ON THE THEORY OF DIRECT NUCLEAR REACTIONS INVOLVING POLARIZED PARTICLES

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The theory of direct nuclear reactions (stripping and pick-up reactions) involving polarized particles is considered. The angular distributions and polarizations of the products of direct nuclear reactions induced by polarized particles in oriented nuclei are determined by the distorted wave method without inclusion of the spin-orbit coupling.

1. INTRODUCTION

THE theory of direct nuclear reactions (stripping and pick-up reactions) is widely used in nuclear spectroscopy to determine the properties of nuclei. The angular distribution of the products of the direct reactions permits the determination of the spin and the parity of the final state of the remaining nucleus, if the spin and the parity of the initial state of the initial nucleus are known. Additional information about the structure of the nucleus can be obtained in the investigation of the polarization phenomena accompanying the direct reactions.

Newns[1] was the first to point out the possibility of a polarization of the protons in stripping reactions. He also showed that the sign of the polarization of the proton determines the value of the total angular momentum of the captured neutron ($j_n = l + \frac{1}{2}$ or $j_n = l - \frac{1}{2}$). Assuming the nucleus to be completely black for protons, Newns found that the polarization is negative if the neutron is absorbed with $j_n = l + \frac{1}{2}$, and positive if $j_n = l - \frac{1}{2}$. The positive direction is taken along the vector $\mathbf{k}_d \times \mathbf{k}_p$. Horowitz and Messiah[2] determined the polarization of the protons in stripping reactions, using for the nucleus the model of a hard sphere. They obtained the same sign for the polarization as in reference 1. The experimental sign of the polarization was in disagreement with these predictions.[3]

Later Tobocman, and also Newns and Refai[4] showed that the correct sign of the polarization of the protons can be obtained if the scattering of the deuteron wave by the nucleus is taken into account. The experimental results[5,6] are in agreement with reference 4.

Cheston[7] and Sawicki[8] discussed the effect of the spin-orbit interaction on the polarization of the protons, which is, however, small.

Dalitz[9] and Lakin[10] considered the theory of the reactions of particles with spin 1. The stripping reaction with polarized deuterons was also investigated by Satchler.[11,12]

In the present paper we obtain the angular distribution and the polarization of the products of stripping and pick-up reactions in the interaction of polarized particles with arbitrarily oriented nuclei. The angular distribution of the protons produced in the stripping reaction with polarized deuterons has an azimuthal asymmetry. A study of this asymmetry leads to the possibility of determining the spin of the final state of the remaining nucleus. We also consider other possibilities of using the stripping reaction with polarized deuterons to obtain additional information about the structure of the nucleus. In particular, it will be possible to determine the reduced widths for states with different values of the orbital momentum of the absorbed neutron.

The formation of deuterons in the interaction of polarized protons with nuclei is also characterized by an angular distribution with an azimuthal asymmetry. The produced deuterons are polarized. The capture reaction with polarized nucleons can be used for the production of polarized deuterons.

We use the method of distorted waves. The spin-orbit interaction was neglected, since it gives a relatively small contribution to the cross section.[4] We also neglect Coulomb effects, which are insignificant for sufficiently high energies.

2. THE STRIPPING REACTION (d,p) WITH POLARIZED PARTICLES

We shall describe the polarization effects in the stripping reaction (d, p) with the help of a density matrix, whose elements completely determine the spin states of the particles participating in the reaction.

The density matrix of the total system in the initial state (deuteron + initial nucleus) is given in the form of the direct product of the density ρ^d and ρ^A, referring to the deuteron and the nucleus. The deuteron density matrix has three rows and columns: $\rho^d = \rho_{\mu_d\mu_d'}$ (μ_d and μ_d' are the possible values of the spin projection of the deuteron). The nuclear density matrix has $(2i+1)$ rows and columns: $\rho^A = \rho_{\mu_i\mu_i'}$ (i is the spin of the initial nucleus, μ_i and μ_i' are the possible values of the spin projection of the nucleus).

Instead of giving the elements of the density matrix, we can expand it in terms of spin-tensors. These form an orthogonal system of matrices which transform according to an irreducible representation of the rotation group when the system of coordinates is rotated. The spin states of the system are then given by the coefficients of this expansion. We expand the 3×3 deuteron density matrix $\rho_{\mu_d\mu_d'}$ in terms of the spin-tensors

$$T^{JM}_{\mu_d\mu_d'} = (-1)^{1+\mu_d'}(11\mu_d - \mu_d' \,|\, JM)$$

of rank J = 0, 1, and 2:

$$\rho_{\mu_d\mu_d'} = \sum_{JM} \langle T^{JM+}\rangle\, T^{JM}_{\mu_d\mu_d'}. \tag{1}$$

The $(2i+1)$-rowed density matrix of the initial nucleus $\rho_{\mu_i\mu_i'}$ is expanded in terms of the spin-tensors $T^{LQ}_{\mu_i\mu_i'} = (-1)^{i+\mu_i'}(ii\mu_i - \mu_i' \,|\, LQ)$ of rank $L = 0, 1, \ldots, 2i$:

$$\rho_{\mu_i\mu_i'} = \sum_{LQ} \langle T^{LQ+}\rangle\, T^{LQ}_{\mu_i\mu_i'}. \tag{2}$$

The density matrix of the total system in the initial state is normalized to unity: $\mathrm{Sp}\,\rho = 1$.

The density matrix of the system in the final state (proton + remaining nucleus), ρ', is connected with the initial matrix, ρ, by the relation

$$\rho' = S\rho S^+,$$

where S is the reaction matrix. It can be shown that, neglecting the spin-orbit interaction, the reaction matrix has the form

$$S_{\mu_p\mu_j;\,\mu_d\mu_i} = \sum_{lsm\mu_s\mu_n} (1/2\, 1/2\, \mu_p\mu_n \,|\, 1\mu_d)$$
$$\times\, (i\, 1/2\, \mu_i\mu_n \,|\, s\mu_s)(sl\mu_s m \,|\, j\mu_j)\sqrt{\gamma_{ls}}\, J_l^m, \tag{3}$$

if the state of the captured neutron in the nucleus is given in the $l-s$ coupling scheme. Here l and m are the orbital angular momentum of the absorbed neutron and its projection, s and μ_s are the channel spin and its projection, j and μ_j are the spin of the remaining nucleus and its projection, μ_n and μ_p are the spin projections of the neutron and the proton in the deuteron, and γ_{ls} is the reduced width. The quantity J_l^m is defined by

$$J_l^m = \sqrt{\frac{4M\alpha}{\pi\hbar^2 R}} \int_{r>R} \psi^*_{\mathbf{k}_p}(\mathbf{r}) \frac{k_l(k_n r)}{k_l(k_n R)} Y^*_{lm}(\vartheta, \varphi)\, \psi_{\mathbf{k}_d}(\mathbf{r})\, d\mathbf{r}.$$

$\psi_{\mathbf{k}_p}$ is the wave function of the liberated proton with the wave vector $\mathbf{k}_p$; $\psi_{\mathbf{k}_d}$ is the wave function of the incoming deuteron; $k_n = \sqrt{2M\,|E_n|}/h$; E_n is the energy of the neuteron in the nucleus.

Using the expansions (1) and (2), we write the density matrix of the final state in the form

$$\rho'_{\mu_p\mu_j;\,\mu_p'\mu_j'} = \sum_{JM,\,LQ} \langle T^{JM+}\rangle \langle T^{LQ+}\rangle$$
$$\times \sum_{\mu_d\mu_d'\mu_i\mu_i'} S_{\mu_p\mu_j;\,\mu_d\mu_i} T^{JM}_{\mu_d\mu_d'} T^{LQ}_{\mu_i\mu_i'} S^*_{\mu_d'\mu_i';\,\mu_p'\mu_j'}. \tag{5}$$

Summing (5) over the spin projections of the remaining nucleus, we find the density matrix for the proton liberated in the stripping of the deuteron by the nucleus:

$$\rho'_{\mu_p\mu_p'} = \sum_{\mu_j} \rho'_{\mu_p\mu_j;\,\mu_p'\mu_j}.$$

The trace of the density matrix $\rho'_{\mu_p\mu_p'}$ determines the angular distribution of the protons. With the wave functions $\psi_{\mathbf{k}_p}$ and $\psi_{\mathbf{k}_d}$ normalized to unity in the incoming wave, we obtain for the reaction cross section

$$d\sigma / do = (v_p / v_d)\, \mathrm{Sp}\,\rho',$$

where v_d and v_p are the velocities of the deuteron and the proton.

To find the polarization of the protons liberated in the stripping process, we must expand the density matrix (normalized to unity, $\rho'/\mathrm{Sp}\,\rho'$) in terms of the spin-tensors

$$T^{RT}_{\mu_p\mu_p'} = (-1)^{1/2+\mu_p'}(1/2\, 1/2\, \mu_p - \mu_p' \,|\, RT)$$

of rank R = 0 and 1:

$$\rho'_{\mu_p\mu_p'} / \mathrm{Sp}\,\rho' = \sum_{RT} \langle T^{RT+}\rangle\, T^{RT}_{\mu_p\mu_p'}.$$

The coefficients of this expansion, $\langle T^{RT+}\rangle$ will then determine the spin states of the liberated protons.

Using the normalization condition for the spin-tensors and the expression (3) for the reaction matrix, we obtain the coefficient $\langle T^{RT+}\rangle$ in the form

$$\langle T^{RT+}\rangle \operatorname{Sp}\rho' = \sum_{JM,\,LQ} \langle T^{JM+}\rangle \langle T^{LQ+}\rangle$$
$$\times \sum_{ls l's' m} \sqrt{\gamma_{ls}\gamma_{l's'}}\, J_l^m J_{l'}^{m+M+Q-T*} \sum_{\mu_d \mu_n \mu_j} (-1)^{1/2+i+\mu_j-m-Q-T-M}$$
$$\times (\tfrac{1}{2}\tfrac{1}{2}\mu_d-\mu_n\mu_n \mid 1\mu_d)(i\tfrac{1}{2}\mu_j-m-\mu_n\ \mu_n \mid s\mu_j-m)$$
$$\times (s\ l\mu_j-m\ m \mid j\mu_j)(11\mu_d M-\mu_d \mid JM)$$
$$\times (ii\mu_j-m-\mu_n\ Q-\mu_j+m+\mu_n \mid LQ)$$
$$\times (\tfrac{1}{2}\tfrac{1}{2}\mu_d-\mu_n-T\ \mu_n+T-M \mid 1\mu_d-M)$$
$$\times (i\tfrac{1}{2}\mu_j-m-\mu_n-Q\ \mu_n+T-M \mid s'\mu_j-m+T-M-Q)$$
$$\times (s'l'\mu_j-m+T-M-Q\ m+M+Q-T \mid j\mu_j)$$
$$\times (\tfrac{1}{2}\tfrac{1}{2}\mu_d-\mu_n\ T-\mu_d+\mu_n \mid RT).$$

With the help of known sum rules[13] we carry out the summation over the spin projections μ_d, μ_n, and μ_j. We have

$$\langle T^{RT+}\rangle \operatorname{Sp}\rho' = \sum_{JM,\,LQ} \langle T^{JM+}\rangle \langle T^{LQ+}\rangle \sum_{ls l's'} \sqrt{\gamma_{ls}\gamma_{l's'}}$$
$$\times \sum_{r,P} (-1)^{-j-s+R-T+P}\, 3(2j+1)(2J+1)^{1/2}(2L+1)^{1/2}$$
$$\times (2R+1)^{1/2}(2s+1)^{1/2}(2s'+1)^{1/2}(2r+1)^{1/2}$$
$$\times (RJ-TM \mid rM-T)(LrQM-T \mid PQ+M-T)$$
$$\times W(ll'ss';Pj)\,X(\tfrac{1}{2}R\tfrac{1}{2};\ \tfrac{1}{2}r\tfrac{1}{2};\ 1J1)\,X(\tfrac{1}{2}r\tfrac{1}{2};\ iLi;\ sPs')$$
$$\times \sum_m (-1)^m (ll'-mm+M+Q-T \mid PM+Q-T)$$
$$\times J_l^m J_{l'}^{m+M+Q-T*}, \qquad (6)$$

where X are the Fano functions.[14]

For $R = 1$ this formula determines the polarization of the protons. The components of the polarization vector $\mathbf{P}$ are related to the coefficients $\langle T^{RT+}\rangle$ by

$$P_x = \langle T^{11+}\rangle - \langle T^{1-1+}\rangle,\quad P_y = i(\langle T^{11+}\rangle + \langle T^{1-1+}\rangle),$$
$$P_z = -\sqrt{2}\langle T^{10+}\rangle. \qquad (7)$$

We note that $\langle T^{00+}\rangle = -1/\sqrt{2}$; with $R = T = 0$ we then find from (6) the angular distribution of the protons:

$$\operatorname{Sp}\rho' = \sum_{JM,\,LQ} \langle T^{JM+}\rangle \langle T^{LQ+}\rangle \sum_{ls l's'} \sqrt{\gamma_{ls}\gamma_{l's'}} \sum_P (-1)^{-j-s+J+P}$$
$$\times 3(2j+1)(2J+1)^{1/2}(2L+1)^{1/2}(2s+1)^{1/2}(2s'+1)^{1/2}$$
$$\times (LJQM \mid PQ+M)\,W(ll'ss';Pj)\,W(\tfrac{1}{2}\tfrac{1}{2}11;\ J\tfrac{1}{2})\,X$$
$$\times (\tfrac{1}{2}J\tfrac{1}{2};\ iLi;\ sPs') \sum_m (-1)^m$$
$$\times (ll'-mm+M+Q \mid PM+Q)\, J_l^m J_{l'}^{m+M+Q*} \qquad (8)$$

Formulas (6) and (8) are the most general formulas giving the angular distribution and the polarization of the protons liberated in the stripping process for arbitrary polarization of the incoming deuterons and the initial nuclei, given by the quantities $\langle T^{JM+}\rangle$ and $\langle T^{LQ+}\rangle$.

We now consider the simplest special cases of (8) and (6).

Nuclei and Deuterons Unpolarized. In this case

$$\langle T^{JM+}\rangle = \frac{1}{\sqrt{3}}\delta_{J0}\delta_{M0},\qquad \langle T^{LQ+}\rangle = \frac{(-1)^{2i}}{\sqrt{2i+1}}\delta_{L0}\delta_{Q0}.$$

Substituting these expressions in (8) and (6), we obtain the following expressions for the angular distribution and the polarization of the protons:

$$(\operatorname{Sp}\rho')_0 = \frac{2j+1}{2(2i+1)}\sum_{ls}\frac{\gamma_{ls}}{2l+1}\sum_m |J_l^m|^2, \qquad (9)$$

$$\langle T^{RT+}\rangle_0 (\operatorname{Sp}\rho')_0 = \sum_{lss'}\sqrt{\gamma_{ls}\gamma_{ls'}}\,(-1)^{-i-j-s-s'-1/2-T}$$
$$\times \frac{2j+1}{2i+1}(2s+1)^{1/2}(2s'+1)^{1/2}$$
$$\times W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2};R1)\,W(\tfrac{1}{2}\tfrac{1}{2}ss';Ri)\,W(llss';Rj)$$
$$\times \sum_m (-1)^m (ll-mm-T \mid R-T)\, J_l^m J_l^{m-T*}. \qquad (10)$$

If the scattering of the deuteron and proton waves in the field of the nucleus is neglected, the differential cross section will be given by the well-known Butler formula

$$\left(\frac{d\sigma}{do}\right)_B = \frac{2j+1}{2i+1}\frac{k_p}{k_d}\frac{4M\alpha R^3}{\hbar^2}\{\alpha^2+(\mathbf{k}_d/2-\mathbf{k}_p)^2\}^{-2}$$
$$\times \sum_{ls}\gamma_{ls}\left|\frac{dj_l(kR)}{dR} - j_l(kR)\frac{d}{dR}\ln k_l(k_nR)\right|^2 \qquad (11)$$

($\mathbf{k} = \mathbf{k}_d - \mathbf{k}_p$). In this approximation there is no polarization of the protons.

If the scattering of the deuteron and the proton waves in the field of the nucleus is taken into account, we obtain a somewhat different angular distribution from that given by (11):

$$\left(\frac{d\sigma}{do}\right)_0 = \frac{2j+1}{2i+1}\frac{k_p}{k_d}\sum_{ls}\frac{\gamma_{ls}}{2l+1}\sum_m |J_l^m|^2, \qquad (12)$$

Furthermore, the protons will be polarized.

It can be shown that the protons are polarized in the direction perpendicular to the plane of the reaction, i.e., in the direction of the vector $\mathbf{k}_d \times \mathbf{k}_p$. Indeed, the angular dependence of the wave functions $\psi_{\mathbf{k}_d}$ and $\psi_{\mathbf{k}_p}$ in the expression for J_l^m is given by

$$\psi_{\mathbf{k}_d}(\mathbf{r}) = \sum_{l_d m_d} R_{l_d}(r)\,Y_{l_d m_d}(\vartheta,\varphi)\,Y^*_{l_d m_d}(\vartheta_{\mathbf{k}_d},\varphi_{\mathbf{k}_d}),$$
$$\psi_{\mathbf{k}_p}(\mathbf{r}) = \sum_{l_p m_p} R_{l_p}(r)\,Y_{l_p m_p}(\vartheta,\varphi)\,Y^*_{l_p m_p}(\vartheta_{\mathbf{k}_p},\varphi_{\mathbf{k}_p}).$$

Substituting these expressions in J_l^m, choosing the z axis along $\mathbf{k}_d \times \mathbf{k}_p$, and integrating over ϑ and φ, we obtain

$$J_l^m \sim \sum_{l_d l_p m_d m_p} A^l_{l_d l_p} (l_p l 00 \mid l_d 0)(l_p l m_p m \mid l_d m_d) \times Y_{l_p m_p}\left(\frac{\pi}{2}, \varphi_{k_p}\right) Y^*_{l_d m_d}\left(\frac{\pi}{2}, \varphi_{k_d}\right).$$

$Y_{lm}(\pi/2, \varphi)$ is different from zero for even $l + m$, or what is the same thing, for even $l - m$. Contributions to J_l^m come only from terms with even $l_p + m_p$ and $l_d - m_d$. On the other hand, the coefficient $(l_p l 00 \mid l_d 0)$ is different from zero only for even $l_p + l + l_d$. Therefore $l - m$ must also be even. Formula (10) contains the product $J_l^m J_l^{m-T*}$. The requirement that $l - m$ and $l - m + T$ be even limits the possible values of T to the single value T = 0. We therefore have for the polarization of the proton in this system of coordinates

$$\langle T^{11+}\rangle = \langle T^{1-1+}\rangle = 0, \qquad \langle T^{10+}\rangle \neq 0,$$

i.e., the polarization vector is directed along the vector $\mathbf{k}_d \times \mathbf{k}_p$,

$$P_z = \sqrt{\frac{2}{3}} \sum_{lss'} \sqrt{\gamma_{ls}\gamma_{ls'}} (-1)^{-i-j-s-s'-1/2+l} (2s+1)^{1/2} \times (2s'+1)^{1/2} (2l+1)^{-1/2} W(^1/_2\, ^1/_2 ss'; 1i)\, W(llss'; 1j) \times \sum_m \frac{m}{\sqrt{l(l+1)}} |J_l^m|^2 \Big/ \sum_{lsm} \frac{\gamma_{ls}}{2l+1} |J_l^m|^2. \quad (13)$$

In the derivation of formula (13) we assumed that the l-s coupling scheme is valid for the neutron absorbed in the nucleus. In the case of j-j coupling we can obtain an expression for the polarization by using the relation between the reduced widths for the two types of coupling,[11]

$$\sqrt{\gamma_{ls}} = \sum_{j_n} (-1)^{j_n - l - 1/2} (2s+1)^{1/2} (2j_n+1)^{1/2} \times W(i^1/_2 jl; sj_n) \sqrt{\gamma_{lj_n}}.$$

Substituting this expression in (13) and summing over s and s′, we find

$$P_z = \frac{1}{3} \sum_{lj_n} \frac{\gamma_{lj_n}}{2l+1} \frac{j_n(j_n+1) - l(l+1) - 3/4}{l(l+1)} \times \sum_m m |J_l^m|^2 \Big/ \sum_{lj_n} \frac{\gamma_{lj_n}}{2l+1} \sum_m |J_l^m|^2. \quad (14)$$

By measuring experimentally the sign of the polarization of the protons in a number of stripping reactions, we can uniquely determine the spin of the remaining nucleus.

Deuterons Polarized, Nuclei Unpolarized. In this case the general formulas can also be greatly simplified. Substituting in (6)

$$\langle T^{LQ+}\rangle = (-1)^{2i}(2i+1)^{-1/2}\delta_{L0}\delta_{Q0},$$

we find

$$\langle T^{RT+}\rangle \operatorname{Sp}\rho' = \sum_{JM} \langle T^{JM+}\rangle \sum_{lss'} \sqrt{\gamma_{ls}\gamma_{ls'}} \times \sum_p (-1)^{-i-j-s-s'-1/2+R-T}\, 3(2j+1)(2i+1)^{-1}(2J+1)^{1/2} \times (2R+1)^{1/2}(2s+1)^{1/2}(2s'+1)^{1/2} (RJ - TM \mid PM - T) \times W(^1/_2\, ^1/_2 ss'; Pi)\, W(llss'; Pj)\, X(^1/_2\, ^1/_2 R; 11J; ^1/_2\, ^1/_2 P) \times \sum_m (-1)^m (ll - mm' \mid PM - T) J_l^m J_l^{m+M-T*}.$$

Using now equation (10) as the definition of $\langle T^{RT+}\rangle_0$, we obtain

$$\langle T_p^{RT+}\rangle \operatorname{Sp}\rho' = \Big\{-{}^1/_2(3+R)^{1/2}(2-R)^{1/2}\langle T_d^{RT+}\rangle + \langle T_p^{10+}\rangle \times \Big[\delta_{R1}\delta_{T0} + \sum_{J=1,2} (-1)^{R-J+1} 2\cdot 3^{1/2}(2R+1)^{1/2}(R1T0 \mid JT) \times X(^1/_2 R ^1/_2; ^1/_2 1 ^1/_2;\ 1J1)\langle T_d^{JT+}\rangle\Big]\Big\}(\operatorname{Sp}\rho')_0. \quad (15)$$

In particular, if R = T = 0, we have

$$\operatorname{Sp}\rho' = \{1 - 6\langle T_p^{10+}\rangle_0 \langle T_d^{10+}\rangle\}(\operatorname{Sp}\rho')_0. \quad (16)$$

Expression $\langle T_d^{10+}\rangle$ through the polarization vector of the deuteron $\mathbf{P}_d$, we obtain the following formula for the angular distribution of the protons produced in the stripping reaction with polarized deuterons:

$$d\sigma/do = (1 + 3\mathbf{P}_p\mathbf{P}_d)(d\sigma/do)_0. \quad (17)$$

The study of the azimuthal asymmetry in the angular distribution of the protons in the stripping reaction with polarized deuterons leads to the same information about the structure of the nucleus as the polarization of the protons. The cross section is maximal if the polarization vectors $\mathbf{P}_p$ and $\mathbf{P}_d$ are parallel; it is minimal if they are in opposite directions. This dependence of the cross section on the mutual orientation of the polarization vectors can be easily understood by recalling the mechanism of the polarization. The polarization of the protons in the stripping reaction due to the scattering of the proton and the deuteron waves is related to the difference of the neutron absorption cross sections for the two possible spin orientations. Since the spins of the neutron and the proton in the deuteron are parallel, the direction of polarization of the protons will correspond to the orientation of the neutron spin leading to the greater absorption probability. The yield of the reaction becomes, of course, greater if the incoming deuterons are polarized. The direction of this polarization coincides with the direction of polarization of the protons in the stripping process.

The polarization of the protons produced by polarized deuterons is given by the expression

$$\langle T_p^{1T+}\rangle = \{-\langle T_d^{1T+}\rangle + \langle T_p^{10+}\rangle_0 (\delta_{T0} + \sqrt{6(4-T^2)} \times \langle T_d^{2T+}\rangle)\} / (1 - 6\langle T_p^{10+}\rangle_0 \langle T_d^{10+}\rangle). \quad (18)$$

Hence the polarization of the protons depends not only on the polarization, but also on the alignment of the deuterons.

Deuterons Unpolarized, Nuclei Polarized. In this case the angular distribution and the polarization of the protons are given by the expressions

$$\mathrm{Sp}\,\rho' = \sum_{LQ} \langle T^{LQ+}\rangle \sum_{lsl's'} \sqrt{\gamma_{ls}\gamma_{l's'}} (-1)^{-j+i+1/2} 2^{-1}(2j+1) \times (2s+1)^{1/2}(2s'+1)^{1/2} W(ll'ss'; L\tfrac{1}{2}) W(iiss'; L\tfrac{1}{2}) \times \sum_m (-1)^m (ll'-mm+Q\,|\,LQ) J_l^m J_{l'}^{m+Q*}. \quad (19)$$

$$\langle T^{RT+}\rangle \mathrm{Sp}\,\rho' = \sum_{LQ} \langle T^{LQ+}\rangle \sum_{lsl's'} \sqrt{\gamma_{ls}\gamma_{l's'}} \sum_P (-1)^{-j-s-T+P} \times (2j+1)(2L+1)^{1/2}(2R+1)^{1/2}(2s+1)^{1/2}(2s'+1)^{1/2} \times (LRQ-T\,|\,PQ-T)\, W(ll'ss'; Pj)\, W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}; R1) \times X(\tfrac{1}{2}R\tfrac{1}{2}; iLi; sPs') \times \sum_m (-1)^m (ll'-mm+Q-T\,|\,PQ-T) J_l^m J_{l'}^{m+Q-T*}. \quad (20)$$

These formulas can be considerably simplified for the actual values of the spin of the initial nucleus.

3. THE PICK-UP REACTION (p, d) WITH POLARIZED PARTICLES

We consider the polarization effects in the pick-up reaction (p, d). The spin of the initial nucleus is denoted by j, the spin of the remaining nucleus, by i. The density matrix of the total system in the initial state is written in the form

$$\rho_{\mu_p\mu_j;\,\mu_p'\mu_j'} = \sum_{RT,\,LQ} \langle T^{RT+}\rangle \langle T^{LQ+}\rangle T^{RT}_{\mu_p\mu_p'} T^{LQ}_{\mu_j\mu_j'},$$

where the expansion coefficients $\langle T^{RT+}\rangle$ $(R = 0, 1)$ and $\langle T^{LQ+}\rangle$ $(L = 0, 1, \ldots, 2j)$ determine the initial spin states of the proton and the initial nucleus. The initial density matrix is normalized to unity: $\mathrm{Sp}\,\rho = 1$.

Using the definition of the reaction matrix,

$$S'_{\mu_d\mu_i;\,\mu_p\mu_j} = 2S_{\mu_p\mu_j;\,\mu_d\mu_i},$$

we obtain the following expression for the deuteron density matrix in the final state:

$$\rho'_{\mu_d\mu_d'} = \sum_{RT,\,LQ} \langle T^{RT+}\rangle \langle T^{LQ+}\rangle \times \sum_{\mu_p\mu_p'\mu_j\mu_j'\mu_i} S'_{\mu_d\mu_i;\,\mu_p\mu_j} T^{RT}_{\mu_p\mu_p'} T^{LQ}_{\mu_j\mu_j'} S'^{*}_{\mu_d'\mu_i;\,\mu_p'\mu_j'}. \quad (22)$$

The trace of this matrix determines the angular distribution of the deuterons produced in the pick-up process:

$$d\sigma / do = (v_d / v_p)\, \mathrm{Sp}\,\rho'$$

The matrix (22), normalized to unity, is expanded in terms of the spin-tensors T^{JM} $(J = 0, 1, 2)$. The expansion coefficients determine the polarization of the produced deuterons:

$$\rho'_{\mu_d\mu_d'} / \mathrm{Sp}\,\rho' = \sum_{JM} \langle T^{JM+}\rangle T^{JM}_{\mu_d\mu_d'}.$$

By calculations similar to those in the previous case we obtain the following general formulas for the angular distribution and the polarization of the deuterons formed in the pick-up reaction with polarized protons and oriented nuclei:

$$\mathrm{Sp}\,\rho' = \sum_{RT,\,LQ} \langle T^{RT+}\rangle \langle T^{LQ+}\rangle \sum_{lsl's'} \sqrt{\gamma_{ls}\gamma_{l's'}} \times \sum_P (-1)^{i+s+1/2-l'+L+T-Q+R}\, 12(2j+1)(2R+1)^{1/2}(2L+1)^{1/2} \times (2s+1)^{1/2}(2s'+1)^{1/2}(LRQT\,|\,PQ+T)\, W(ss'\tfrac{1}{2}\tfrac{1}{2}; Ri) \times W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}; 1R)\, X(s'sR; l'lP; jjL) \sum_m (-1)^m \times (ll'-mm-T-Q\,|\,P-T-Q) J_l^{m*} J_{l'}^{m-T-Q}; \quad (23)$$

$$\langle T^{JM+}\rangle \mathrm{Sp}\,\rho' = \sum_{RT,\,LQ} \langle T^{RT+}\rangle \langle T^{LQ+}\rangle \sum_{lsl's'} \sqrt{\gamma_{ls}\gamma_{l's'}} \times \sum_{pP} (-1)^{i+s+1/2-l'+J+L+T-Q}\, 12(2j+1)(2J+1)^{1/2}(2R+1)^{1/2} \times (2L+1)^{1/2}(2s+1)^{1/2}(2s'+1)^{1/2}(2p+1)^{1/2} \times (JR-MT\,|\,pT-M)\,(LpQT-M\,|\,PQ+T-M) \times W(ss'\tfrac{1}{2}\tfrac{1}{2}; pi)\, X(\tfrac{1}{2}\tfrac{1}{2}1; \tfrac{1}{2}\tfrac{1}{2}1; RpJ)\, X(s'sp; l'lP; jjL) \times \sum_m (-1)^m (ll'-mm+M-T-Q\,|\,PM-T-Q) \times J_l^{m*} J_{l'}^{m+M-T-Q}. \quad (24)$$

We consider a few actual cases.

Nuclei and Protons Unpolarized. In this case the initial spin state of the system is characterized by the quantities

$$\langle T^{LQ+}\rangle = (-1)^{2j}(2j+1)^{-1/2}\delta_{L0}\delta_{Q0}, \quad \langle T^{RT+}\rangle = -\delta_{R0}\delta_{T0}/\sqrt{2}. \quad (25)$$

Substituting these values in (23) and (24), we find

$$(\mathrm{Sp}\,\rho')_0 = 3\sum_{lms} \frac{\gamma_{ls}}{2l+1} |J_l^m|^2,$$

$$\langle T_d^{JM+}\rangle_0 (\mathrm{Sp}\,\rho')_0 = \sum_{lss'} \sqrt{\gamma_{ls}\gamma_{ls'}} (-1)^{-j+i+s-s'+1/2+J}\, 6(2s+1)^{1/2} \times (2s'+1)^{1/2} W(\tfrac{1}{2}\tfrac{1}{2}ss'; Ji)\, W(\tfrac{1}{2}\tfrac{1}{2}11; J\tfrac{1}{2})\, W(llss'; Jj) \times \sum_m (-1)^m (ll-mm+M\,|\,JM) J_l^{m+M} J_l^{m*}. \quad (26)$$

We note that $\langle T_d^{JM+}\rangle$ reduces to zero for J = 2.

The differential cross section for the pick-up reaction (p, d) is equal to

$$\left(\frac{d\sigma}{do}\right)_0 = \frac{3}{2}\frac{k_d}{k_p}\sum_{lsm}\frac{\gamma_{ls}}{2l+1}|J_l^m|^2. \quad (27)$$

The deuterons formed in the pick-up process are polarized in the direction perpendicular to the plane of the reaction:

$$P_d = \sum_{lss'}\sqrt{\gamma_{ls}\gamma_{ls'}}(-1)^{l-j+l+s-s'+1/2}2^{5/2}3^{-1/2}(2s+1)^{1/2}$$
$$\times (2s'+1)^{1/2}(2l+1)^{-1/2}W(1/2\,1/2ss';1l)W(llss';1j)$$
$$\times \sum_m \frac{m}{\sqrt{l(l+1)}}|J_l^m|^2 \Big/ \sum_{lsm}\gamma_{ls}(2l+1)^{-1}|J_l^m|^2. \quad (28)$$

If the scattering of the deuteron and the proton waves is neglected, there is no polarization.

Protons Polarized, Nuclei Unpolarized. Substituting $\langle T^{LQ+}\rangle$ from (25) into (23) and (24), and using (26) as the definition of $\langle T_d^{JM+}\rangle_0$, we find

$$\mathrm{Sp}\,\rho' = (1-\langle T_p^{10+}\rangle\langle T_d^{10+}\rangle_0)(\mathrm{Sp}\,\rho')_0, \quad (29)$$

$$\langle T_d^{JM+}\rangle = \{(-1)^J 2W(1/2\,1/2\,1\,1;J\,1/2)\langle T_p^{JM+}\rangle + [\delta_{J1}\delta_{M0}$$
$$+ 2(-1)^{J-M}3^{3/2}(2J+1)^{1/2}(J1-MM|10)$$
$$\times X(1/2\,1/2\,1;1/2\,1/2\,1;11J)\langle T_p^{1M+}\rangle]\langle T_d^{10+}\rangle_0\}$$
$$\times (1-\langle T_p^{10+}\rangle\langle T_d^{10+}\rangle_0)^{-1}. \quad (30)$$

Thus the angular distribution of the deuterons formed in the pick-up process with polarized protons and unoriented nuclei is given by the cross section

$$\frac{d\sigma}{do} = (1+1/2\,\mathbf{P}_p\mathbf{P}_d)\left(\frac{d\sigma}{do}\right)_0, \quad (31)$$

where P_d and $(d\sigma/do)_0$ are determined by (28) and (27).

The polarization of the deuterons is given by the expressions

$$\langle T_d^{1M+}\rangle = (-2/3\langle T_p^{1M+}\rangle$$
$$+\langle T_d^{10+}\rangle_0\delta_{M0})/(1-\langle T_p^{10+}\rangle\langle T_d^{10+}\rangle_0); \quad (32)$$

$$\langle T_d^{2M+}\rangle = -\sqrt{2/3(4-M^2)}$$
$$\times\langle T_p^{1M+}\rangle\langle T_d^{10+}\rangle_0/(1-\langle T_p^{10+}\rangle\langle T_d^{10+}\rangle_0). \quad (33)$$

We note that the use of polarized protons in the pick-up reaction can lead to the formation of aligned deuterons.

Protons Unpolarized, Nuclei Polarized. In this case we have the following formulas for the angular distribution and the polarization of the deuterons:

$$\mathrm{Sp}\,\rho' = \sum_{LQ}\langle T^{LQ+}\rangle\sum_{ll's}\sqrt{\gamma_{ls}\gamma_{l's}}(-1)^{j+s-Q}3(2j+1)W(ll'jj;Ls)$$
$$\times\sum_m(-1)^m(ll'-mm-Q|L-Q)J_l^{m*}J_{l'}^{m-Q}, \quad (34)$$

$$\langle T^{JM+}\rangle\,\mathrm{Sp}\,\rho' = \sum_{LQ}\langle T^{LQ+}\rangle\sum_{lsl's'}\sqrt{\gamma_{ls}\gamma_{l's'}}$$
$$\times\sum_P(-1)^{j+s+1/2-l'+L-Q}6(2j+1)$$
$$\times(2J+1)^{1/2}(2L+1)^{1/2}(2s+1)^{1/2}$$
$$\times(2s'+1)^{1/2}(LJQ-M|PQ-M)W(1/2\,1/2\,1\,1;J\,1/2)$$
$$\times W(1/2\,1/2ss';Ji)X(ll'P;ss'J;jjL)\sum_m(-1)^m$$
$$\times(ll'-mm-Q+M|PM-Q)J_l^{m*}J_{l'}^{m-Q+M} \quad (35)$$

4. THE EXAMPLE $B^{11}(d,p)B^{12}$

As an example, we consider the angular distribution and the polarization of the protons produced in the reaction $B^{11}(d,p)B^{12}$ (reference 15). In the calculation we use the following parameters:

$$E_d = 8\text{ Mev},\quad Q = 1.14\text{ Mev},$$
$$R = 4.4\cdot10^{-13}\text{ cm},\quad \varepsilon = 2.23\text{ Mev}.$$

The orbital angular momentum of the absorbed neutron is equal to unity, $l = 1$. The initial state of B^{11} is odd (−), the spin is 3/2, the final state is even (+), with the possible spin values j = 0, 1, 2, 3.

In the calculation of the integrals J_l^m entering into the reaction matrix we choose for the wave function of the proton a plane wave $\psi_{\mathbf{k}_p} = e^{i\mathbf{k}_p\mathbf{r}}$; in the deuteron wave function, on the other hand, we take account of the scattering of the deuteron by the nucleus:

$$\psi_{\mathbf{k}_d}(\mathbf{r}) = e^{i\mathbf{k}_d\mathbf{r}} - 4\pi$$
$$\times\sum_{l_dm_d}\eta_{l_d}i^{l_d}h_{l_d}^{(1)}(k_dr)Y^*_{l_dm_d}(\vartheta_{\mathbf{k}_d},\varphi_{\mathbf{k}_d})Y_{l_dm_d}(\vartheta,\varphi).$$

Regarding the nucleus as a hard sphere of radius R, we have for the coefficients η_l:

$$\eta_{l_d} = j_{l_d}(k_dR)[j_{l_d}(k_dR)$$
$$-in_{l_d}(k_dR)]/[j_{l_d}^2(k_dR)+n_{l_d}^2(k_dR)].$$

Substituting these functions in the integral J_l^m, we obtain

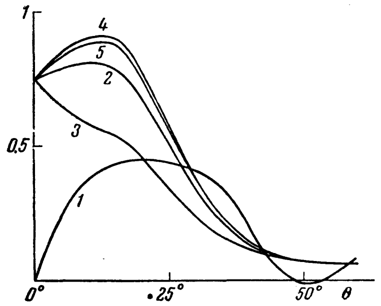
$$J_l^m = \sqrt{\frac{4\alpha M}{\pi\hbar^2R}}\left\{\frac{4\pi i^lR^2}{k^2+k_n^2}Y^*_{lm}(\vartheta_{\mathbf{k}}\varphi_{\mathbf{k}})\right.$$
$$\times\left[\frac{dj_l(kR)}{dR}-j_l(kR)\frac{d}{dR}\ln k_l(k_nR)\right]-(4\pi)^{3/2}\sum_{l_dl_p}\eta_{l_d}i^{l_d-l_p}$$
$$\times\sqrt{\frac{(2l+1)(2l_p+1)}{2l_d+1}}(ll_p00|l_d0)(ll_pmm_p|l_dm_d)$$
$$\left.\times J_{ll_pl_d}Y_{l_pm_p}(\vartheta_{\mathbf{k}_p},\varphi_{\mathbf{k}_p})Y^*_{l_dm_d}(\vartheta_{\mathbf{k}_d},\varphi_{\mathbf{k}_d})\right\},$$

where $\mathbf{k} = \mathbf{k}_d - \mathbf{k}_p$. We also introduced the notation

$$J_{ll_pl_d} = \int_R^\infty j_{l_p}(k_pr)\frac{k_l(k_nr)}{k_l(k_nR)}h_{l_d}^{(1)}(k_dr)\,r^2dr.$$

In the calculations we included the deuteron scattering states $l_d = 0, 1, 2, 3$, and 4.

The resulting angular distribution of the protons is given in the figure.



The function $m(\theta)$ (curve 1) and the angular distributions in arbitrary units for the reaction $B^{11}(d, p)B^{12}$ with unpolarized deuterons (curve 2) and completely polarized deuterons (curves 3, 4, 5). The angular distributions were obtained under the assumptions $j = 1$, and $\beta = 0.1$ and ∞, respectively.

The polarization of the protons is in this case given by the expression

$$P = c(j)\, m(\theta), \quad m(\theta) = \sum_m m\, |J_1^m|^2 \Big/ \sum_m |J_1^m|^2.$$

The functional dependence of $m(\theta)$ is also shown in the figure. The coefficient $c(j)$ depends on the value of the spin of the nucleus in the final state. For different possible values of the nuclear spin this coefficient is equal to

$$c(0) = {}^1/_6, \quad c(1) = \frac{\beta^2 + 2\sqrt{5}\beta - 3}{12(\beta^2+1)},$$

$$c(2) = -\frac{\beta^2 - 6\beta + 1}{12(\beta^2+1)}, \quad c(3) = {}^1/_6,$$

where $\beta^2 = \gamma_{11}/\gamma_{12}$. We note that a measurement of the absolute value of the polarization makes it possible to determine the ratio of the reduced widths for a given spin of the final state.

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