ple considered here. We also note that the assumption that the heavy-particle temperature is stationary, necessary for the validity of the entire analysis (cf. reference 1), is always satisfied in a weakly ionized plasma.

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ON SOME SYMMETRY PROPERTIES OF THE EIGENFUNCTIONS OF THE SCHRÖDINGER EQUATION

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In the present note we wish to call attention to two facts which have not, so far as we know, been noted in the literature: the fact that the symmetry groups of the eigenfunctions of the Schrödinger equation are subgroups of the symmetry group G_H of the corresponding Hamiltonian $\hat{H}$, and the fact that the converse statement is not valid, i.e., the existence of subgroups of the group G_H that are not symmetry groups of the eigenfunctions of the given Schrödinger equation.

The first statement has been made by Melvin,[1] who introduced the concept of the cokernel K^{ij} of the i-th row of the j-th irreducible representation of the group G_H, to which there correspond in the j-th irreducible representation matrices with all the elements in the i-th row equal to zero except for the diagonal element, which is unity. It is easy to see that the symmetry transformations occurring in the cokernel K^{ij} leave the i-th function in the list of eigenfunctions $\psi_1, \psi_2, \ldots, \psi_l$ invariant, and that they form a subgroup of the group G_H.[1] Contrary to Melvin's statement, however, this still does not mean that the cokernel K^{ij} is identical with the symmetry group of the functions ψ_i, since it remains unproved that an eigenfunction of the Hamiltonian $\hat{H}$ with the symmetry group G_H cannot be invariant with respect to some symmetry operator s which does not belong to the group G_H.

We shall prove this last assertion on the assumption that the set (L) of the nodal points of the eigenfunctions of the equation

$$\hat{H}\psi = (\hat{T} + \hat{V})\psi = E\psi, \quad (1)$$

has no internal points and that the value of the potential at any point ζ of the configuration space can be represented as the limit of the values of the potential at a sequence of points ζ_n that converges to ζ, i.e.,

$$V(\zeta) = \lim V(\zeta_n) \quad \text{for } \zeta_n \to \zeta.$$

Suppose that s does not belong to G_H. We shall show that no eigenfunction of the operator $\hat{H}$, which satisfies our assumptions, can be invariant with respect to the symmetry operation s. Let us assume the opposite, i.e., that there exists a function (whose set of nodal points has no internal points) for which $s\psi = \psi$. Then $s\hat{H}\psi = s(\hat{T} + \hat{V})\psi = \hat{T}s\psi + s\hat{V}\psi = E\psi$ and, on the other hand, $\hat{T}s\psi + \hat{V}s\psi = Es\psi$. Consequently $s\hat{V}\psi = \hat{V}s\psi = \hat{V}\psi$, so that $sV = V$ at points where $\psi \neq 0$.*

On our assumption about the set L of the nodal points, for any point ζ in L we can always find a set of points ζ_n not belonging to L that converge to ζ. Obviously $\psi(\zeta_n) \neq 0$. Therefore from $sV(\zeta_n)\psi(\zeta_n) = V(\zeta_n)\psi(\zeta_n)$ it follows that $sV(\zeta_n) = V(\zeta_n)$. Going to the limit, we get $sV(\zeta) = V(\zeta)$, and consequently the equation $sV = V$ is valid for every point of the configuration space, which contradicts the hypothesis of our argument.

Thus on the assumptions indicated it has been shown that the symmetry groups of the eigenfunctions of the Schrödinger equation are subgroups of the symmetry group G_H of the Hamiltonian $\hat{H}$, namely they are the corresponding cokernels.

We shall show the incorrectness of the converse statement for the example of a Hamiltonian with the symmetry group C_{6v}. The group C_{6v} has as one of its subgroups the group $(E, C_3^{\pm})$. We shall show that this subgroup cannot be a cokernel of the group C_{6v}. From the table of characters[2] of the group C_{6v} it can be seen that the only subgroups that are cokernels corresponding to one-dimensional representations are: for $A_1(C_{6v})$, for $A_2(E, C_2, C_3^{\pm}, C_6^{\pm})$, for $B_1(E, C_3^{\pm}, \sigma_{d_1}, \sigma_{d_2}, \sigma_{d_3})$, and for $B(E, C_3^{\pm}, \sigma_{v_1}, \sigma_{v_2}, \sigma_{v_3})$. In the two-dimensional representation E_1 there correspond to the elements C_3, independently of the choice of basis, matrices of the form

$$\begin{vmatrix} -1/2 & \pm\sqrt{3/2} \\ \mp\sqrt{3/2} & -1/2 \end{vmatrix},$$

which follows from the value of the character $\chi^{E_1}(C_3^{\pm}) = -1$ and the unitary nature of the matrices; this means that these elements cannot occur in the cokernel of the representation E_1. In the two-dimensional representation E_2, with any basis, a unit matrix corresponds to the element C_2. This follows from the value of the character $\chi^{E_2}(C_2) = 2$ and the fact that $C_2^2 = E$. Consequently any cokernel corresponding to the representation E_2 will contain the element C_2, and the subgroup $(E, C_3^{\pm})$ cannot be identical with it.

Thus the subgroup $(E, C_3^{\pm})$ cannot be a cokernel of the group C_{6V}, i.e., cannot be a symmetry group of any of the solutions of the Schrödinger equation with a Hamiltonian of that symmetry.

In conclusion we note that solutions of the Schrödinger equation that possess the full symmetry of the system of eigenfunctions (for the case of a finite number of particles see reference 3) have as their symmetry groups all possible cokernels of the symmetry group of the Hamiltonian.

*Division by the function ψ is possible because the effect of the potential energy operator $\hat{V}$ reduces to multiplication by the potential function V.

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ON THE PROBLEM OF TESTING PARITY CONSERVATION IN THE STRONG INTERACTIONS

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IN the analysis of the problem of the conservation of parity in individual interactions we shall start from the following postulates: (1) the law of conservation of the combined parity reflects fundamental properties of space-time and is the basic symmetry law in nature; (2) the conservation of spatial parity in individual interactions is a consequence of additional invariance requirements.

In fact, as has been shown in references 1—3, in the case of the renormalized quantum electrodynamics, owing to the gauge-invariance condition, the requirement of invariance with respect to the combined-inversion operation PC (or the time reversal T) leads to invariance with respect to the spatial-inversion operation P. In the case of the renormalized pseudoscalar meson theory, owing to the condition of isotopic invariance, the requirement of invariance with respect to the combined-inversion transformation PC also leads to invariance with respect to the spatial-inversion operation P. The requirement of the invariance with respect to the transformation PC of the renormalized and isotopically invariant interaction Lagrangian of the K mesons and baryons does not lead to invariance with respect to the operation P. In this connection it is of interest to examine whether parity is conserved in processes of production of K mesons and hyperons.

It is known that parity is conserved with great precision in nucleon-nucleon collisions and nuclear reactions. If there is no departure from isotopic invariance, then parity nonconservation in these processes can appear both as a consequence of the participation of virtual K mesons and hyperons, and also owing to the nonrenormalizability (nonlocal character) of the interaction. As is shown by a calculation carried out in reference 5, the contribution of the K-meson forces to the nucleon-nucleon potential is small, so that the (very precise) parity conservation in nucleon-nucleon interactions is not in contradiction with violation of parity conservation in interactions involving K mesons and hyperons.*

Let us consider the process $\pi + N \rightarrow K + Y$ with the subsequent decay $Y \rightarrow N + \pi$ (Y can be a Λ or a Σ hyperon). As has been shown in reference 4, if parity is not conserved in the production of the K meson and hyperon, there is a longitudinal component of the polarization vector of the hyperon, and this leads to the appearance of an asymmetry in the distribution of the π mesons from the decay of the hyperons (in the center-of-mass system), both relative to the plane perpendicular to the plane of production and containing the direction of the initial π meson, and also relative to the plane perpendicular to the plane of production and perpendicular to the direction of the initial π meson. It is found[4,5] that if there is a longitudi-