PENETRATION OF AN ELECTROMAGNETIC FIELD INTO A PLASMA

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The depth of penetration of an electromagnetic field into a semi-infinite plasma in a magnetic field perpendicular to the plasma boundary is calculated.

SILIN[1] and Shafranov[2] have considered the penetration of an electromagnetic field into a uniform electron plasma which fills a half-space in the presence of an external magnetic field perpendicular to the plasma boundary, However, the ion motion, which is important at low frequencies, was not taken into account in references 1 and 2.

In the present work we determine the depth of penetration of circularly polarized electromagnetic waves for the case of perpendicular incidence with the ion motion taken into account. The plasma fills the half-space $z > 0$. The external magnetic field H_0 is perpendicular to the plasma boundary. It is assumed that the time dependence of all quantities can be written in the form $e^{-i\omega t}$ ($\mathrm{Im}\,\omega = -0$). It is also assumed that the frequency ω is so large that "close" collisions can be neglected. In the case of specular reflection of the electrons and ions from the plasma boundary, using the kinetic equations for the electron and ion distribution functions and Maxwell's equations we find (cf. reference 1) for the electric field in the plasma:

$$E^{(\pm)}(z) = E_x(z) \pm iE_y(z)$$

$$= -\frac{2cE^{(\pm)\prime}(0)}{\pi\omega}\int_0^\infty \frac{\cos(\omega nz/c)}{n^2 - \varepsilon^{(\pm)}(n)}\,dn, \qquad (1)$$

where

$$\varepsilon^{(\pm)}(n) = 1 + i\sqrt{\pi}\sum_{\alpha=e,\,i}\frac{\Omega_\alpha^2}{\omega^2} z_\alpha w(z_\alpha^{(\pm)}),$$

$$w(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right),$$

$$z_\alpha = (\sqrt{2}\beta_\alpha n)^{-1}, \qquad z_\alpha^{(\pm)} = (1 \pm \omega_H^\alpha/\omega)/\sqrt{2}\beta_\alpha n,$$

$$\beta_\alpha = v_T^\alpha/c = (T_\alpha/m_\alpha c^2)^{1/2}, \qquad \Omega_\alpha^2 = 4\pi e^2 n_0/m_\alpha,$$

$$\omega_H^\alpha = e_\alpha H_0/m_\alpha c.$$

When $\alpha = e$ the subscript refers to electrons, and when $\alpha = i$ to ions. The quantity T_α is the temperature of the gas of particles of type α, mass m_α and electric charge e_α ($e_i = e > 0$), n_0 is the equilibrium electron density. $E^{(+)}(z)$ is the electric field associated with the extraordinary wave, $E^{(-)}(z)$ is the field of the ordinary wave.

In the general case, when spatial dispersion due to the thermal motion of the electrons and ions must be taken into account, Eq. (1) for $E^{(+)}(z)$ is extremely complicated. However, it is possible to find an asymptotic expression for $E^{(+)}(z)$ for large values of z which is determined essentially by the equilibrium distribution functions for the charged particles at high velocities v ($v \gg v_T$, where v_T is the average thermal velocity).

We denote the depth of penetration of the magnetic field into the plasma by the complex quantity $\delta_H^{(\pm)}$, which has the dimensions of length

$$\delta_H^{\pm} = \int_0^\infty [H_x(z) \pm iH_y(z)]\,dz\,/\,[H_x(0) \pm iH_y(0)]. \qquad (2)$$

Similar expressions are used for the depth of penetration of the electric field (E), the electron current (j_e), and the ion current (j_i). Except for the factor $i\omega/c$ the quantity in (2) is the surface impedance of the plasma. Using Eq. (1), from Maxwell's equations we have

$$\delta_H^{(\pm)} = \frac{2c}{\pi\omega}\int_0^\infty \frac{dn}{n^2 - \varepsilon^{(\pm)}(n)}, \qquad (3)$$

$$\delta_E^{(\pm)} = -(c^2/\omega^2\varepsilon^{(\pm)}(0))/\delta_H^{(\pm)}, \qquad (4)$$

$$\delta_{j_\alpha}^{(\pm)} = -\frac{i\sqrt{\pi}}{2}\frac{c}{\omega\varepsilon^{(\pm)}(0)}\left\{\int_0^\infty \frac{z_\alpha w(z_\alpha^{(\pm)})}{n^2 - \varepsilon^{(\pm)}(n)}\,dn\right\}^{-1}, \qquad (5)$$

where

$$\varepsilon^{(\pm)}(0) = \varepsilon^{(\pm)}(n)\big|_{n=0}$$

$$= 1 - \Omega_e^2/\omega(\omega \mp |\omega_H^e|) - \Omega_i^2/\omega(\omega \pm \omega_H^i). \qquad (6)$$

We determine the quantities in Eqs. (3) — (5) for a number of limiting cases.

1. We assume that $|z_{e,i}^{(\pm)}| \gg 1$ for those values

of n which make an important contribution in the integrals (3), (5), (spatial dispersion is neglected). Then, from Eqs. (3) — (5), assuming that $\epsilon^{(\pm)}(n) \approx \epsilon^{(\pm)}(0)$, we have

$$\delta_H^{(\pm)} = \delta_E^{(\pm)} = \delta_{j_e}^{(\pm)} = \delta_{j_i}^{(\pm)} = \delta_0^{(\pm)} \equiv ic/\omega\sqrt{\varepsilon^{(\pm)}(0)}. \quad (7)$$

In particular, at gyromagnetic resonances

$$\delta_0^{(+)} = (c/\Omega_e)(1 - |\omega_H^e|/\omega)^{1/2}, \qquad \omega \approx |\omega_H^e|,$$

$$\delta_0^{(-)} = (c/\Omega_i)(1 - \omega_H^i/\omega)^{1/2}, \qquad \omega \approx \omega_H^i.$$

Since $n \sim |\epsilon^{(\pm)}(0)|^{1/2}$ in Eqs. (3) and (5), the condition $|z^{(\pm)}| \gg 1$ is satisfied if

$$c/v_T^e |\varepsilon^{(\pm)}(0)|^{1/2} \gg 1. \quad (8)$$

The inequality in (8) is not satisfied for very large values of $|\epsilon^{(\pm)}(0)|$.

Below we consider the cases opposite to that given in (8). In these cases the depth of penetration of the electromagnetic field into the plasma is determined essentially by the thermal motion of the electrons and ions.

2. We assume that the chief contribution in the integral in (3) is due to those values of n for which $|z_e^{(\pm)}| \ll 1$. Then

$$\varepsilon^{(\pm)}(n) \approx i\sqrt{\pi/2}\,\Omega_e^2/\omega^2\beta_e n.$$

Whence it follows that in (3) the values $n \sim (\Omega_e^2/\omega^2\beta_e)^{1/3}$, are important. The condition $|z_e^{(\pm)}| \ll 1$ is satisfied if

$$(c\omega/v_T^e\Omega_e)\,|1 \mp |\omega_H^e|/\omega|^{1/2} \ll 1. \quad (9)$$

The depth of penetration of the magnetic field in this case is given by

$$\delta_H^{(\pm)} = \delta_a \equiv \frac{2}{3}\left(1 + \frac{i}{\sqrt{3}}\right)\left(\sqrt{\frac{2}{\pi}}\frac{c^2 v_T^e}{\Omega_e^2\omega}\right)^{1/3}. \quad (10)$$

A comparison of Eqs. (10) and (7) shows that because of (9) $|\delta_a| \gg |\delta_0^{(\pm)}|$. Equation (10) determines the surface impedance of the plasma when there is an anomalous skin effect (cf. also reference 1).

Furthermore, if (9) is satisfied we have

$$\delta_E^{(\pm)} = \frac{9}{8}\left(1 - \frac{i}{\sqrt{3}}\right)\left(\sqrt{\frac{\pi}{2}}\frac{c\omega}{v_T^e\Omega_e}\right)^{1/3} \times\left(1 \mp \frac{|\omega_H^e|}{\omega}\right)\left(1 \pm \frac{\omega_H^i}{\omega}\right)\frac{c}{\Omega_e}, \quad (11)$$

$$\delta_{j_e}^{(\pm)} = \frac{27}{16}\left(1 \mp \frac{|\omega_H^e|}{\omega}\right)\left(1 \pm \frac{\omega_H^i}{\omega}\right)\delta_a, \quad (12)$$

$$\delta_{j_i}^{(\pm)} = (v_T^i/v_T^e)\,\delta_{j_e}^{(\pm)} \quad \text{for } |\delta_a(\omega \pm \omega_H^i)| \ll v_T^i, \quad (13)$$

$$\delta_{j_i}^{(\pm)} = \delta_E^{(\pm)} \quad \text{for } |\delta_a(\omega \pm \omega_H^i)| \gg v_T^i. \quad (14)$$

It follows from Eqs. (9), (10) and (11) that $|\delta_E^{(\pm)}| \ll |\delta_0^{(\pm)}| \ll |\delta_a|$. In the case of an electronic gyromagnetic resonance $\omega \approx |\omega_H^e|$ and the depths of penetration of the electron current, the ion current, and the electric field of the extraordinary wave are reduced markedly.

3. Suppose that $\omega \ll |\omega_H^e|$. We assume that $|z_e^{(\pm)}| \gg 1$ and $|z_i^{(\pm)}| \ll 1$. Then

$$\varepsilon^{(\pm)}(n) \approx \pm\Omega_i^2/\omega\omega_H^i + i\sqrt{\pi/2}\,\Omega_i^2/\omega^2\beta_i n. \quad (15)$$

If the second term on the right side of Eq. (15) is much smaller than the first $\beta_i n \gg \omega_H^i/\omega$; then the important values in the integrals in Eqs. (3) and (5) are $n \sim \Omega_i/\sqrt{\omega\omega_H^i}$. In this case

$$\delta_H^{(\pm)} = (ic/\Omega_i)(\pm\omega_H^i/\omega)^{1/2}, \quad (16)$$

$$\delta_E^{(\pm)} = \delta_{j_e}^{(\pm)} = \pm(ic/\Omega_i)(1 \pm \omega_H^i/\omega)(\pm\omega_H^i/\omega)^{1/2}, \quad (17)$$

$$\delta_{j_i}^{(\pm)} = \mp i\sqrt{\pi/2}\,(v_T^i/\omega)(1 \pm \omega_H^i/\omega)/\varkappa^{(\pm)}, \quad (18)$$

where

$$\varkappa^{(+)} = \ln\alpha, \quad \varkappa^{(-)} = -\ln\alpha - i\pi/2,$$

$$\alpha = \sqrt{\pi/2}\,c\omega_H^i\sqrt{\omega_H^i}/v_T^i\Omega_i\sqrt{\omega}.$$

The condition $\beta_i n \gg \omega_H^i/\omega$ is satisfied if $\alpha \ll 1$. The inequalities $|z_e^{(\pm)}| \gg 1$ and $|z_i^{(\pm)}| \ll 1$ are satisfied when

$$\alpha m_i v_T^i/m_e v_T^e \gg 1, \quad \alpha|1 \pm \omega/\omega_H^i| \ll 1. \quad (19)$$

In the case of a gyromagnetic resonance $\omega \approx \omega_H^i$

$$\delta_H^{(-)} = c/\Omega_i, \qquad \delta_E^{(-)} = (c/\Omega_i)(1 - \omega_H^i/\omega). \quad (20)$$

If, however $\alpha \gg 1$, when $\omega \ll |\omega_H^e|$, even in the absence of resonance $\omega \approx \omega_H^i$ we can use Eq. (7) for the ordinary wave. When $\omega \approx \omega_H^i$ we can neglect the first term in Eq. (15) as compared with the second (for the ordinary wave). In this case in the integrals in Eqs. (3) and (5) the values $n \sim (\Omega_i^2/\omega^2\beta_i)^{1/3}$ become important. The condition $|z_i^{(-)}| \ll 1$ is satisfied if, when $\alpha \gg 1$,

$$\alpha|\omega_H^i/\omega - 1|^{3/2} \ll 1. \quad (21)$$

In this case the depth of penetration is given by

$$\delta_H^{(-)} = \frac{2}{3}\left(1 + \frac{i}{\sqrt{3}}\right)\left(\sqrt{\frac{2}{\pi}}\frac{c^2 v_T^i}{\Omega_i^2\omega}\right)^{1/3},$$

$$\delta_E^{(-)} = \delta_{j_e}^{(-)} = \frac{c^2}{\Omega_i^2\delta_H^{(-)}}\left(1 - \frac{\omega_H^i}{\omega}\right),$$

$$\delta_{j_i}^{(-)} = -\frac{27}{16}\left(1 - \frac{\omega_H^i}{\omega}\right)\delta_H^{(-)}. \quad (22)$$

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[1] V. P. Silin, Tr. ФИАН (Trans. Physics Inst. Acad. Sci.) **6**, 199 (1955).

[2] B. D. Shafranov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1475 (1958), Soviet Phys. JETP **7**, 1019 (1958).

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