The calculation is carried out under the assumption that the radius of the short-distance repulsion forces r_0 and the averaged scattering amplitude e'^2/T in the Coulomb field e'^2/r are considerably smaller than the average distance between particles $\bar{r} = \nu^{-1/3}$. These conditions signify that the system is close to ideal, i.e., the correction to the free energy, due to the interaction, are small compared with the free energy of the ideal gas. It was also assumed that the probability of molecule formation is small and the contribution of molecules to the free energy of the system can be neglected. The electrolyte was assumed to consist of two types of particles with charges Z_1 and Z_2.

The expression used for the paired correlation function was that given in reference 1 (correct for distances $|\mathbf{x}| \gg e^2T$), and was "joined," at distances much smaller than the Debye radius, with the expression $\exp\{-\beta V(\mathbf{x})\}$ for the correlation function at small distances.

With all the foregoing conditions satisfied, the free energy of a strong electrolyte is an expansion in the particle density ν, given, with accuracy to terms up to the second power in ν inclusive, by the following formula (for the free energy per unit volume):

$$F = F_0 - \frac{T\varkappa^3}{12\pi}$$
$$+ \lim_{R\to\infty}\Big\{-2\pi T\int_0^R r^2 dr \sum_{\alpha\beta} \nu_\alpha \nu_\beta \left(e^{-\beta V_{\alpha\beta}(r)} - 1\right) + \frac{T\varkappa^4}{16\pi} R$$
$$- \frac{\pi}{3}\beta^2 e'^6 \left(\sum \nu Z^3\right)^2 \ln \varkappa R\Big\} + \pi\beta^2 e'^6 \Big[\left(\sum \nu Z^3\right)^2 \frac{1}{3}\left(C - \ln 3\right)$$
$$- \sum \nu Z^2 \sum \nu Z^4\Big], \quad \varkappa = \sqrt{4\pi\beta e'^2 \sum_\alpha \nu_\alpha Z_\alpha^2}. \tag{1}$$

Here F_0 is the free energy of an ideal gas, κ is the reciprocal of the Debye radius, ν_α and Z_α are the density and charge of particles of type α, $\beta = 1/T$ is the reciprocal of the temperature, and C is Euler's constant. The summation in (1) is by particle type.

We note that the expression obtained in reference 2 for the free energy F for the case of charged ideally-hard spheres is incorrect, since the expression used there for the paired correlation function is inaccurate, and, furthermore, the limit for the case of ideally hard spheres was approached incorrectly, and as a result the contribution of the non-electric (i.e., repulsive short-range) forces to the free energy was lost.

I thank Academician L. D. Landau for valuable comments, made during an examination of the results.

[1] A. A. Vedenov, Dokl. Akad. Nauk SSSR, in press.

[2] V. V. Tolmachev and S. V. Tyablikov, Dokl. Akad. Naul SSSR 119, 314 (1958). Научные доклады высшей школы (Scientific Reports of the Higher Schools) 1, 101 (1958).

Translated by J. G. Adashko
179

ON THE POSSIBLE MULTIPLE PRODUCTION OF MUONS

I. L. ROZENTAL'

P. N. Lebedev Physics Institute, Academy of Sciences, U. S.S.R.

Submitted to JETP editor November 26, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) 36, 943-945 (March, 1959)

In a study of the curve of the spreading of shower μ mesons of very high energy ($\sim 10^{12}$ ev) Barrett and others[1] observed a remarkable break in the curve at small distances ($\sim 1 - 2$ m) between the counter systems used for the measurements. This break was at once interpreted as an indication of two different processes. In the opinion of the authors of the paper in question multiple production of μ mesons from the decay of π mesons is responsible for the coincidences at large distances ($> 1 - 2$ m), whereas the sharp rise in the number of coincidences at small distances is due to local showers in the earth (the thickness of earth in these experiments was 1600 m water equivalent). The latter conclusion was based on the following chain of argument: the production of μ mesons in the air should occur at distances of about a nuclear range from the boundary of the atmosphere, which means a height of about 10 km. Consequently, the angle of divergence of the particles responsible for the rise of the spreading curve is $\sim 10^{-4}$ rad. If we assume that when a primary particle makes a collision the secondaries are distributed isotropically in the center-of-mass system, such values of the angle correspond to primary particle energies of the order of 10^{17} ev, which considerably exceeds the observed value of the energy of the showers accompanying the μ mesons ($\sim 3 \times 10^{15}$ ev). Therefore the authors of reference 1 reject the "air hypothesis" of the origin of the break. In the light of the latest data such an argument does not seem convincing, since it has now been established (see, e.g., reference 2) that the angular distribution in the center-of-mass system is anisotropic, and the angle of 10^{-4} rad corresponds to an energy $\sim 10^{14} - 10^{15}$ ev. Therefore additional analysis is needed to settle the question of the origin of the break in the curve.

Possible causes of local showers at great depths are: (1) pairs of μ mesons produced by photons; (2) μ mesons arising from the decay of π mesons produced by nuclear interactions in the ground; (3) secondary particles accompanying high-energy π mesons. The first process is of negligible importance because of the fact that the probability for the production of an electron pair is much larger (by a factor of about 40,000) than tnat for the production of a pair of μ mesons. The second process must be rejected, since for a π meson of energy $\sim 10^{12}$ ev the probability of interaction is larger than that of disintegration by a factor 10^5.

Let us examine the third process in more detail. μ mesons can produce: (a) δ showers or radiative showers, (b) electron-nuclear showers, (c) μ-meson pairs. The phenomena involving electrons can be of no importance, since the apparatus used in the work[1] was screened by about 10 cm of lead. To estimate the second effect we assume that the cross-section for production of electron-nuclear showers by μ mesons is $\sim 10^{-29}$ cm^2 per nucleon.[3] Assuming that the range of nuclear-active particles in the ground is ~ 1 m, we easily find that only one out of 1000 μ mesons will be accompanied by a shower. This ratio is already smaller than the experimental value by about an order of magnitude; actually, because of the narrow angular distribution of the secondary particles[3] the effective value must be still smaller.

Although an estimate of the direct production of μ-meson pairs by μ mesons gives a value of the ratio in agreement with experiment (one shower per hundred particles), nevertheless there is a difficulty in explaining the increase of the coincidences at small distances by means of this process. In fact, in passing through earth to a depth of 1600 m water equivalent μ mesons are scattered in transverse directions to distances of about 1 m.[4] Two particles fell on a detector of area ~ 0.5 m^2.* Assuming for our estimate that the μ mesons are uniformly distributed over a circle of radius 2 m, we readily see that the total number of μ mesons falling on this circle is ~ 10. Such a large number cannot be explained by the direct production of μ-meson pairs.

Assuming that the increase at small distances is caused by "air" μ mesons, we can estimate the possibility of their production in the decay of π mesons. We adopt a plausible model,[5] according to which in an individual nuclear interaction one-third of the energy of the interacting particles goes into π mesons, and assume that the μ mesons are uniformly distributed over a circle of radius 10 m (the experimental data[1] indicate this). Then the calculated number of μ mesons in a circle of radius ~ 2 m is smaller than that observed by a factor 50.

Attention has been called to the discrepancy between the calculated and observed values of the density of μ mesons in the columns of broad showers by Hayakawa[6] in connection with experiments by Japanese physicists.[7] To explain it he suggested that the high-energy primary component consists of heavy nuclei, which cause a high multiplicity of π mesons from collisions (the multiplicity increasing in proportion to the atomic weight A). But a comparison made by Zhdanov of the multiplicities of showers produced by α particles and nucleons in photographic emulsions led to the conclusion that the multiplicity increases much more slowly (approximately as $A^{1/5}$). Therefore it is hard to believe that Hayakawa's hypothesis could be correct. Accordingly we must suppose that there is an additional source of μ mesons, and in particular that direct multiple production of μ mesons can occur.†

It must be noted that this conclusion is only of a preliminary nature, since the estimates made here are based on not very precise data (in particular on the energy distributions). For a final decision we need more exact studies of high-energy μ-meson showers and the accompanying electrons.

The writer expresses his gratitude to N. A. Dobrotin for a number of valuable comments.

*A comparison of the frequency of passage of pairs of particles through the apparatus used in reference 1 with the expected frequency of passage of the axes of broad showers of energy $\sim 3 \times 10^{15}$ ev shows that the probability is close to unity that such pairs of particles accompany the showers.

†Some justification of such an assumption can be found in arguments based on extrapolation of the theory of weak interactions to very small distances.[8,9]

[1] Barrett, Bollinger, Cocconi, Eisenberg, and Greisen, Revs. Modern Phys. **24**, 133 (1952).

[2] I. L. Rozental' and D. S. Chernavskiĭ, Usp. Fiz. Nauk **52**, 185 (1954).

[3] D. Kessler and R. Maze, Nuovo cimento **5**, 1540 (1957).

[4] S. Z. Belen'kiĭ, Лавинные процессы в космических лучах (Cascade Processes in Cosmic Rays), p. 187, Gostekhizdat, 1948.

[5] Vernov, Grigorov, Zatsepin, and Chudakov, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 493 (1955), Columbia Tech. Transl. p. 445.

[6] S. Hayakawa, Nuovo cimento **5**, 608 (1957).

[7] Higashi, Oshio, Shibata, Watanabe, and Watase, Nuovo cimento **5**, 597 (1957).

[8] W. Heisenberg, Z. Physik **101**, 533 (1956).

[9] D. I. Blokhintsev, Usp. Fiz. Nauk **62**, 381 (1957).

Translated by W. H. Furry
180

ON THE MAGNITUDE OF THE RATIO σ^-/σ^+ NEAR THE THRESHOLD OF MESON PHOTOPRODUCTION

S. P. KHARLAMOV, M. I. ADAMOVICH, and V. G. LARIONOVA

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor November 26, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 945-947 (March, 1959)

THE magnitude of the ratio of the yields of positive to negative photo mesons from deuterium $\eta = N_d^-/N_d^+$ can differ appreciably from the ratio σ^-/σ^+ for the photoproduction of π mesons from free nucleons. As we have shown earlier[1] this can be due to a difference in the final state interaction of the particles after photoproduction of a π^- or a π^+ meson on deuterium. Furthermore one has to keep in mind that the difference of the thresholds for π^- and π^+ production on deuterium can have a strong influence on the quantity η if one measures it utilizing photons close to the high energy limit of the bremsstrahlung spectrum.

In the present note the ratio σ^-/σ^+ is deduced from experimental values of η taking into account the above mentioned effects. The experimental results are given in the Table. The ratio of the π^- to π^+ yields on deuterium were measured at an angle of 73° with respect to the photon beam with bremsstrahlung of maximum energy ν_m = 300 Mev[2] and at angle of 60° with bremsstrahlung of ν_m = 165 Mev[3] (first row). In the second row the deduced values for the ratio σ^-/σ^+ are given. They were obtained from the experimental data by applying corrections for the Coulomb interaction of the π^- meson with the protons and of the protons with themselves, and by applying corrections to take into account that π^- and π^+ mesons of the same energy have been produced by photons of different energy.

The Coulomb corrections have been computed on the basis of Baldin's calculations[4] concerning our earlier experiments[1] on the distribution of the relative momenta, p, of the protons, and of the recoil momenta, q, in the reaction $\gamma + d \rightarrow \pi^- + p + p$ for the photon energies given in the table.

We now shall discuss in greater detail the corrections which have to be applied to account for the difference in the threshold energies for π^- and π^+ photoproduction considering the strong energy dependence of the bremsstrahlung spectrum near the upper energy tip. We will do so for the case ν_m = 165 Mev.* Figure 1 shows the experimental momentum distribution of the π^- mesons from

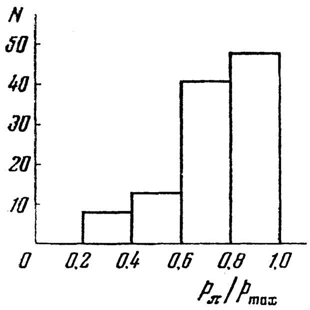

FIG. 1

the reaction $\gamma + d \rightarrow \pi^- + p + p$ for photon energies 155 — 165 Mev. It has been obtained by our method described in reference 1. On the abscissa we have plotted the ratio of the meson momentum p_π to the maximum possible meson momentum p_{max} (given by the meson emission angle θ and the photon energy). The curves in Fig. 2 show the yields $N_d^{\mp}(\nu)$ of π^- and π^+ mesons of energies between 6.7 and 11.7 Mev emitted at an angle of 60° with respect to the photon direction as a function of the photon energy (when ν_m = 165 Mev). For the π^- mesons the momentum distribution of Fig. 1 and the cross sections for the $\gamma + d \rightarrow \pi^- + p + p$ reaction from reference 1 was utilized. For the π^+ mesons the same momentum distribution was taken and it was assumed that the cross sections are given by

ν_m	165 Mev	310 Mev			
ν	159	170	180	190	200
N_d^-/N_d^+	2.10±0.17	1.50±0.15	1.41±0.10	1.41±0.09	1.28±0.09
σ^-/σ^+	1.30±0.11	1.39±0.14	1.33±0.09	1.35±0.09	1.25±0.09