THE RESONANCE OF CHARGE CARRIERS PRODUCED BY AN ULTRASONIC WAVE

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The interaction of charge carriers situated in a magnetic field with the electric field produced by an ultrasonic wave is considered. The power absorbed in unit volume is calculated for charges with scalar and tensor effective masses. The curve for absorbed energy has peaks where the ultrasonic frequency is $n\omega_0$ (n being an integer and ω_0 the cyclotron frequency of the carrier), provided the relaxation time $\tau \gg 1/\omega_0$. Because the ultrasonic wavelength is about 10^5 times smaller than that of light at the same frequency, polarization effects should be absent in experiments on ultrasonic resonance; this prevents the use of cyclotron resonance in semiconductors with a high concentration of free electrons.

THE experimental successes of the ultrasonic technique have made the problem of the interaction of charge carriers with sound waves one of current importance.

The transport of electrons by ultrasonic waves has been studied by Gurevich[1] and Parmenter.[2] In the present communication we consider the effect of ultrasonic waves on the electron gas of a semiconductor situated in a magnetic field.

We shall solve the problem in the classical approximation for which the de Broglie electron wavelength must be smaller than the sound wavelength:

$$\lambda_e \ll \lambda_s. \qquad (1)$$

Using inequality (1) and the known values of the effective mass of the charge carriers and of the speed of sound c_0, one can derive a limiting temperature when the carriers are non-degenerate and a limiting concentration when they are degenerate; below these limits the formulae derived below are justified.

We use the method of the deformation potential[3,4] and write the interaction energy of an electron with an isotropically deformed crystal in the form

$$W = a(u_{xx} + u_{yy} + u_{zz}), \qquad (2)$$

where a is the deformation potential and u_{ii} are the components of the deformation tensor. Then the force acting on an electron is

$$\mathbf{F} = \varkappa a u_{0yy} \cos(\omega t - \varkappa y + \beta_1), \qquad (3)$$

if the deformation is caused by a monochromatic wave moving along the y axis. The magnetic field, H, is taken to be along the z axis. The radius vector of the electron $\mathbf{r}$ can be written as the sum of two components: the vector $\mathbf{R}$, referring the electron to the center of its orbit, and $\mathbf{r}_1$, the displacement of the center of the orbit in the time t from the instant of its last collision. The problem of finding $\mathbf{r}_1$ in the general case for the force (3) reduces to the solution of a non-linear differential equation. However, if

$$|\varkappa y_1| \ll 1, \qquad (4)$$

the additional phase can be neglected, leaving only κR_y in (3). The value of y_1 can be obtained by considering the limiting case of a uniform electric field: $\mathbf{E} = \mathbf{E}_0 \cos\omega t$. Solving the kinetic equation with this force, we obtain for the magnitude of the displacement of the center of the orbit under resonance conditions

$$|y_{10}| = \frac{eE_0\tau}{m\omega_0}\sqrt{\frac{1+\tau^2\omega_0^2}{1+4\tau^2\omega_0^2}}, \quad eE_0 = \varkappa a u_{0yy}. \qquad (5)$$

From (5) it is seen that if $au_{0yy}\tau\omega_0/mc_0^2 \ll 1$, inequality (4) is satisfied. Not having data for comparison with experiment, we will assume that the parameters lie within the limits necessary for the latter inequality to be satisfied. As an example, it can be shown that if $c_0 = 5 \times 10^5$ cm/sec, $m = m_e$, $\tau\omega_0 = 5$, then au_{0yy} should be smaller than 10^{-4} ev.

1. THE POWER ABSORBED IN UNIT VOLUME

Because the field created by an ultrasonic wave is strongly inhomogeneous, instead of using the kinetic equation, as was done, for example, in references 5 and 6, we resort to a direct average of the energy absorbed by the individual particles.

In the interval of time Δt between two successive collisions, let $u(\beta_1, \beta, \Delta t, v_\perp)$ be the energy change of the charged particle with velocity component $v_\perp$ perpendicular to the magnetic field, with β the initial phase of the motion and with β_1 the phase of the force. After the time Δt, having changed its energy by u, the particle moves from the energy level $\epsilon - u$ to the level ϵ. In unit time and unit volume the number of such transitions will be

$$\frac{f(\varepsilon - u)\exp(-\Delta t/\tau)}{\Delta t}\frac{d\Delta t}{\tau}\frac{d\beta}{2\pi}\frac{d\beta_1}{2\pi}$$

$$\approx \frac{1}{4\pi^2}\left(f(\varepsilon) - u\frac{\partial f}{\partial \varepsilon}\right)\frac{\exp(-\Delta t/\tau)}{\Delta t\tau}\,d\beta_1 d\beta, \tag{6}$$

where $f(\epsilon)$ is the distribution function normalized to the number of electrons in unit volume. Multiplying (6) by u and integrating with respect to β and β_1 from 0 to 2π, with respect to Δt from 0 to ∞, and over all the energy levels, we obtain the expression for the absorbed power U:

$$U = -\frac{1}{4\pi^2}$$

$$\times \int_0^\infty\int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} \frac{u^2}{\Delta t}\frac{\partial f}{\partial \varepsilon}\exp(-\Delta t/\tau)\frac{d\Delta t}{\tau}\,d\varepsilon_\perp\, d\varepsilon_\parallel\, d\beta_1 d\beta. \tag{7}$$

Here, $\epsilon_\perp$ is the orbital and $\epsilon_\parallel$ the residual part of the energy, and $\rho d\epsilon_\perp d\epsilon_\parallel$ is the number of states in the energy interval $d\epsilon_\perp d\epsilon_\parallel$.

According to (3), when (4) is satisfied:

$$u(\beta_1, \beta, \Delta t, v_\perp) = F_{0y}v_{0\perp}$$

$$\times \int_0^{\Delta t} \cos(\omega_0 t + \beta)\cos(\omega t - \varkappa R\sin(\omega_0 t + \beta) + \beta_1)\,dt. \tag{8}$$

Substituting (8) in (7) and completing the integrations with respect to β_1, β, and Δt, we obtain

$$U = -\frac{4F_{0y}^2}{m}$$

$$\times \sum_{n=1}^\infty n^2 \iint \frac{J_n^2(z)}{z^2}\frac{\tau(1 + \tau^2\omega_n^2 + \tau^2\omega^2)}{(1 + \tau^2\omega_n^2 - \tau^2\omega^2)^2 + 4\tau^2\omega^2}\frac{\partial f}{\partial \varepsilon}\rho\, d\varepsilon_\perp d\varepsilon_\parallel, \tag{9}$$

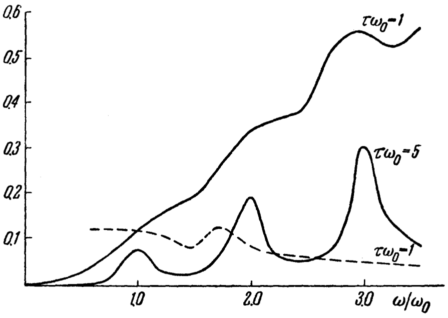
where $z = \kappa R$, $\omega_n = n\omega_0$, and J_n is the Bessel function of the n-th order.

As expected, in the limiting case in which $\kappa \to 0$, when $\tau = \text{const}$, $F_{0y} = \text{const}$, and classical statistics are used, (9) goes over to the well-known formula of the classical theory of cyclotron resonance (see reference 5).

To simplify calculations with Eq. (9), we will assume that $\tau = \text{const}$ and use classical statistics, After straightforward manipulation we find that

$$U = \frac{2a^2u_{0yy}^2\omega_0^2\tau N}{kT}\sum_{n=1}^\infty \frac{n^2(1 + \tau^2\omega_n^2 + \tau^2\omega^2)}{(1 + \tau^2\omega_n^2 - \tau^2\omega^2)^2 + 4\tau^2\omega^2}e^{-x}I_n(x). \tag{10}$$

where I_n is the Bessel function of argument $x = (\omega/\omega_0)^2 kT/mc_0^2$.



In the figure, curves constructed with the aid of (10) are given for $\tau\omega_0 = 5$ and 1, and $kT/mc_0^2 = 0.5$; they show the relative amount of absorbed energy: $UkT/2a^2u_{0yy}^2\omega_0^2\tau N$, where N is the concentration of electrons.

For $\tau\omega_0 = 5$, resonance peaks of the first, second and higher orders are clearly visible. For $\tau\omega_0 = 1$, resonance maxima are observed starting with the third order. The increasing height of the peaks with increasing order is explained by the linear dependence of the force (3) on the frequency of the ultrasonic wave for constant deformation amplitude. The broken line is the curve for constant force amplitude. The more complicated nature of the absorption curve compared with cyclotron resonance is connected with the presence in (9) of the additional variable $z = \kappa R$.

2. ULTRASONIC RESONANCE FOR AN ELLIPSOIDAL SURFACE OF CONSTANT ENERGY

We will consider a hexagonal crystal and limit ourselves to the simple case in which the energy minimum is in the center of the Brillouin zone. The calculation for a more complicated surface can be carried out when the practical need arises.

The z axis is taken parallel to the principal symmetry axis of the crystal. The x and y axes are chosen so that κ is directed along the y axis. The magnetic field is taken along a line lying in the xz plane. The equations of motion of an electron in this case are (see reference 5):

$$\dot p_x = \omega_t\cos\vartheta\, p_y,\quad \dot p_y = \omega_l\sin\vartheta\, p_z - \omega_t\cos\vartheta\, p_x,$$
$$\dot p_z = -\omega_t\sin\vartheta\, p_y,\qquad \omega_{t,l} = eH/cm_{t,l}, \tag{11}$$

The quantities m_t and m_l are determined from the formula for the energy, which, under the assumptions made above, must have the form

$$\varepsilon = (p_x^2 + p_y^2)/2m_t + p_z^2/2m_l. \tag{12}$$

Integrating (11), we find

$$x = R_x\cos(\omega_0 t + \beta) + V_{0x}t,$$
$$y = -R_y\sin(\omega_0 t + \beta),$$
$$z = R_z\cos(\omega_0 t + \beta) + V_{0z}t; \tag{13}$$

$$R_x = (2\varepsilon_\perp / m_t)^{1/2} (\omega_t / \omega_0^2) \cos \vartheta,$$
$$R_y = (2\varepsilon_\perp / m_t)^{1/2} (1 / \omega_0),$$
$$R_z = (2m_t \varepsilon_\perp)^{1/2} (\omega_t / \omega_0^2) \sin \vartheta,$$
$$\omega_0^2 = \omega_t^2 \cos^2 \vartheta + \omega_l \omega_t \sin^2 \vartheta. \quad (14)$$

We introduce the new variables

$$\xi_x = m_t^{-1/2} p_x, \quad \xi_y = m_t^{-1/2} p_y, \quad \xi_z = m_l^{-1/2} p_z. \quad (15)$$

In ξ phase space the vector $\xi_{||}$ describing the residual motion of the center of the electron orbit is perpendicular to $\xi_\perp$ describing the orbital motion. Therefore, introducing a cylindrical system of co-ordinates with axis along $\xi_{||}$, we obtain for the number of states in the energy interval $d\epsilon_\perp d\epsilon_{||}$,

$$\rho d\varepsilon_\perp d\varepsilon_{||} = 4\pi (m_t^2 m_l)^{1/2} \xi_\perp d\xi_\perp d\xi_{||} / (2\pi\hbar)^3. \quad (16)$$

The force (3) in the case under consideration is

$$F = \varkappa u_{0yy} a_1 \cos (\omega t - \varkappa R_y + \beta_1), \quad (17)$$

(here a_1 is one of the two unequal components of the deformation potential tensor: $a_{xx} = a_{yy} = a_1 \neq a_{zz} = a_2$)

$$u = \varkappa u_{0yy} a_1 \omega_0 R_y \int_0^{\Delta t} \cos (\omega_0 t + \beta) \cos (\omega t - \varkappa R_y + \beta_1)\, dt. \quad (18)$$

After substituting for u, the integration of (7) with respect to angular variables leads to a formula differing from (9) only in the replacement of a^2/m by a_1^2/m_t.

Using (16) and integrating further with respect to $\xi_\perp$ and $\xi_{||}$, under the same assumptions as were used to obtain (10), we arrive at an expression which can be obtained from (10) by substituting

$$\omega_0^2 = (eH / mc)^2 \rightarrow \omega_0^2 = \omega_t^2 \cos^2 \vartheta + \omega_t \omega_l \sin^2 \vartheta,$$
$$x \rightarrow x_1 = (\omega / \omega_0)^2 kT / m_t c_{0\perp}^2,$$

where $c_{0\perp}$ is the velocity of sound in a direction perpendicular to the symmetry axis.

If the ultrasonic wave is directed along the z axis and the magnetic field along the x axis, we find $U \sim a_2^2$. If τ is weakly dependent on the direction of the magnetic field, then measurements of the heights of the resonance maxima for various directions of $\boldsymbol{\kappa}$ and **H** enable one to evaluate the relative magnitudes of the components of the deformation potential tensor.

In conclusion I regard it as a pleasant duty to express my gratitude to Yu. E. Perlin for useful discussion and valuable observations.

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