ABSORPTION OF μ^- MESONS BY POLARIZED NUCLEI. ANGULAR DISTRIBUTION OF THE NEUTRONS

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The angular distribution of neutrons emitted in the absorption of unpolarized μ^- mesons by polarized nuclei is calculated.

I. INTRODUCTION

In this article we consider the angular distribution of neutrons emitted in the absorption of unpolarized μ^- mesons by polarized nuclei. The shape of the angular distribution and, in particular, the asymmetry relative to the direction of polarization of the nucleus, depend on the variant of four-fermion interaction between μ mesons and nucleons, the degree of nonconservation of parity in μ^- capture, the degree of polarization of the nucleus and, of course, on specific properties of the nucleus considered. Results are calculated for nuclei in which the spin arises complete from a single proton outside filled proton subshells of the nucleus (this proton will be called the 'outer' proton in the future).

The process of nuclear absorption of μ^- mesons proceeds through an intermediate stage with formation of a mesic atom, and then through the reaction

$$\mu^- + P \to N + \nu, \quad (1)$$

in which the proton (P) absorbs the μ^- meson and turns into a neutron (N) with emission of a neutrino (ν). In considering this effect, it is especially important to take into account the hyperfine structure of the μ^- meson in the K orbit of the mesic atom, since, owing to the slowness of the process[1] ($\tau \sim 10^{-6}$ to 10^{-7} sec),* unpolarized μ^- mesons falling into the K orbit of the mesic atom can acquire a substantial polarization in the direction of polarization of the atom because of the hyperfine structure, and, in the process, they depolarize the nucleus appreciably.* This leads, in particular, to an anisotropy in the angular distribution of neutrons emitted in the absorption of μ^- mesons by protons in closed subshells of the atom, which has the form

$$dW_\mu(E_N, \theta) \sim 1 + P_\mu \alpha(E_N) \cos\theta,$$

where θ is the angle between the direction of emission of the neutron and that of the polarization of the nucleus, and $\alpha(E_N)$ is the asymmetry coefficient calculated by L. D. Blokhintsev and the author.[1]

It should be noted that, together with the anisotropic angular distribution of neutrons emitted by the nucleus immediately after absorption by the μ^- meson ('direct' process), there will be an isotropic background of neutrons which are decay products of the compound nucleus formed in reaction.[1] However, as shown in reference 1, selection of neutrons of energies $E_N \gtrsim 3$ Mev significantly decreases the background.

Thus, the resulting angular distribution of neutrons of given energy E_N emitted in the absorption of unpolarized μ^- mesons by unpolarized nuclei, will have the form

*The frequency corresponding to the energy of hyperfine splitting ΔE in the K orbit of the mesic atom is equal to $\omega = \Delta E/h \sim 10^{14} Z^3$ sec^{-1} so that $\tau\omega \gg 1$.

*It is easy to show that the degree of polarization acquired by the μ^- meson in the K orbit of a mesic atom is equal to $P_\mu = [4j/(2j+1)^2] P_i$, where P_i is the initial degree of polarization of the nucleus, and j is the spin of the nucleus. The resulting final polarization of the nucleus is equal to $P_f = [1 - 2/(2j+1)^2] P_i$. It should be noted that the hyperfine structure interaction can, in special cases, be important also for μ^- mesons in excited states of mesic atoms. However, if one tries to take this property into account, one meets a series of essential difficulties connected, in the main, with the absence of sufficiently reliable information about the initial stages of capture of μ^- mesons from the continuous spectrum into mesic atom orbits and about the way the μ^- meson makes transition from the excited states to the ground state.

$$dW(E_N, \theta) = dW_b(E_N) + dW_{\rm nuc}(E_N, \theta) + dW_\mu(E_N, \theta), \qquad (2)$$

where $dW_b(E_N)$ denotes the isotropic background, and $dW_{\rm nuc}(E_N, \theta)$ and $dW_\mu(E_N, \theta)$ denote the angular distributions of neutrons, emitted from the nucleus as a result of the direct process on the outer proton and on the protons of the closed subshells, respectively. Formulas for dW_μ were given in reference 1.* The present work is devoted to consideration of $dW_{\rm nuc}(E_N, \theta)$.

2. NEUTRON ANGULAR DISTRIBUTION

The Hamiltonian for the interaction between μ mesons and nucleons, the form of the wave functions of the particles, and the assumptions at the basis of the calculation coincide with those of Sec. 2 of reference 1. No difficulties arise in taking account of the hyperfine structure of the mesic atom. It is easy to show that, in the presence of hyperfine structure, the angular distribution of neutrons emitted in the absorption of unpolarized μ^- mesons by the outer proton of the nucleus, of spin j and projection j_z on the z axis, should be calculated from the formula

$$dW_{\rm nuc}(E_N, \theta;\ njlj_z) = {}^1\!/_2 \sum_{s_z} \sum_{I} \left[C^{Ij_z+s_z}_{jj_z ss_z}\right]^2 dW_{\rm nuc}(E_N, \theta;\ Ij_z + s_z), \qquad (3)$$

where $dW_p(E_N, \theta; I\, I_z)$ is the angular distribution of neutrons emitted in μ^- capture by the outer proton of the mesic atom, in state with spin I and projection I_z; s_z is the projection of the μ^--meson spin, which is averaged over; summation is carried out over $I = j \pm 1/2$; $s = 1/2$.

The general formula for the angular distribution of neutrons emitted in absorption of unpolarized μ^--mesons by the outer proton of a polarized nucleus with arbitrary spin j, is rather complex and is given in the appendix [Eq. (A.1)]. In the special case of absorption of unpolarized μ^- mesons by polarized nuclei, the spin of which comes from the proton in state $n\, s_{1/2}$ ($j = 1/2$, $l = 0$) Eq. (A.1) simplifies considerably, and has the form:†

$$dW_{\rm nuc}(E_N, \theta; n\,{}^1\!/_2\,0) = \{(f_{11} + 3f_{22} + \gamma_v^2 f_{pp} - 2\gamma_v \operatorname{Re} f_{2p})\, a_n(E_N) + {}^1\!/_2 P^i_{\rm nuc}[h_{11} + h_{22} + 2\operatorname{Re} h_{12} + \gamma_v^2 h_{pp} - 2\gamma_v \operatorname{Re}(h_{1p} + h_{2p})]\, b_n(E_N)\cos\theta\}\, dE_N d\Omega_N / 4\pi, \qquad (4)$$

*Equation (9) in reference 1. Summation over n, j, l in Eqs. (A.2)–(A.4) is now carried only over the closed proton subshells.

†The notation used in this article is given in the appendix.

where $P^i_N = 2\langle j_z\rangle$ is the initial (previous to formation of the mesic atom) degree of polarization of the nucleus, and the quantities $a_n(E_N)$ and $b_n(E_N)$ are function of the neutron energy, depending also on properties of the nucleus considered. We note, for comparison, that in the case of μ^- capture by a free polarized proton, the neutron angular distribution is described by a formula analogous to Eq. (4), with $\gamma_\nu = 0.053$ and $a_n(E_N)/b_n(E_N) = 1$.

If the interaction between μ^- mesons and nucleons is described by the theory of Feynman and Gell-Mann,[2] which assumes vector (v) and pseudovector (a) variants of interaction, then the formula for the neutron angular distribution again simplifies considerably. Neglecting renormalization of the pseudovector coupling constant g_a, we obtain from Eq. (A.1):

$$dW_{\rm nuc}(E_N, \theta;\ njl) = \frac{2}{\pi}|g|^2[A_{1njl}(E_N, \theta) - A_{2njl}(E_N, \theta)]\, dE_N d\Omega_N, \qquad (5a)$$

if $g_v = -g_a = g,\quad g'_v = g_v,\quad g'_a = g_a;$

$$dW_{\rm nuc}(E_N, \theta;\ njl) = \frac{2}{\pi}|g|^2[A_{1njl}(E_N, \theta) - B_{2njl}(E_N, \theta)]\, dE_N d\Omega_N, \qquad (5b)$$

if $g_v = g_a = g$, $g'_v = g_v$, $g'_a = g_a$. In case $g'_v = -g_v$, $g'_a = -g_a$, the sign preceding B_{2njl} should be changed from minus to plus in Eq. (5b). We remember that in the theory of Feynman and Gell-Mann for $|g_v| = |g_a|$, as noted in reference 1, the angular distribution of neutrons emitted in the absorption of polarized μ^--mesons by nuclei with zero spin is isotropic. Therefore, in Eq. (2), $dW_\mu = dW_\mu(E_N)$ and all of the dependence upon angle in the angular distribution of neutrons emitted in the absorption of unpolarized μ^- mesons by polarized nuclei will be contained in the term $dW_p(E_N, \theta)$, which arises from the absorption of the μ^--meson by the outer proton. Insofar as A_{1njl} and A_{2njl} are even functions of $\cos\theta$, and B_{2njl} an odd function of $\cos\theta$, then it is clear from Eqs. (5a) and (5b) that in the case $g_v = -g_a$, there is no asymmetry in the angular distribution of neutrons relative to the direction of the polarization of the nucleus, and, in the case $g_v = g_a$, the indicated asymmetry occurs. In the case of μ^- capture by an $ns_{1/2}$ proton, Eqs. (5a) and (5b) take on the especially simple form

$$dW_{\rm nuc}(E_N, \theta;\ n\,{}^1\!/_2\,0) = \frac{2}{\pi}|g|^2 a_n(E_N)\, dE_N d\Omega_N \qquad (6a)$$

and

$$dW_{\rm nuc}(E_N, \theta;\ n\,{}^1\!/_2\,0) = \frac{2}{\pi}|g|^2[a_n(E_N) + {}^1\!/_2 P^i_{\rm nuc}\, b_n(E_N)\cos\theta]\, dE_N d\Omega_N, \qquad (6b)$$

i.e., for $g_v = -g_a$, the neutron angular distribution is isotropic, and for $g_v = g_a$, anisotropic.

We note the following, relevant to the absorption of polarized μ^- mesons by unpolarized nuclei. We assume that the neutrino is longitudinal (e.g., for definiteness, $g_k' = g_k$) and do not consider the pseudoscalar variant. We use the notation

$$x = |g_t + g_a| / |g_s + g_v|,$$

$$\Delta = \arg(g_t + g_a) - \arg(g_s + g_v).$$

Then, from Eq. (11) of reference 1 for the coefficient of asymmetry in the angular distribution of neutrons arising in the direct process of absorption of polarized μ^- mesons by nuclei with zero spin, we obtain the following expression

$$\alpha_0 = [(-1 + x^2) B_0 + 2x \sin \Delta \cdot G_0] [(1 + 3x^2) A_0]^{-1}. \quad (7)$$

On the other hand, from the results of the present work and those of reference 1 it is easy to show that the coefficient of asymmetry in the angular distribution of neutrons from the direct process of absorption of polarized μ^- mesons by unpolarized nuclei, the spin of which comes from a single proton in state $ns_{1/2}$ above filled proton subshells, is given by

$$\alpha_{1/2} = [(-1 + x^2) B_0 + 2x \sin \Delta \cdot G_0 + (1 + x^2 + 2x \cos \Delta) b_n] [(1 + 3x^2)(A_0 + a_n)]^{-1}. \quad (8)$$

In both cases the angular distribution has the form $1 + P_\mu \alpha \cos\theta$ ($\alpha = \alpha_0$ or $\alpha_{1/2}$) where P_μ denotes the polarization of the μ^- meson in the K orbit of the mesic atom.* The quantities A_0, B_0, G_0, a_n and b_n entering into Eqs. (7) and (8) can be calculated theoretically (we note that the quantities A_0, B_0 and G_0 in Eqs. (7) and (8) are, in general, different, since they refer to different nuclei).

Thus, measurement of the coefficients of asymmetry in the angular distribution of neutrons emitted in the direct part of the absorption of polarized μ^- mesons by nuclei of the type considered, would make it possible to determine the ratio of moduli and relative phase of the Fermi and Gamow-Teller constants for the interaction of μ^- mesons with nucleons.

In particular, for the interaction proposed by Feynmann and Gell-Mann (where $x = 1$ and $\Delta = 0, \pi$), $\alpha_0 = 0$ and $\alpha_{1/2} \sim (1 \pm x)^2$. Therefore, measurement of the coefficient of asymmetry of the angular distribution of the direct neutrons from the absorption of polarized μ^- mesons by unpolarized nuclei, the spin of which comes from a proton in the $ns_{1/2}$ state, would make it possible to differentiate directly between the case $g_v = -g_a$ ($\alpha_{1/2} \neq 0$).

*In the derivation of Eq. (8), we have assumed that the only depolarizing factor for the μ^- meson, falling into the K orbit of the mesic atom, is the interaction giving rise to the hyperfine structure of the mesic atom. In this case, for a nucleus of spin $j = 1/2$, the μ^- meson falling into the K orbit of the mesic atom, depolarizes by 50%.

3. NUMERICAL ESTIMATES FOR μ^- CAPTURE IN F^{19}

The quantities entering into Eq. (4) were evaluated for μ^- capture by the outer proton in the F^{19} nucleus. This proton was assumed to be in the state $2s_{1/2}$ ($n = 2$, $j = 1/2$, $l = 0$). The assumptions in the calculation were the same as the assumptions noted in Sec. 3 of reference 1. The following parameters were used in the calculation: $R = 1.45 \times 10^{-13} A^{1/3}$ cm, $U_P = 44.7$ Mev, $U_N = 42$ Mev, $\zeta = 0$ and -0.15, where R is the radius of the square-well potential, U_P and U_N are the potential depths for protons and for neutrons, ζ is the ratio of the imaginary part of the complex potential for the neutron to its real part.

The angular distribution, integrated over energy, for neutrons emitted in the absorption of unpolarized F^{19} nucleus, can be represented to a good accuracy as

$$dW_{\text{nuc}}(\theta) = \text{const}\,(1 + P^i_{\text{nuc}}\, \beta \alpha_H \cos\theta)\, d\Omega_N, \quad (9)$$

where α_H is the asymmetry coefficient for μ^- capture in mesic hydrogen, calculated including the hyperfine structure of the mesic atom [Eq. (A.16) of the appendix], and $\beta = 0.52$ for $\zeta = 0$, $\beta = 0.76$ for $\zeta = -0.15$. Eq. (9) also describes the angular distribution of the neutrons in the absorption of polarized μ^- mesons by the outer proton in unpolarized F^{19}, if P^i_N is replaced by $2P_\mu$, where P_μ is the polarization of the μ^- meson in the K orbit of the mesic atom. In so far as $2P_\mu$ is a quantity of order[3] ~ 0.2, and $0 \leq |\alpha_H| \leq 1$, then for $\beta \sim 0.7$ we find the upper limit of the quantity $|2P_\mu \beta \alpha_H|$ to be of the ~ 0.14. From the calculations given in reference 1, one can expect the maximum asymmetry in the angular distribution of direct neutrons from the absorption of polarized μ^- mesons in unpolarized F^{19} to be of the order of $\sim 5\%$.

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APPENDIX

The probability of emission of a neutron in the direct process Eq. (1) with kinetic energy between

E_N and $E_N + dE_N$ into the solid angle $d\Omega_N$ at given angle θ to the direction of polarization of the nucleus, in the absorption of a μ^- meson by an outer proton in subshell (n, j, l) is of the form:

$$dW_{\text{nuc}}(E_N, \theta; njl) = \{[f_{11}A_1 + f_{22}(3A_1 - 2A_2) + \gamma_\nu^2 f_{pp}A_1 + 2\operatorname{Re} f_{12}\cdot A_2 - 2\gamma_\nu \operatorname{Re} f_{1p}\cdot A_3 - 2\gamma_\nu \operatorname{Re} f_{2p}\cdot(A_1 - A_2 + A_3)]$$
$$+ [h_{11}B_1 - h_{22}(B_1 + 2B_2) + \gamma_\nu^2 h_{pp}B_1 - 2\operatorname{Re} h_{12}\cdot B_2 + 2\operatorname{Im} h_{12}\cdot B_3$$
$$+ \gamma_\nu \cdot 2\operatorname{Re} h_{1p}\cdot B_2 - 2\gamma_\nu \operatorname{Re} h_{2p}\cdot B_1 - 2\gamma_\nu \operatorname{Im} h_{2p}\cdot B_3]\}\, dE_N d\Omega_N / 4\pi. \qquad \text{(A.1)}$$

where the following notation has been employed

$$A_m = A_{mnjl}(E_N, \theta) = \sum_{(k=0,1,2,\ldots)} a^{2k}_{mnjl}(E_N)\, f^{2k} P_{2k}(\cos\theta); \qquad \text{(A.2)}$$

$$B_m = B_{mnjl}(E_N, \theta) = \sum_{(k=0,1,2,\ldots)} b^{2k+1}_{mnjl}(E_N)\, f^{2k+1} P_{2k+1}(\cos\theta) \quad (m = 1, 2, 3); \qquad \text{(A.3)}$$

$$a^k_{1\,njl}(E_N) = [1 - k(k+1)/(2j+1)^2]\, W(lskj;\ jl) \sum_{LL'\Lambda} (-)^{(L+L')/2} (2\Lambda + 1)\, W(L\Lambda kl;\ lL') \operatorname{Re} F^k_{njlL\Lambda L'\Lambda}(E_N); \qquad \text{(A.4)}$$

$$a^k_{2\,njl}(E_N) = 3 \sum_{LL'\Lambda f} (-)^{(L+L')/2+1} (2\Lambda + 1)(2f+1)\, W(L\Lambda kl;\ lL')\, V(jkf)\, X(jfj;\ lkl;\ s1s) \operatorname{Re} F^k_{njlL\Lambda L'\Lambda}(E_N); \qquad \text{(A.5)}$$

$$a^k_{3\,njl}(E_N) = 3 \sum_{LL'\Lambda\Lambda'\tau fg} (-)^{(L+\Lambda+L'+\Lambda')/2} (2\Lambda + 1)(2\Lambda' + 1)(2f+1)$$
$$\times (2g+1)\, C^{\tau 0}_{1010} C^{\tau 0}_{\Lambda 0 \Lambda' 0}\, V(jkg)\, W(kg\tau 1;\ 1f)\, X(jgj;\ lfl;\ s1s)\, X(LkL';\ lfl;\ \Lambda\tau\Lambda') \operatorname{Re} F^k_{njlL\Lambda L'\Lambda'}(E_N); \qquad \text{(A.6)}$$

$$b^k_{1\,njl}(E_N) = 3^{1/2} \sum_{LL'\Lambda f} (-)^{(L+L'+1)/2} [(2\Lambda+1)(2\Lambda+2)(2\Lambda+3)]^{1/2} (2f+1)$$
$$\times\ V(jkf)\, W(jsfl;\ lj)\, X(LkL';\ lfl;\ \Lambda 1 \Lambda + 1) \operatorname{Re} F^k_{njlL\Lambda L'\Lambda+1}(E_N); \qquad \text{(A.7)}$$

$$b^k_{2\,njl}(E_N) = 2\cdot 3^{1/2} [1 - k(k+1)/(2j+1)^2] \sum_{LL'\Lambda f} (-)^{(L+L'+3)/2}$$
$$\times [(2\Lambda+1)(2\Lambda+2)(2\Lambda+3)]^{1/2} (2f+1)\, X(jkj;\ lfl;\ s1s)\, X(LkL';\ lfl;\ \Lambda 1\Lambda + 1) \operatorname{Re} F^k_{njlL\Lambda L'\Lambda+1}(E_N); \qquad \text{(A.8)}$$

$$b^k_{3\,njl}(E_N) = 3\,[2(2k+1)]^{1/2} \sum_{LL'\Lambda f} (-)^{(L+L'+1)/2} [(2\Lambda+1)(2\Lambda+2)(2\Lambda+3)]^{1/2}$$
$$\times \varepsilon_f V(jkf)\, X(LkL';\ lkl;\ \Lambda 1 \Lambda + 1)\, X(jfj;\ lkl;\ s1s) \operatorname{Im} F^k_{njlL\Lambda L'\Lambda+1}(E_N); \qquad \text{(A.9)}$$

$$F^k_{njlL\Lambda L'\Lambda'}(E_N) = \check{c}\, j^k (2j+1) \binom{2k}{k} \left[\frac{(2k+1)(2j-k)!}{(2j+k+1)!}\right]^{1/2} (2L+1)$$
$$\times (2L'+1)\, C^{k0}_{L0L'0}\, C^{l0}_{L0\Lambda 0}\, C^{l0}_{L'0\Lambda' 0}\, [b^*_{L\Lambda njl}(E_N)\, b_{L'\Lambda' njl}(E_N)]\, \rho_{njl}(E_N); \qquad \text{(A.10)}$$

$$V(jkf) = \sum_I (2I+1)^2 W(jskI;\ Ij)\, X(jfj;\ IkI;\ s1s); \qquad \text{(A.11)}$$

$$\varepsilon_{k+1} = (2k+3)\,[k/(k+1)]^{1/2}; \quad \varepsilon_{k-1} = -(2k-1)\,[(k+1)/k]^{1/2}; \qquad \text{(A.12)}$$

$$f^k = j^{-k} \binom{2k}{k}^{-1} \sum_{j_z=-j}^{j} p_{j_z} \sum_{r=0}^{r=k} (-)^r \frac{(j-j_z)!\,(j+j_z)!}{(j-j_z-r)!\,(j+j_z-k+r)!} \binom{k}{r}^2; \qquad \text{(A.13)}$$

$$a_n(E_N) = C \sum_{L=0}^{\infty} (2L+1)\, |b_{LLn\frac{1}{2}0}(E_N)|^2 \rho_{n\frac{1}{2}0}(E_N); \qquad \text{(A.14)}$$

$$b_n(E_N) = C \sum_{L=0}^{\infty} (2L+2) \operatorname{Re}[b^*_{LLn\frac{1}{2}0}(E_N)\, b_{L+1L+1n\frac{1}{2}0}(E_N)]\, \rho_{n\frac{1}{2}0}(E_N); \qquad \text{(A.15)}$$

$$\alpha_H = \tfrac{1}{2}[h_{11} + h_{22} + \gamma^2 h_{pp} + 2\operatorname{Re} h_{12} - 2\gamma \operatorname{Re}(h_{1p} + h_{2p})]\,(f_{11}$$
$$+ 3f_{22} + \gamma^2 f_{pp} - 2\gamma \operatorname{Re} f_{2p})^{-1}; \quad \gamma = v_N/2c = 0.053; \qquad \text{(A.16)}$$

$$g_1 = g_s + g_v, \quad g_1' = g_s' + g', \quad g_2 = g_t + g_a, \quad g_2' = g_t' + g_a';$$

$$f_{ik} = g_i^* g_k + g_i'^* g_k'; \quad h_{ik} = g_i^* g_k' + g_i'^* g_k; \quad (i, k \equiv 1, 2, p); \quad \gamma_\nu \equiv \gamma_\nu^{njl} = E_\nu^{njl}/2Mc^2; \quad C = (2M)^{3/2} \hbar^{-7} c^{-3}; \qquad \text{(A.17)}$$

where M is the nucleon mass, E_ν^{njl} is the neutrino energy.

In the case of a longitudinal neutrino ($g'_k = g_k$) we have: $\alpha_H = \frac{1}{2}$ for s, v and p variants and $\alpha_H = \frac{1}{6}$ for the t and a variants of interaction. The parameters f^k, which were introduced in reference 4, characterize the orientation of the nuclear spin; p_{j_z} is the probability of finding the nucleus in a state with spin projection on the z axis equal to j_z. The quantities $b_{L\Lambda njl}(E_N)$, $\rho_{njl}(E_N)$, W and X were given in the appendix to reference 1.

We note that for $f^1 \gg f^2, f^3, \ldots$ the neutron, angular distribution takes on the form: $1 + f^1 \alpha_{njl}(E_N) \cos\theta$, where f^1 is the degree of polarization of the nucleus, and $\alpha_{njl}(E_N)$ is a function of neutron energy only, depending on properties of the nucleus considered and on the properties of the nucleus considered and on the variant of interaction between μ^- mesons and nucleons.

[1] E. I. Dolinskiĭ and L. D. Blokhintsev, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1488 (1958), Soviet Phys. JETP **8**, 1040 (1959).

[2] R. P. Feynmann and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[3] Egorov, Ignatenko, Khalupa and Chultēm, Joint Institute of Nuclear Research, IV Session of the Scientific Conference, Dubna, May (1958).

[4] H. A. Tolhoek and J. A. M. Cox, Physica **19**, 101 (1953).

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