angular azimuthal symmetry relations are obtained for cascades of reactions, of the type of triple scattering of protons; these relations follow from parity conservation in the reactions of the cascade. It is pointed out that experimental confirmation of the simplest of these relations, which is well known — the symmetry of the particles from the second scattering with respect to the plane of the first scattering — is not an exhaustive check on parity conservation. Experiments suggested here are a more thorough check of this law, and in some cases amount to a complete proof.

In the present paper the following azimuthal symmetry relations are obtained.

1. Suppose there is triple scattering by unpolarized targets (the incident beam for the first reaction is also unpolarized). If "parity is conserved"* in all the reactions, then the number of particles that undergo second scattering in the direction (ϑ, φ) and third scattering in the direction (ϑ', φ') is equal to the number of particles undergoing second scattering at the angles $(\vartheta, -\varphi)$ and further scattering (by a suitably placed target) at the angles $(\vartheta', -\varphi')$:

$$\sigma_{\vartheta,\varphi}(\vartheta', \varphi') = \sigma_{\vartheta,-\varphi}(\vartheta', -\varphi'). \tag{1}$$

The angles ϑ, φ are measured in the following coordinate system: the z axis is along the direction of the first scattering, and the y axis is perpendicular to the plane of the first reaction. For the angles ϑ', φ' the z axis is parallel to the direction (ϑ, φ), and the y axis is perpendicular to the plane of the second reaction (for details see Sec. 3).

The symmetry (1) is valid for any cascade of the type

$$a + b \rightarrow c + d, \quad (\vartheta_0, \varphi_0), \quad c + e \rightarrow f + g, \ (\vartheta, \varphi), \quad f + h \rightarrow i + j, \ (\vartheta', \varphi'), \tag{2}$$

if the particles a and the targets b, e, h are unpolarized. a, b, c, etc. can be nuclei or "elementary" particles (including γ-ray quanta) with arbitrary spins.

2. If "parity is conserved" in the reactions of the cascade

$$a + b \rightarrow a + b, \quad \begin{matrix} a + e_1 \rightarrow a + e_1, & (\vartheta_1, \varphi_1) \\ b + e_2 \rightarrow b + e_2, & (\vartheta_2, \varphi_2) \end{matrix} \tag{3}$$

with unpolarized beam a and targets b, e_1, e_2, the number of coincidences for counters of the twice-scattered particles a and b placed in the directions (ϑ_1, φ_1) and (ϑ_2, φ_2) must equal the number of coincidences with the counters placed in the directions $(\vartheta_1, -\varphi_1)$ and $(\vartheta_2, -\varphi_2)$:

$$\sigma(\vartheta_1, \varphi_1; \vartheta_2, \varphi_2) = \sigma(\vartheta_1, -\varphi_1; \vartheta_2 - \varphi_2). \tag{4}$$

Some of the reactions in the sequences (2) and (3) can be replaced by particle-decay reactions of the type $a \rightarrow c + d$. For example, Eq. (1) holds for the cascade $K^- + n \rightarrow \Xi^- + K^+$, $\Xi^- \rightarrow \Lambda + \pi$, $\Lambda \rightarrow p + \pi$, and Eq. (4) holds for the cascade $\pi^- + p \rightarrow \Sigma^- + K^+$, $\Sigma^- \rightarrow n + \pi$, $K \rightarrow \pi + \pi$.

The proposed experiments essentially serve to complete the set of experiments required to reconstruct the transition matrix (S matrix) of the reactions; this set is usually laid out on the assumption that parity is conserved. Of course, if any of these symmetries (or any already known) is violated, then parity is not conserved.

1. GENERAL FORMULAS

Formulas are known in the literature that express, in terms of the elements of the reaction matrix $R = S - 1$, the angular distribution of the products of the reaction of a polarized beam and polarized target, and also the polarization vector of these products (and if their spins are larger than $\frac{1}{2}$, also the polarization tensors).

Introducing instead of rectangular components of the spin vector the cyclic components,

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*We shall say for brevity that "parity is conserved" if: 1) in the real three-dimensional space there is no distinction between a right-handed and a left-handed screw; 2) all the particles taking part in the reactions have definite parities. "Parity is not conserved" if any one of these postulates does not hold.

$$\sigma_{-1} = (\sigma_x + i\sigma_y)/\sqrt{2}, \qquad \sigma_0 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\sigma_{+1} = -(\sigma_x - i\sigma_y)/\sqrt{2},$$

we can write these formulas for the case of scattering of a particle with spin $\frac{1}{2}$ by a spinless particle (cf. references 1 and 2) as follows:

$$I = \frac{1}{2}\left\{\mathrm{Sp}\, RR^+ + \sum_{\tau=-1}^{+1} P_\tau^{in}\, \mathrm{Sp}\, R\tilde{\sigma}_\tau R^+\right\} \tag{5}$$

is the angular distribution for polarized incident beam; and

$$P'_{\tau'} I = \frac{1}{2}\left\{\mathrm{Sp}\, \sigma_{\tau'} RR^+ + \sum_{\tau=-1}^{+1} P_\tau^{in}\, \mathrm{Sp}\, \sigma_{\tau'} R\tilde{\sigma}_\tau R^+\right\} \tag{6}$$

gives the cyclic components of the polarization vector of the scattered particles, for polarized incident beam (~ denotes transposition).

Let us write Eqs. (5) and (6) compactly in the form

$$\rho'(\vartheta, \varphi; q'\tau') = \sum_{q=0}^{1} \sum_{\tau=-q}^{+q} (q'\tau' | W(\vartheta, \varphi) | q\tau)\, \rho(q, \tau), \tag{7}$$

where $\rho(0, 0) = 1$ (one particle in the target and unit flux density of incident particles); $\rho(1, \tau) \equiv P_\tau^{in}$ are the cyclic components of the polarization vector of the beam; $\rho'(\vartheta, \varphi; 00) \equiv I(\vartheta, \varphi)$ gives the angular distribution; $\rho'(\vartheta, \varphi; 1\tau') \equiv P'_{\tau'}(\vartheta, \varphi) \times I(\vartheta, \varphi)$; and

$$(1\tau' | W(\vartheta, \varphi) | 1\tau) \equiv {}^1\!/_2\, \mathrm{Sp}\, \sigma_{\tau'} R(\vartheta, \varphi)\, \tilde{\sigma}_\tau R^+(\vartheta, \varphi). \tag{8}$$

For any reaction a + b → c + d (the spins are arbitrary) a formula of this kind also holds (cf. reference 3, and also references 4 – 6):

$$\rho'(\vartheta, \varphi; q_c\tau_c q_d\tau_d)$$

$$= \sum_{q_a\tau_a q_b\tau_b} (q_c\tau_c q_d\tau_d | W(\vartheta, \varphi) | q_a\tau_a q_b\tau_b)\, \rho(q_a\tau_a q_b\tau_b); \tag{9}$$

$$(q_c\tau_c q_d\tau_d | W(\vartheta, \varphi) | q_a\tau_a q_b\tau_b)$$

$$= [(2i_c + 1)(2i_d + 1)]^{1/2} [(2i_a + 1)(2i_b + 1)]^{-1/2}$$

$$\times \sum_{m_c\, m'_c, m_d, m'_d} (-1)^{i_c - m'_c} (i_c i_c m_c - m'_c | q_c\tau_c) (-1)^{i_d - m'_d}$$

$$\times (i_d i_d m_d - m'_d | q_d\tau_d) \sum_{m_a, m_a', m_b, m_b'} (m_c m_d | R(\vartheta, \varphi) | m_a m_b)$$

$$\times (m_c' m_d' | R(\vartheta, \varphi) | m_a' m_b')^* (-1)^{i_a - m'_a}$$

$$\times (i_a i_a m_a - m_a' | q_a\tau_a) (-1)^{i_b - m'_b} (i_b i_b m_b - m'_b | q_b\tau_b). \tag{10}$$

Letters i denote the spins of the particles, m their z components, q the rank of polarization tensors (for definition see reference 4). In this general case the Clebsch-Gordan coefficients (iim – m′ | qτ) appear instead of the matrices σ_τ of Eqs. (5), (6), and (8).

There is an analogous formula for the decay reaction a → c + d (cf. reference 4):

$$\rho'(\vartheta, \varphi; q_c\tau_c q_d\tau_d) = \sum_{q_a\nu_a} (q_c\tau_c q_d\tau_d | W(\vartheta, \varphi) | q_a\nu_a)\, \rho(q_a\nu_a).$$

If the total angular momentum is conserved, then for the elements of the transition matrix of the reaction a + b → c + d in the representation of momenta and spin components we have the well known expression [cf., e.g., reference 9, Eq. (4)]:

$$(\mathbf{p}_c n_c n_d | R | \mathbf{p}_a n_a n_b) = N \sum Y_{l'\mu'}(\vartheta_c, \varphi_c)\, C^{s'n'}_{i_c n_c i_d m_d} C^{JM}_{l'\mu's'n'}$$

$$\times (s'l' | R^J | sl)\, C^{JM}_{l\mu sn} C^{sn}_{i_a n_a i_b n_b} Y_{l\mu}(\vartheta_a, \varphi_a), \tag{11}$$

N is a normalizing factor. The sum is taken over all indices that occur twice. In this formula all the quantities are referred to a single coordinate system zyx.

In Eq. (10) and everywhere hereafter it is assumed that the spin components m_a and m_b of particles a and b are referred to a reference system A with the axis $z_a \parallel \mathbf{p}_a$. The y axis is chosen, for example, parallel to the polarization vector of a or b. The components m_c and m_d, on the other hand, are referred to the system C: $z_c \parallel \mathbf{p}_c$, $y_c \parallel [\mathbf{p}_a\mathbf{p}_c]$.[4] Going over from the old z axis of quantization to the new axes z_a and z_c, we get from Eq. (11) the following expression for the elements of R in the representation of these spin components:*

$$(m_c m_d | R(\vartheta, \varphi) | m_a m_b) = \frac{N}{4\pi} \sum_{s', l', J, l, s} C^{s'm_c + m_d}_{i_c m_c i_d m_d} C^{Jm_c + m_d}_{s'm_c + m_d\, l'0}$$

$$\times [(2l' + 1)(2l + 1)]^{1/2} (s'l' | R^J | sl)\, C^{Jm_a + m_b}_{sm_a + m_b\, l0} C^{sm_a + m_b}_{i_a m_a i_b m_b}$$

$$\times D^J_{mc + m_d, m_a + m_b}(-\pi, \vartheta, \pi - \varphi). \tag{12}$$

The function D is defined in reference 4. It follows from Eq. (12) such elements of R do not depend on $\mathbf{p}_c$ and $\mathbf{p}_a$ separately, but on the Euler angles $\{-\pi, \vartheta, \pi - \varphi\}$ of the rotation that makes the axes A coincide with the axes C; ϑ and φ are the spherical angles of the vector $\mathbf{p}_c$ relative to the axes A (cf. reference 4, Sec. 2). In addition, the dependence on φ is known:

$$(m_c m_d | R(\vartheta, \varphi) | m_a m_b)$$

$$= (m_c m_d | R(\vartheta, 0) | m_a m_b)\, e^{i(m_a + m_b)\varphi}. \tag{13}$$

2. THE PARITY SELECTION RULE

In Eq. (12) let us make the substitution

$$D^J_{m_c + m_d, m_a + m_b}(-\pi, \vartheta, \pi - \varphi) = (-1)^{m_a + m_b - m_c - m_d}$$

$$\times D^J_{-m_c - m_d, -m_a - m_b}(-\pi, \vartheta, \pi + \varphi),$$

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*Analogous calculations are carried out in Sec. 2 of reference 4 [derivation of Eq. (2.7) from Eq. (2.4)].

use the property

$$C^{c\gamma}_{a\alpha b\beta} = (-1)^{\pm c \mp a \mp b} C^{c-\gamma}_{a-\alpha\, b-\beta}$$

of the Clebsch-Gordan coefficients, and also use the well known expression of the law of conservation of parity (cf. footnote*): $\pi_c \pi_d (-1)^{l'} = \pi_a \pi_b (-1)^{l}$. We finally get

$$(m_c m_d | R(\vartheta, \varphi) | m_a m_b)$$
$$= \overset{*}{\pi}_c \overset{*}{\pi}_d \pi_a \pi_b (-1)^{i_c+i_d+i_a+i_b-m_c-m_d-m_a-m_b}$$
$$\times (-m_c - m_d | R(\vartheta, -\varphi) | -m_a - m_b). \qquad (14)$$

By the same method as in Sec. 1 of reference 5 we get the corresponding relation for the coefficients W:

$$(q_c\tau_c q_d\tau_d | W(\vartheta, \varphi) | q_a\tau_a q_b\tau_b) = (-1)^{q_c+q_d+q_a+q_b+\tau_c+\tau_d+\tau_a+\tau_b}$$
$$\times (q_c - \tau_c q_d - \tau_d | W(\vartheta, -\varphi) | q_a - \tau_a q_b - \tau_b). \qquad (15)$$

The components are referred to the directions of the respective relative momenta $\mathbf{p}_c$ and $\mathbf{p}_a$.

This "selection rule" was first obtained by Chou Kuang-Chao[7] in a different formulation. L. G. Zastavenko has pointed out that it is completely equivalent to the rule "$\nu_c + \nu_d + \nu_a + \nu_b$ even" obtained in reference 4. For decay reactions a → c + d we have an entirely analogous relation; we have only to remove the labels q_b and τ_b in Eq. (15).

Since the dependence of W on φ is known [cf. reference 5, Eq. (6)] both sides of Eq. (15) can be divided by $\exp\{i(\tau_a + \tau_b)\varphi\}$, i.e., without any loss of generality we can set $\varphi = 0$ in Eq. (15) [and Eq. (14), see Eq. (13)].

Taking the complex conjugates of both members of Eq. (10), and noting also that $(iim - m' | q\tau) = (iim' - m | q - \tau)$ and $(-1)^{-m'} = (-1)^{\tau - m}$, we get

$$(q_c\tau_c q_d\tau_d | W(\vartheta, \varphi) | q_a\tau_a q_b\tau_b)^* = (-1)^{\tau_c+\tau_d+\tau_a+\tau_b}$$
$$\times (q_c - \tau_c q_d - \tau_d | W(\vartheta, \varphi) | q_a - \tau_a q_b - \tau_b). \qquad (16)$$

From Eqs. (16) and (15) there follows one more formulation of the parity "selection rule"

$$(q_c\tau_c q_d\tau_d | W(\vartheta, 0) | q_a\tau_a q_b\tau_b)$$
$$= (-1)^{q_c+q_d+q_a+q_b} (q_c\tau_c q_d\tau_d | W(\vartheta, 0) | q_a\tau_a q_b\tau_b)^*, \qquad (17)$$

i.e., the coefficients $W(\vartheta, 0)$ are purely real if $q_c + q_d + q_a + q_b$ is even, and purely imaginary if this sum is odd.

3. DERIVATION OF THE AZIMUTHAL SYMMETRIES

Let us express the above selection rule in terms of properties of the angular distributions that are directly observable in experiments.

If a and b in the cascade (2) are unpolarized, then

$$\rho_{ab}(q_a\tau_a q_b\tau_b) = 1 \cdot \delta_{q_a,0} \delta_{q_b,0}$$

(if the flux density of the incident particles is unity and there is one target particle). The polarization tensors of particle c (we do not concern ourselves with particle d) are then independent of the azimuthal angle φ_0 and have the property (cf. reference 4)

$$\rho_0(\vartheta_0;\ q_c\tau_c 00) = (q_c\tau_c 00 | W_0(\vartheta_0, 0) | 0000)$$
$$= (-1)^{q_c+\tau_c} \rho_0(\vartheta_0;\ q_c - \tau_c 00). \qquad (18)$$

In order to find the polarization tensors of the product f of the second reaction of the cascade (2), we have to substitute in Eq. (9) the polarization tensors $\tilde{\rho}_0$ of particle c, referred to the axes $\tilde{C}$ belonging to the initial state of the second reaction (stationary relative to its center of mass). But the tensors (18) are referred to C_0, the axes belonging to the first reaction (stationary relative to the center of mass of the first reaction). To bring the axes C_0 into coincidence with the axes $\tilde{C}$ it is enough just to rotate the axis z_{c0}, parallel to the momentum $\mathbf{p}_c$, around y_{c0} by such an angle α that it becomes parallel to the direction $\mathbf{p}^{l}_c$ of this momentum in the laboratory system (we note that this direction is the same as its direction in the center-of-mass system of the second reaction, since the target e is at rest).

Therefore

$$\tilde{\rho}_0(\vartheta_0;\ q_c\tilde{\tau}_c) = \sum_{\tau_c} D^{q_c}_{\tilde{\tau}_c, \tau_c}(0, \alpha, 0)\, \rho_0(\vartheta_0;\ q_c\tau_c). \qquad (19)$$

We now insert in Eq. (19) instead of $\rho_0(\vartheta_0;\ q_c\tau_c)$ the quantities $(-1)^{q_c+\tau_c} \rho_0(\vartheta_0;\ q_c - \tau_c)$ which are equal to the former quantities by Eq. (18), change the sign of the summation index, $\tau_c \rightarrow -\tau_c$, and use the relation

$$D^{q}_{\tilde{\tau}, -\tau}(0, \alpha, 0) = (-1)^{-\tau-\tilde{\tau}} D^{q}_{-\tilde{\tau}, \tau}(0, \alpha, 0).$$

We find in this way that $\tilde{\rho}_0$ has the same property (18) as ρ_0. And in general rotations of the sets of axes belonging to the products of a reaction around the perpendicular to the plane of the reaction do not change the formulation (15) of the parity selection rule.

The relativistic spin rotation[9] is also applied around the perpendicular to the plane of the reaction. We can assume that α already includes the angle Ω of this rotation, and therefore the azimuthal symmetries obtained in what follows will be relativistic results.

Thus we have for the polarization tensors f

$$\rho(\vartheta, \varphi; q_f\tau_f 00) = \sum_{q_c\tau_c} (q_f\tau_f 00 | W(\vartheta, \varphi)| q_c\tau_c 00)\, \widetilde{\rho}_0(\vartheta_0; q_c\tau_c 00)\cdot 1,$$

if the target e is unpolarized (so that $\rho(q_e\tau_e) = 1\cdot\delta_{q_e,0}$). Since τ_c ranges from $-q_c$ to $+q_c$, we can replace the summation index τ_c by $-\tau_c$ (regrouping of the terms of the sum). Then applying Eq. (15) and for brevity not writing out the indices q and τ when they are equal to zero we have

$$\rho(\vartheta, \varphi; q_f\tau_f) = (-1)^{q_f+\tau_f} \sum_{q_c\tau_c} (q_f -\tau_f | W(\vartheta, -\varphi)| q_c\tau_c) \times \widetilde{\rho}_0(\vartheta_0; q_c\tau_c) = (-1)^{q_f+\tau_f} \rho(\vartheta, -\varphi; q_f -\tau_f).$$

Use has been made of the equation $(-1)^{q_c+\tau_c} \times (-1)^{q_c+\tau_c} = 1$. The angles ϑ, φ are measured from the respective axes $\widetilde{z}_c \parallel \mathbf{p}_c^l$; $\widetilde{y}_c \parallel \mathbf{p}_a \times \mathbf{p}_c^l$.

In particular, for $q_f = 0$ we get $\sigma(\vartheta, \varphi) = (\vartheta, -\varphi)$, i.e., symmetry of the angular distribution of the products of the second reaction with respect to the plane of the first reaction.

For the polarization tensors of the products of the third reaction we get in the same way, if the target h is unpolarized:

$$\rho'_{\vartheta,\varphi}(\vartheta', \varphi'; q_i\tau_i q_j\tau_j) = \sum_{q_f\tau_f} (q_i\tau_i q_j\tau_j | W'(\vartheta', \varphi')| q_f\tau_f) \times \sum_{q_c\tau_c} (q_f\tau_f | W(\vartheta, \varphi)| q_c\tau_c)(q_c\tau_c | W_0(\vartheta_0, 0)|)$$
$$= (-1)^{q_i+\tau_i+q_j+\tau_j} \rho'_{\vartheta,-\varphi}(\vartheta', -\varphi'; q_i -\tau_i q_j -\tau_j). \quad (20)$$

The indices ϑ, φ on ρ' mean that the particles f incident on h are those that emerged at the angles ϑ, φ with respect to the axes $\widetilde{C}$ of the second reaction. The angles ϑ', φ' are measured relative to the axis system $\widetilde{F}$: $\widetilde{z}_f$ is parallel to $\mathbf{p}_f^l$, the momentum of f in the laboratory system, and $\widetilde{y}_f \parallel \mathbf{p}_c^l \times \mathbf{p}_f^l$. In particular, for the angular distribution in the third reaction we get the symmetry (1). The extension to cascades with an arbitrary number of reactions is obvious (all the azimuthal angles φ in the right members of the equations are replaced by $-\varphi$).

In establishing the symmetry (1) we have used out of all the relations (15) only those of the form

$$(q_c\tau_c 00 | W(\vartheta, \varphi)| q_a\tau_a 00) = (-1)^{q_c+q_a+\tau_c+\tau_a} \times (q_c -\tau_c 00 | W(\vartheta, -\varphi)| q_a -\tau_a 00), \quad (21)$$

where q_c, q_a take the values 0, 1, ... $2i_c$ and 0, 1, ... $2i_a$, respectively.

Some of the other relations of Eq. (15) are used in proving the symmetry (4). In our arguments so far we have not been concerned at all with the second products of the reactions. But the common origin of the products of the reaction a + b → a + b makes the angular distributions of the particles a and b after second scattering correlated (regarding the correlation of the polarizations cf., e.g., reference 8). Namely, let us select among all the particles a after their first scattering only those that emerged together with particles b that have undergone subsequent scattering in the direction ϑ_2, φ_2. The angular distribution $\sigma(\vartheta_1, \varphi_1)$ from the second scattering of this subensemble of particles a will depend on ϑ_2 and φ_2 as parameters. The selection is made by the usual method of coincidences. For this joint angular distribution of the second scatterings we get by using Eq. (15):

$$\sigma(\vartheta_1, \varphi_1; \vartheta_2, \varphi_2) = \sum_{q_a\tau_a q_b\tau_b} (|W_1(\vartheta_1, \varphi_1)| q_a\tau_a)(|W_2(\vartheta_2, \varphi_2)| q_b\tau_b) \times (q_a\tau_a q_b\tau_b | W_0(\vartheta_0, 0)|) = \sigma(\vartheta_1, -\varphi_1; \vartheta_2, -\varphi_2).$$

By the same method we get for the cascade

$$a + b \to c + d, \quad c + e \to f + g, \ (\vartheta, \varphi)$$
$$f + h_1 \to i_1 + j_1, \ (\vartheta_1', \varphi_1')$$
$$g + h_2 \to i_2 + j_2, \ (\vartheta_2', \varphi_2')$$

with unpolarized a, b, e, h_1, h_2 the result

$$\sigma_{\vartheta,\varphi}(\vartheta_1', \varphi_1'; \vartheta_2', \varphi_2') = \sigma_{\vartheta,-\varphi}(\vartheta_1', -\varphi_1'; \vartheta_2', -\varphi_2') \quad (22)$$

(the comparison is made between the numbers of coincidences in the last reactions of the cascade).

Since the azimuthal angles φ do not change when transferred from the center-of-mass system to the laboratory system, we can suppose that the azimuthal angles in Eqs. (1), (4), and (22) are those of the momenta of the particles in the laboratory system. We can also insert the polar angles of these momenta (referred to the same axes $\widetilde{z}_c$ and $\widetilde{z}_f$) instead of ϑ and ϑ', which were defined as the polar angles of the momenta $\mathbf{p}_f$ and $\mathbf{p}_i$ in the respective center-of-mass systems (i.e., instead of the numbers ϑ and ϑ' we can insert the corresponding numbers ϑ_l and ϑ_l'). Thus ϑ, φ and ϑ', φ' in Eqs. (1), (4), and (22), for example, can be taken to be the spherical angles (measured in the axes $\widetilde{C}$ and $\widetilde{F}$, respectively) of the particle tracks directly observed in a chamber or emulsion.

The proof of the symmetries (1), (4), and (22) for cascades involving γ-ray quanta or for cascades including reactions of decay of one particle into two particles is obtained in just the same way (all formulas and relations needed for the proof in cases involving neutrinos and photons are contained in reference 6).

4. THE TEST OF PARITY CONSERVATION AND THE AZIMUTHAL SYMMETRIES (TRIPLE SCATTERING OF PROTONS BY SPINLESS TARGETS)

Let us now pose the inverse problem: how one can experimentally test "parity conservation" in a given reaction. Strictly speaking, one must check all of the relations (14).* The validity of only part of them could be either accidental or due to some other symmetry property of the interaction (examples below).

The summetry $\sigma(\vartheta, \varphi) = (\vartheta, -\varphi)$ is a consequence of only part of the set of relations (15):

$$(0000 \mid W(\vartheta, \varphi) \mid q_c\tau_c 00) = (-1)^{q_c+\tau_c} (0000 \mid W(\vartheta, -\varphi) \mid q_c - \tau_c 00). \tag{23}$$

Conversely, if this symmetry has been confirmed experimentally and it is suggested that the relations (15) hold for the coefficients W_0, then for the reaction $c + e \rightarrow f + g$ no more is verified than this part of the whole set of relations (15) (more exactly, $2i_c$ relations). The number of the relations (21) is considerably larger, and for this reason alone we can expect that a check of (1) is a more thoroughgoing test of parity conservation. It can be objected, however, that since not all of the relations (15) are independent (see note*) it is not excluded that Eq. (23) may contain just the same number of independent relations as Eq. (21). Our assertion that Eq. (1) gives a more complete check of parity conservation that just symmetry with respect to the plane of the first reaction will now be proved by a concrete analysis of a cascade of three very simple reactions of the type of the scattering of a particle of spin ½ by a spinless particle (for example, triple scattering of protons by helium targets).

The scattering matrix $(m'|R|m)$ (m' and m refer to different axes; cf. Sec. 1) has in this case just four elements:

$$\begin{pmatrix} (1/2 \mid R(\vartheta, 0) \mid 1/2) & (1/2 \mid R(\vartheta, 0) \mid -1/2) \\ (-1/2 \mid R(\vartheta, 0) \mid 1/2) & (-1/2 \mid R(\vartheta, 0) \mid -1/2) \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} Ae^{i\alpha} & Be^{i\beta} \\ Ce^{i\gamma} & De^{i\delta} \end{pmatrix}. \tag{24}$$

$A, B, C, D \geq 0$ and $0 \leq \alpha, \beta, \gamma, \delta \leq 2\pi$.

The relations (14) (with φ set equal to zero) reduce to just two complex equations, $a = d$, $b = -c$, if the product of the intrinsic parities of the particles does not change (as in an elastic reaction), or $a = -d$, $b = c$, if it does change sign.

According to Eqs. (5) — (8) of Sec. 1 we find the expressions for the coefficients $(q'\tau'|W(\vartheta, 0)|q\tau)$ that we shall need in terms of the elements (24):

$$(1-1 \mid W \mid 00) \equiv (-1 \mid W \mid) = [ACe^{i(-\alpha+\gamma)} + BDe^{i(-\beta+\delta)}]/\sqrt{2}, \tag{25}$$

$$(00 \mid W \mid 1-1) \equiv (\mid W \mid -1) = [ABe^{i(-\alpha+\beta)} + CDe^{i(-\gamma+\delta)}]/\sqrt{2}, \tag{26}$$

$$(1-1 \mid W \mid 1-1) \equiv (-1 \mid W \mid -1) = ADe^{i(-\alpha+\delta)}, \tag{27}$$

$$(1-1 \mid W \mid 1+1) \equiv (-1 \mid W \mid +1) = -BCe^{i(-\beta+\gamma)}, \tag{28}$$

$$(1-1 \mid W \mid 10) = [ACe^{i(-\alpha+\gamma)} - BDe^{i(-\beta+\delta)}]/\sqrt{2}. \tag{29}$$

Using Eq. (16), we can write the angular distribution of the second scattering in the form

$$\sigma(\vartheta, \varphi) = \sum_{q=0}^{1} (\mid W(\vartheta, 0) \mid q0)\rho_0(q0) + 2\mathrm{Re}(\mid W(\vartheta, 0) \mid 1-1)\rho_0(1-1)\cos\varphi + 2\mathrm{Im}(\mid W(\vartheta, 0) \mid 1-1)\rho_0(1-1)\sin\varphi. \tag{30}$$

To simplify the further analysis we assume that Eq. (15) holds for the coefficients W_0 of the first reaction and W' of the third, i.e., in this sense the first reaction is the polarizer and the third is the analyzer.* Then if for even a single value of $\varphi = \varphi_0 \neq 0$ one finds that $\sigma(\vartheta, \varphi_0) = \sigma(\vartheta, -\varphi_0)$, this means that $\mathrm{Im}(|W|1-1)\ \rho_0(1-1) = 0$, and thus $\mathrm{Re}(|W|1-1) = 0$, since $\rho_0(1-1) = (1-1|W_0|)$ is purely imaginary by hypothesis. As can be seen from Eq. (26), the equation $\mathrm{Re}(|W|1-1) = 0$ means the existence of a single restricting relation between the elements of the matrix R,† $\mathrm{Re}(a^*b + c^*d) = 0$, from which one cannot get the four relations $a = d$, $b = -c$. In fact, we can point out the following simple possible symmetry properties of the interaction that have the same character as the

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*All of these relations are independent. If all of the spins i_c, i_d, i_a, i_b, or two of them, are half-integral, there are in all $\frac{1}{2}(2i_a + 1)(2i_b + 1)(2i_c + 1)(2i_d + 1)$ complex relations (14), or twice that many real ones. If all the spins are integral, the number of real relations is smaller by one if $\pi_c^* \pi_d^* \pi_a \pi_b = (-1)^{-i_c-i_d+i_a+i_b}$, and is larger by one if this relation does not hold. The number of the relations (15) is larger than the number of the relations (14), and therefore not all of the set (15) are independent.

*More complicated experiments (with rotation of the spin between the successive scatterings) would clearly make it possible to test parity conservation without this simplification (and provide us with polarizers and analyzers).

†Or between the coefficients K, L, M, N in the expression $R = K + L(\sigma \cdot p' \times p) + M(\sigma \cdot p') + N(\sigma \cdot p)$ (cf. references 1, 2); that is, the symmetry $\sigma(\vartheta, \varphi) = \sigma(\vartheta, -\varphi)$ can exist even when R contains both scalar and pseudoscalar terms.

law of parity conservation and "imitate" it in the sense of making the quantity $\mathrm{Re}\,(a^*b + c^*d)$ vanish.

1) Apart from sign, the probability amplitude for the transition from the state with spin component $+\frac{1}{2}$ along the direction of the initial momentum to the state with spin component $+\frac{1}{2}$ along the final momentum is equal to the amplitude for the transition from $\frac{1}{2}$ to $-\frac{1}{2}$ (or, $a = \pm c$). The transition amplitudes $\frac{1}{2} \leftarrow -\frac{1}{2}$ and $-\frac{1}{2} \leftarrow -\frac{1}{2}$ are also equal (or $b = \mp d$).

In these terms the law of parity conservation is expressed as the equality (apart from sign) of the transition amplitudes $+\frac{1}{2} \leftarrow +\frac{1}{2}$ and $-\frac{1}{2} \leftarrow -\frac{1}{2}$ $(a = \pm d)$, and $+\frac{1}{2} \leftarrow -\frac{1}{2}$ and $-\frac{1}{2} \leftarrow +\frac{1}{2}$ $(b = \mp c)$.

2) $a = \pm id$, $b = \mp ic$ — the amplitudes for the corresponding transitions are equal, but unlike the case of parity conservation they differ in phase by $\pi/2$.

Doubts of parity conservation mean, in particular, that it is to be regarded as one of various possible symmetry properties (and on an equal footing with them). Only further experiments can show which of these properties exists in reality. One such experiment can be a test of the symmetry (1) in triple scattering. Let us write $\sigma_{\vartheta\varphi}(\vartheta', \varphi')$ in a form analogous to Eq. (30) [cf. Eq. (20)]:

$$\begin{aligned}\sigma_{\vartheta,\varphi}(\vartheta', \varphi') &= f(\vartheta', \vartheta, \vartheta_0) + 2\rho w_1' \{-\mathrm{Im}\,(-1 \mid W \mid) \cos\varphi' \\ &+ \mathrm{Re}\,(-1 \mid W \mid) \sin\varphi'\} + 2w'\rho_1\{-\mathrm{Im}\,(\mid W \mid -1) \cos\varphi \\ &+ \mathrm{Re}\,(\mid W \mid -1) \sin\varphi\} - 2w_1'\rho_1 \{\mathrm{Re}\,(-1 \mid W \mid -1) \\ &\times \cos(\varphi' + \varphi) + \mathrm{Im}\,(-1 \mid W \mid -1) \sin(\varphi' + \varphi) \\ &+ \mathrm{Re}\,(-1 \mid W \mid +1) \cos(\varphi' - \varphi) \\ &+ \mathrm{Im}\,(-1 \mid W \mid +1) \sin(\varphi' - \varphi)\}, \end{aligned} \tag{31}$$

where

$$\rho \equiv \rho_0(\vartheta_0, 00), \quad i\rho_1 \equiv \rho_0(\vartheta_0;\ 1-1),$$

$$w' \equiv (00 \mid W'(\vartheta', 0) \mid 00), \quad iw_1' \equiv (00 \mid W'(\vartheta', 0) \mid 1-1).$$

On verifying that Eq. (1) holds at four points (φ', φ), for example $(0, \pi/2)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$, $(\pi/2, -\pi/2)$, we get the results

$$\mathrm{Re}\,(-1 \mid W \mid) = \mathrm{Re}\,(\mid W \mid -1) = 0,$$

$$\mathrm{Im}\,(-1 \mid W \mid -1) = \mathrm{Im}\,(-1 \mid W \mid +1) = 0$$

or [cf. Eqs. (25) – (28)]

$$\begin{gathered} AD \sin(\alpha - \delta) = 0, \quad BC \sin(\beta - \gamma) = 0, \\ AC \cos(\alpha - \gamma) + BD \cos(\beta - \delta) = 0, \\ AB \cos(\alpha - \beta) + CD \cos(\gamma - \delta) = 0. \end{gathered} \tag{32}$$

In addition to the "parity conservation" solutions $a = d$, $b = -c$, and $a = -d$, $b = c$, Eq. (32) also has several other solutions (for example, solutions with one or two of the parameters A, B, C, D equal to zero).

Strictly, an additional solution is needed to eliminate these other solutions. They all make the real part of the coefficient (29) vanish (which does not follow at all from parity conservation). If after the first reaction the polarization vector is subjected to a rotation (by a magnetic field, for example) around $[\mathbf{p}_c \times \mathbf{p}_l]\mathbf{p}_c^l$ so that its component in the direction of the first scattering, $\rho_0(\vartheta_0;\ 1\ 0)$, becomes different from zero, then there is added to Eq. (31) a term

$$2\rho_0(\vartheta_0; 10)\, w_1' \{\mathrm{Im}\,(1-1 \mid W \mid 10) \cos\varphi' + \mathrm{Re}\,(1-1 \mid W \mid 10) \sin\varphi'\}.$$

If this destroys the symmetry (1), then $\mathrm{Re}\,(1-1 \mid W \mid 1\ 0) \neq 0$ and there remain the two "parity conservation" solutions. Since the intrinsic parities of the particles are not involved in Eq. (15), one cannot find out from the azimuthal symmetry whether or not the product of these particle symmetries changes.

We note that if parity conservation is established, this at the same time will have the meaning that for the elastic scattering of a particle of spin $\frac{1}{2}$ by a spinless particle there is invariance under time reversal.[2]

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