ONE-PARTICLE MECHANISM IN PHOTONUCLEAR REACTIONS

G. M. SHKLYAREVSKIĬ

Physico-Technical Institute, Academy of Sciences, U.S.S.R.

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The one-particle (direct) mechanism of photonuclear reactions at high energies is investigated on the basis of the shell model. It is shown that the ground state momentum distribution obtained from that model allows one to explain the forward shift of the maximum in the angular distribution of the photoprotons and leads to a correct magnitude of the reaction cross section.

1. INTRODUCTION

THE study of photonuclear reactions at large energies ($\gtrsim 20$ Mev) of the emitted nucleons is important because it gives a direct method of investigating the correlations of the nucleons and their momentum distribution in the nuclear ground state. Of equal interest is the question on the interaction of the electromagnetic field with the nucleus at high photon and reaction product energies. The characteristic peculiarity of photonuclear reactions lies in the fact that the photon carries a momentum which is several times smaller than the momentum of the emitted particle. One therefore has to provide for such a reaction mechanism which will secure the fulfilment of the conservation rules. The two nucleon ("quasi-deuteron") model which was proposed by Levinger[1] and further developed by Dedrik[2] and Gottfried[3] has been since fully confirmed experimentally[4,5] and is generally accepted as the model of photonuclear reactions at high energies. However, there exist also experimental data not contained within the framework of the two nucleon mechanism. These are: (i) the presence of photoprotons with energy almost equal to the maximum energy of the x-ray spectrum;[5,6] (ii) forward shift of the peak in the angular distribution into the range $20-50°$;[6,7] (iii) direct observations of the (γ, p) and (γ, n) reactions at high photon energies.[8]

In connection with the indicated facts the importance of the (one-nucleon) direct photoeffect has been repeatedly mentioned. In particular, this idea has been successfully applied to the region of the giant resonance.[9]

In the present paper the one-nucleon direct photoeffect will be investigated on the basis of the shell model in its simplest form. The conservation of momentum will be taken care of by considering that a bound nucleon possesses momentum (the internal momentum distribution). Furthermore the forward shift of the angular distribution will find an explanation analogous to the case of the atomic photoeffect.[10] The well known success of the shell model justifies the hope that its wave functions will sufficiently accurately give the ground state momentum distribution of the nucleons.

2. THE REACTION MECHANISM AND THE CROSS SECTIONS

The basic assumptions which we will make are the following:

1. The nucleus in its ground state can be described as a system of nucleons moving independently in a certain spherically symmetrical potential. The state of each nucleon is described by the orbital angular momentum l, its projection m, and the energy ϵ_l. The spin and magnetic moment of the nucleon will not be taken into account. This in the present case is equivalent to the neglect of the spin-orbit interaction. It should be mentioned that the disregard of the nucleon-nucleon interactions allows only to consider excited "hole" states of the daughter nucleus; the excitation energy of these states is included into the binding energy of the particular nucleon.

2. The final state interaction is given by the optical model of the nucleus.

3. The impulse approximation is applicable.

In a system of noninteracting particles only transitions involving the excitation of a single particle can be induced by a one-particle operator like the interaction operator of the electromagnetic field with a system of charged particles. If we separate mentally the particle that makes the transition into an excited state, then we can say that the remainder of the nucleus acts rigidly with respect to the electromagnetic disturbance, i.e., the electromagnetic field acts coherently on the nucleons that do not participate in the transition. It is further known that the demand of keeping the center of mass of the nucleus fixed leads to the appearance of the so-called "effective charge" for both protons and neutrons. This can be understood as due to the absorption of photons by the recoiling remainder of those nucleons which do not make a transition. Thus there are two possible photon absorption processes by nuclei within the framework of the one-particle mechanism: absorption of a photon by the nucleon making the transition and absorption of a photon by the remainder of the nucleons not participating in the transition. Clearly, a process of the first kind can occur only for the case of proton emission. The above can be expressed as the following assumption:

4. The interaction operator of the electromagnetic field with the nucleus can be written in the form

$$V = \mathbf{j}\cdot\left[\frac{e_1}{m_1}\, e^{i\mathbf{k}_\omega\cdot\mathbf{r}_1}\mathbf{p}_1 + \frac{Ze-e_1}{M-m_1}\, e^{i\mathbf{k}_\omega\cdot\mathbf{r}_2}\mathbf{p}_2\right], \qquad (1)$$

where e_1, m_1 and $\mathbf{r}_1$ — the charge, mass and radius vector respectively of the nucleon making the transition, Ze and M are the charge and mass of the nucleus, $\mathbf{r}_2$ is the radius vector of the center of mass of the daughter nucleus, and $\mathbf{j}$ is the polarization vector of the photon. The necessary normalization of the electromagnetic field can be achieved by adding the factor $(2\pi\hbar/c\omega)^{1/2}$ in (1).

We now introduce the center-of-mass system of the photon and the nucleus. In this system the nucleus moves with momentum $\mathbf{K} = -\mathbf{k}_\omega$ and the two parts of the nucleus move with respect to its center of mass with momenta χ_0 and $-\chi_0$. Then the wave function of the initial state can be written as

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = (2\pi)^{-3/2}\psi(\mathbf{r})\, e^{i\mathbf{K}\cdot\mathbf{R}}, \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,$$
$$\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/M.$$

After absorption of the photon, the wave function describing the motion of the photonucleon and the daughter nucleus is $\Phi(\mathbf{r}_1, \mathbf{r}_2) = (2\pi)^{-3/2}\varphi(\mathbf{r})$.

As is well known, the reaction cross section is given by

$$d\sigma = \frac{2\pi}{\hbar}|M|^2\rho(E), \qquad (2)$$

$$M = \int \Phi^+(\mathbf{r}_1\, \mathbf{r}_2)\, V\Psi(\mathbf{r}_1, \mathbf{r}_2)\, d\mathbf{r}_1 d\mathbf{r}_2, \quad \rho(E) = k^2\frac{dk}{dE}\, d\Omega, \qquad (2')$$

where $\mathbf{k}$ is the relative momentum in the final state.

We introduce

$$M' = (2\pi)^3\left[\frac{e_1}{m_1}\int \Phi^+(\mathbf{r}_1, \mathbf{r}_2)\, e^{i\mathbf{k}_\omega\cdot\mathbf{r}_1}\mathbf{j}\cdot\nabla_1\Psi(\mathbf{r}_1, \mathbf{r}_2)\, d\mathbf{r}_1 d\mathbf{r}_2 + \frac{Ze-e_1}{M-m_1}\int \Phi^+(\mathbf{r}_1, \mathbf{r}_2)\, e^{i\mathbf{k}_\omega\cdot\mathbf{r}_2}\mathbf{j}\cdot\nabla_2\Psi(\mathbf{r}_1, \mathbf{r}_2)\, d\mathbf{r}_1 d\mathbf{r}_2\right] \qquad (3)$$

and go over to momentum space.

We write

$$\Phi(\mathbf{r}_1, \mathbf{r}_2) = (2\pi)^{-3}\int C(\mathbf{k}_1, \mathbf{k}_2)\, e^{i(\mathbf{k}_1\cdot\mathbf{r}_1 + \mathbf{k}_2\cdot\mathbf{r}_2)} d\mathbf{k}_1 d\mathbf{k}_2$$

and insert this into (3). Going over to relative coordinates and recalling that the photon polarization is transverse and $\mathbf{K} = -\mathbf{k}_\omega$, we obtain

$$M' = \int d\mathbf{k}_1 C(\mathbf{k}_1, -\mathbf{k}_1)\left[\frac{e_1}{m_1}\int e^{-i(\mathbf{k}-\mathbf{k}''_\omega)\cdot\mathbf{r}}\,\mathbf{j}\cdot\nabla\psi(\mathbf{r})\, d\mathbf{r} - \frac{Ze-e_1}{m_2}\int e^{-i(\mathbf{k}+\mathbf{k}'_\omega)\cdot\mathbf{r}}\,\mathbf{j}\cdot\nabla\psi(\mathbf{r})\, d\mathbf{r}\right], \qquad (4)$$

where

$$\mathbf{k}'_\omega = m_1\mathbf{k}_\omega/M, \quad \mathbf{k}''_\omega = m_2\mathbf{k}_\omega/M,$$
$$\mathbf{k} = (m_2\mathbf{k}_1 - m_1\mathbf{k}_2)/M \qquad (m_2 = M - m_1).$$

Clearly $C(\mathbf{k}_1, -\mathbf{k}_1)$ is the amplitude of the momentum distribution of the nucleon which is making the transition. It can be written in the form

$$C(\mathbf{k}_1, -\mathbf{k}_1) \equiv C(\mathbf{k}_1) = (2\pi)^{-3/2}\int e^{-i\mathbf{k}_1\cdot\mathbf{r}}\varphi(\mathbf{r})\, d\mathbf{r}. \qquad (5)$$

Further, in the spirit of the impulse approximation and taking into account the kinematics of both kinds of processes, we put in (4) $\mathbf{k} = \mathbf{k}_{10}$, where $\mathbf{k}_{10}$ is the momentum of the emitted nucleon right after absorbing the photon: then both $\mathbf{k}_{10} - \mathbf{k}''_\omega$ and $\mathbf{k}_{10} + \mathbf{k}'_\omega$ represent that initial (internal) momentum which is necessary for the fulfilment of the momentum conservation law. Integrating (4) by parts we obtain the matrix element M' in the form

$$M' = (2\pi)^{3/2}\int d\mathbf{k}_1 C^+(\mathbf{k}_1)\left[\frac{e_1}{m_1}G(\mathbf{k}_{10} - \mathbf{k}''_\omega) - \frac{Ze-e_1}{m_2}G(\mathbf{k}_{10} + \mathbf{k}'_\omega)\right]\mathbf{j}\cdot\mathbf{k}_{10}, \qquad (6)$$

where

$$G(\mathbf{p}) = (2\pi)^{-3/2}\int e^{-i\mathbf{p}\cdot\mathbf{r}}\psi(\mathbf{r})\, d\mathbf{r} \qquad (7)$$

is the amplitude of the momentum distribution in the nuclear ground state.

3. COMPARISON WITH EXPERIMENT

For the comparison with experiment one has to make a choice of initial and final state. As is well known for light nuclei the oscillator potential gives good results when applied to the ground state. The solution of the Schrödinger equation in momentum space gives for the momentum distribution of a state with principal quantum number unity the following expression:

$$G(\mathbf{p}_0) = C \exp(-p_0^2/2\mu\hbar\omega_0)\, p_0^l Y_{lm}(\mathbf{p}_0/p_0), \qquad (8)$$

where

$$C = \hbar^{3/2}\sqrt{2/\Gamma(l+3/2)}\,(1/\mu\hbar\omega_0)^{(2l+3)/4},$$

$$p_0^2 = \hbar^2\chi_0^2 = \hbar^2(k_{10}^2 + k_\omega''^2 - 2k_{10}k_\omega''\cos\theta)$$

(absorption of type 1)

$$p_0^2 = \hbar^2\chi_0^2 = \hbar^2(k_{10}^2 + k_\omega'^2 + 2k_{10}k_\omega'\cos\theta)$$

(absorption of type 2)

θ is the angle between $\mathbf{k}_\omega$ and $\mathbf{k}_{10}$, and μ is the reduced mass. The only parameter in (8) — the characteristic oscillator frequency ω_0 — is determined by the condition that the rms radius of the nucleus agree with the experiment.[11]

The determination of $C(\mathbf{k}_1)$ turns out to be a difficult problem. In the considered energy region the known approximate methods are not applicable; the exact computation of $C(\mathbf{k}_1)$ requires the solution of an integral equation of complicated character. It should however be borne in mind that the choice of the form and the parameters of the interaction in the final state at large energies is rather problematical. Therefore an exact complicated calculation would actually not have a greater value than an approximate estimate based on simple concepts. This estimate is easy to perform with the assumption that the final state interaction can be described by an optical model. This provides in essence two effects: (1) The imaginary part of the potential leads to a decrease of the cross section of the direct photoeffect because of the appearance of cascades within the nucleus and (or) the excitation of compound nucleus type excited states; the latter receives also a contribution from the reflection from the barrier. (2) After absorption of the photon, the nucleon experiences elastic scattering due to the real part of the potential, which will lead to a certain smearing out of the angular distribution.

The first effect can be easily estimated from the mean free path of the nucleon within the nucleus, the second — from the experimental elastic scattering cross sections. At high energies the barrier has a minor influence. This way one can as a rough approximation assume a plane wave for the final state wave function: $\varphi(\mathbf{r}) = \exp(i\mathbf{k}\cdot\mathbf{r})$; then $C(\mathbf{k}_1) = (2\pi)^{3/2}\delta(\mathbf{k}-\mathbf{k}_1)$. In order to determine the momentum of the photonucleon within the nucleus, $\mathbf{k}_{10}$, one still needs to know the depth of the potential well. Taking for simplicity a square well we have $k_{10}^2 = 2m\hbar^{-2}(E_p + V - \epsilon_l)$ where V — the depth of the well; its value can be obtained from the analysis of the elastic proton (neutron) scattering in the framework of the optical model.[12]

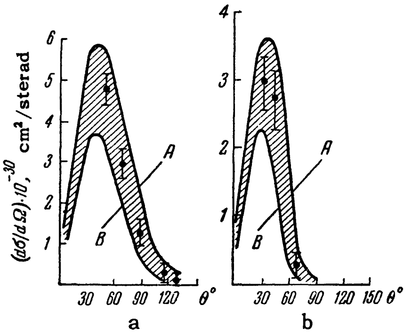
Assuming circular polarization and averaging over the initial states, we obtain for the cross section of the one-particle photoreaction on a particle with orbital angular momentum l, in the center-of-mass system (for the (γ, p) reaction)

$$d\sigma_l = \frac{e^2}{\hbar c}\frac{\pi^{1/2}}{(\mu\hbar\omega_0)^{3/2}}\frac{\hbar^2 p_{10}^2 p_1}{mE_\omega}$$
$$\times \sin^2\theta\, |A^l(1) - A^l(2)|^2 (1-F) R_l\, d\Omega, \qquad (9)$$

where $p_1 = \sqrt{2mE_p}$, E_p is the energy of the photonucleon in the center of mass system, E_ω is the photon energy;

$$A^l(1) = \exp(-p_0^2/2\mu\hbar\omega_0)\, p_0^l(\mu\hbar\omega_0)^{-l/2},$$
$$p_0^2 = p_{10}^2 + p_\omega''^2 - 2p_{10}p_\omega''\cos\theta;$$
$$A^l(2) = \frac{Z-1}{A-1}\exp(-p_0^2/2\mu\hbar\omega_0)p_0^l(\mu\hbar\omega_0)^{-l/2},$$
$$p_0^2 = p_{10}^2 + p_\omega'^2 + 2p_{10}p_\omega'\cos\theta.$$

F is a coefficient describing the absorption of the nucleons, $R_0 = 1$, $R_1 = 2/3$, and $R_2 = 4/15$. For (γ, n) reactions $A^l(1) \equiv 0$, and the factor $(Z-1)/(A-1)$ has to be replaced by $Z/(A-1)$.



Points: experimental data of Whitehead et al.,[6] curve A: calculated with a potential $V = V_{RW} - 5$ Mev, B: calculated with a potential $V = V_{RW} + 5$ Mev, where V_{RW} is the potential (set B) of Riesenfeld and Watson.[13] a: $E_p = 37$ Mev, $E_\omega = 45$ to 56 Mev; b: $E_p = 78$ Mev; $E_\omega = 90$ to 110 Mev.

We shall compare our results with the experiment for the C^{12} nucleus. We further choose those data where the (γ, pn) reaction cannot take place because of energetic reasons. From the figure one can see that one can obtain agreement with the experimental points by varying the depth of the potential well for the final state.

It should be mentioned that according to calculation the transitions from the $1s_{1/2}$ orbit in C^{12} do not contribute more than 10% to the total cross section. Therefore the assumption that the nucleons do not interact, which is particularly bad for nucleons in the $1s_{1/2}$ state, cannot essentially influence the results.

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