SCATTERING OF FAST PIONS BY DEUTERONS

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Shadow effects appearing in the diffraction scattering of fast π mesons were examined.

THE experimental data on the scattering of π mesons by protons show that collisions in the high energy region (> 1 Bev) have a diffraction character, that is, the scattering takes place primarily into small angles.[1-4] Belen'kiĭ carried out an analysis of the diffraction scattering of high energy π mesons by protons on the basis of a general theory, not connected with any concrete model of the nucleus.[5,6] Grishin and Saitov made an analogous examination of the diffraction scattering of high energy protons by protons.[7] Lately, Blokhintsev, Barashenkov, and Grishin used the results of an analysis of diffraction scattering of π's on protons to determine the mean radius of the proton and get knowledge of the nucleon structure.[8]

Data on the interactions of π's at high energies with neutrons can be got by studying π scattering by deuterons. For analyzing the diffraction scattering of π's by deuterons one must necessarily consider the shadow effects, whose existence was first demonstrated by Glauber, using the black sphere model of the nucleon.[9] In the present note the diffraction scattering of fast π's by deuterons is examined within the framework of the hypotheses of reference 5.

For high energy incident π's one can assume that the scattering takes place independently from the neutron and proton. In this case the amplitude for elastic π-d scattering can be written in the form

$$f(\vartheta) = \frac{ik}{2\pi}\int \exp(-i\varkappa\rho_d)$$

$$\times\{1 - S_n(\rho_n)S_p(\rho_p)\}\varphi^2{}_0(\mathrm{r})\,d\mathbf{r}d\rho_d, \qquad (1)$$

where S_n and S_p are the scattering functions of the on the neutron and proton, $\rho_d = \rho_n + \rho_p/2$ is the plane radius vector from the center of mass of the deuteron, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$ is the relative radius vector, $\kappa = \kappa\theta$ (θ = scattering angle), and $\varphi_0(r)$ is the wave function of the ground state of the deuteron.

The scattering cross section σ_s, the absorption cross section σ_a, and the total cross section σ_t are determined by the following formulas:

$$\sigma_s = \iint |1 - S_n(\rho_n)S_p(\rho_p)|^2\varphi_0^2(r)\,d\mathbf{r}d\rho_d,$$

$$\sigma_a = \iint\{1 - |S_n(\rho_n)S_p(\rho_p)|^2\}\varphi_0^2(r)\,d\mathbf{r}d\rho_d,$$

$$\sigma_t = 2\iint\{1 - \mathrm{Re}\,S_n(\rho_n)S_p(\rho_p)\}\varphi_0^2(s)\,d\mathbf{r}d\rho_d. \qquad (2)$$

The cross section σ_s describes the elastic scattering of a π by a deuteron as well as the scattering of a π which accompanies a deuteron aplitting. Analogously the cross section σ_a describes the absorption of a π by a deuteron as well as the absorption of a π accompanying the splitting of a deuteron.

We choose the π-n or π-p scattering functions to be of the form

$$S_n = 1 - \alpha_n\exp(-\rho_n^2/R_n^2),$$

$$S_p = 1 - \alpha_p\exp(-\rho_p^2/R_p^2), \qquad (3)$$

where the parameters α and R must be determined from experiment.

Choosing the wave function for the deuteron ground state in the form of a Gaussian function[10]

$$\varphi_0(r) = Ne^{-\gamma^2r^2}, \quad N^2 = 8/\pi^3R_d^3, \quad \gamma = \sqrt{2/\pi}\,R_d^{-1}, \qquad (4)$$

we get for the amplitude of the elastic π-d scattering the expression

$$f(\vartheta) = \frac{i}{2}\alpha kR^2\Big\{\exp\left[-\tfrac{1}{4}(1+r^2)\varkappa^2R^2\right]$$

$$+\tau\zeta^2\exp\left[-\tfrac{1}{4}(\zeta^2+r^2)\varkappa^2R^2\right]$$

$$-\frac{\alpha\tau\zeta^2}{1+\zeta^2+4r^2}\exp\left[-\frac{1}{4}\,\frac{\zeta^2+(1+\zeta^2)r^2}{1+\zeta^2+4r^2}\varkappa^2R^2\right]\Big\}, \qquad (5)$$

where

$$R = R_p, \quad r^2 = (\pi/16)(R_d/R)^2.$$

$$\tau = \alpha_n/\alpha_p, \quad \zeta = R_n/R_p, \quad \alpha = \alpha_p,$$

The total cross section for the π-d interaction is

$$\sigma_t = 2\pi\alpha R^2 \{1 + \tau\zeta^2 - \alpha\tau\zeta^2 / (1 + \zeta^2 + 4r^2)\}. \quad (6)$$

If the parameter r goes to infinity, then

$$\sigma_t = \sigma_t^{(p)} + \sigma_t^{(n)}, \qquad r \gg 1.$$

For finite r the cross section for the π-d interaction is less than the sum of the interaction cross sections for an individual neutron and proton (shadow effect). This effect becomes strongest for small values of the parameter r:

$$\sigma_t = 2\pi\alpha R^2 \left\{1 + \tau\zeta^2 - \frac{\alpha\tau\zeta^2}{1+\zeta^2}\right\}, \qquad r \ll 1.$$

The scattering and absorption π-d cross sections are:

$$\sigma_s = \frac{\pi}{2}\alpha^2 R^2 \left\{1 + \tau^2\zeta^2 + \frac{4\tau\zeta^2}{1+\zeta^2+4r^2} - \frac{4\alpha\tau\zeta^2}{1+2\zeta^2+8r^2} - \frac{4\alpha\tau^2\zeta^2}{2+\zeta^2+8r^2} + \frac{\alpha^2\tau^2\zeta^2}{1+\zeta^2+8r^2}\right\},$$

$$\sigma_a = 2\pi\alpha R^2 \left\{(1 + \tau^2\zeta^2)\left(1 - \frac{\alpha}{4}\right) - \frac{8\alpha\tau\zeta^2}{1+\zeta^2+4r^2} + \frac{\alpha^2\tau\zeta^2}{1+2\zeta^2+8r^2} + \frac{\alpha^2\tau^2\zeta^2}{2+\zeta^2+8r^2} - \frac{\alpha^3\tau^2\zeta^2}{4(1+\zeta^2+8r^2)}\right\}. \quad (7)$$

The angular distribution for π-d scattering (taking into account possible deutron splitting) is determined by the expression

$$\frac{d\sigma_s}{do} = \frac{k^2}{4\pi^2}\int \varphi_0^2(r) \left|\int \exp(-i\varkappa\rho_d)\{1 - S_n(\rho_n) S_p(\rho_p)\} d\rho_d\right|^2 d\mathbf{r}. \quad (8)$$

Using (3) and (4), we get the following formula for the angular distribution:

$$\frac{d\sigma_s}{do} = \frac{1}{4}\alpha^2 k^2 R^4 \Big\{\exp[-\tfrac{1}{2}\varkappa^2 R^2] + \tau^2\zeta^4 \exp[-\tfrac{1}{2}\zeta^2\varkappa^2 R^2] + 2\tau\zeta^2 \exp[-\tfrac{1}{4}(1+\zeta^2+4r^2)\varkappa^2 R^2] - \frac{2\alpha\tau\zeta^2}{1+\zeta^2+4r^2}\exp\left[-\frac{1}{4}\frac{1+2\zeta^2+8r^2}{1+\zeta^2+4r^2}\varkappa^2 R^2\right] - \frac{2\alpha\tau^2\zeta^4}{1+\zeta^2+4r^2}\exp\left[-\frac{1}{4}\frac{2+\zeta^2+8r^2}{1+\zeta^2+4r^2}\zeta^2\varkappa^2 R^2\right] + \frac{\alpha^2\tau^2\zeta^4}{(1+\zeta^2)(1+\zeta^2+8r^2)}\exp\left[-\frac{1}{2}\frac{\zeta^2}{1+\zeta^2}\varkappa^2 R^2\right]\Big\}. \quad (9)$$

The parameters α and R can be determined by analyzing the experimental data for π scattering by protons. Then, using the experimental data for π scattering by deuterons (σ_s and σ_t), one can determine the quantities τ and ζ, which characterize the difference between the neutron and corresponding proton parameters. The accuracy of the experimental data at present is, however, not sufficient for such an analysis.

Within the limits of present experimental accuracy one can put $\tau = \zeta = 1$. In that case formulas (6), (7), and (9) are simplified in an essential way:

$$\sigma_t = 2\pi\alpha R^2 \left\{2 - \frac{\alpha}{2(1+2r^2)}\right\},$$

$$\sigma_s = \frac{\pi}{2}\alpha^2 R^2 \left\{2 + \frac{2}{1+2r^2} - \frac{8\alpha}{3+8r^2} + \frac{\alpha^2}{2(1+4r^2)}\right\},$$

$$\sigma_a = 2\pi\alpha R^2 \left\{2\left(1 - \frac{\alpha}{4}\right) - \frac{\alpha}{1+2r^2} + \frac{2\alpha^2}{3+8r^2} - \frac{\alpha^3}{8(1+4r^2)}\right\},$$

$$\frac{d\sigma_s}{do} = \frac{1}{4}\alpha^2 k^2 R^4 \Big\{2\exp[-\tfrac{1}{2}\varkappa^2 R^2] + 2\exp[-\tfrac{1}{2}(1+2r^2)\varkappa^2 R^2] - \frac{2\alpha}{1+2r^2} \times \exp\left[-\frac{1}{8}\frac{3+8r^2}{1+2r^2}\varkappa^2 R^2\right] + \frac{\alpha^2}{4(1+4r^2)}\exp\left[-\frac{1}{4}\varkappa^2 R^2\right]\Big\}. \quad (10)$$

Using the experimental values for the full cross section and the diffraction scattering cross section for 1.4 Bev π's on a proton, $\sigma_t^{(p)} = 34.6 \pm 2.7$ millibarns and $\sigma_s^{(p)} = 7.5 \pm 1.0$ millibarns,[1] we get the parameter values $\alpha = 0.86$, $R = 0.8 \times 10^{-13}$ cm. Using these values and letting $R_d = 2.18 \times 10^{-13}$ cm, we get, for the difference $\Delta_t = \sigma_t - 2\sigma_t^{(p)}$, $\Delta_t = -3.8$ millibarns. The experimental value for this difference for $E_\pi = 1.5$ Bev is $\Delta = -3$ millibarns.[2] For the scattering and absorption cross sections we get, respectively, $\Delta_s = 0.7$ and $\Delta_a = -4.5$ millibarns.

We see that the integral π-d scattering cross section is equal, practically, to double the π-p elastic scattering cross section. The shadow effect actually appears in the π-d absorption cross section.

Figure 1 shows the angular distribution for π-p scattering. The points on the graph correspond to the experimental data for π^- scattering on protons at 1.44 Bev.[3] The cross section is given in millibarns/steradian.

Figure 2 shows the angular distribution for π-d

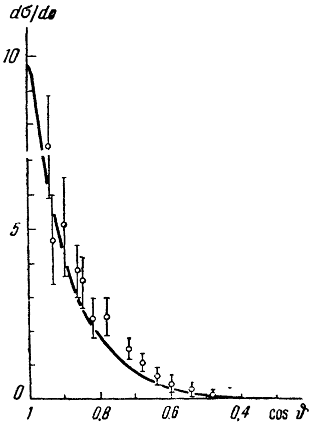

FIG. 1

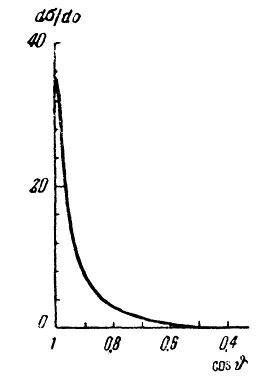

FIG. 2

scattering, calculated according to formula (10).

Obviously, the results obtained can also be applied to the analysis of the high energy scattering of nucleons on deuterons.

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