POLARIZATION OF BETA PARTICLES AND BETA-GAMMA CORRELATION FOR FIRST-FORBIDDEN TRANSITIONS OF ORIENTED NUCLEI

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Explicit formulas are obtained for the polarization of β particles and the β-γ correlation for first-forbidden transitions of oriented nuclei. All five types of β interaction are considered, and nonconservation of parity is taken into account. Effects of the Coulomb field of the extended nucleus are included. Unoriented nuclei are treated as a special case.

IN an earlier paper by one of the writers[1] general formulas were obtained for the angular and polarization correlations of the particles from β decay transitions of any order of forbiddenness. Here we shall examine first-forbidden transitions in detail, and shall give for this case explicit formulas suitable for practical calculations. Since the method of the calculation has been described in reference 1, we shall present at once the final expressions for the correlations.

We take a right-handed system of coordinates with the z axis along the preferred direction of orientation of the nuclear spin $\mathbf{j}_0$. We introduce the notations: $\mathbf{p}(p, \vartheta, \varphi)$ is the momentum of the electron; $\mathbf{k}(k, \theta, \Phi)$ is the momentum of the γ quantum; I is the multipole order of the γ quantum; j_0, j_1, and j_2 are the angular momenta of the nuclear levels for the β-γ transition $j_0(\beta)j_1(\gamma)j_2$; μ_0, μ_1, and μ_2 are the z components of the angular momenta of the nuclear levels; and $\boldsymbol{\zeta}(1, \chi, \omega)$ is the polarization vector of the electron is the system of coordinates in which it is at rest. χ and ω are the polar angles of the vector $\boldsymbol{\zeta}$ in the right-handed system of coordinates formed by the vectors

$$z_p \parallel \mathbf{p}, \quad x_p \parallel \mathbf{p}\times\mathbf{j}_0, \quad y_p \parallel [\mathbf{p}\times\mathbf{j}_0]\times\mathbf{p}. \tag{1}$$

The probability of finding the electron with the polarization $\boldsymbol{\zeta}$ and the momentum $\mathbf{p}$ from the first-forbidden β decay of the oriented nucleus is given by the formula:*

$$W(\mathbf{j}_0\mathbf{p}\boldsymbol{\zeta}) = \sum_{g,\, L\leqslant L'} A^g_{LL'} h_g(\mathbf{j}_0)\, W(j_1L'j_0g; j_0L), \tag{2}$$

$$h_g(\mathbf{j}_0) = \sum_{\mu_0} (-1)^{j_0-\mu_0} C^{g0}_{j_0\mu_0 j_0-\mu_0}\, w(\mu_0), \tag{3}$$

$$A^g_{LL'} = z^g_{LL'}\sqrt{2g+1}\, P_g(\cos\vartheta) + (-1)^g \sum_m y^m_{LL'} f_m(x, \omega, \vartheta). \tag{4}$$

Here L and L′ = 0, 1, and 2; g = 0, 1, 2, 3, but with $g \leq 2j_0$; m = 0, 1, ... 9, with m = 0 for g = 0, m = 1, 2, 3 for g = 1, m = 4, 5, 6 for g = 2, and m = 7, 8, 9 for g = 3. Common factors have been omitted throughout. If the polarization of the electrons is not observed, we have

$$A^g_{LL'} = z^g_{LL'}\sqrt{2g+1}\, P_g(\cos\vartheta). \tag{5}$$

For aligned nuclei (obtained, for example, by the method of Bleaney[3] or Pound[4]) only even values of g are possible. For arbitrarily oriented nuclei (obtained, for example, by the Gorter-Rose method[5]) odd values of g are also possible. $C^{a\alpha}_{b\beta c}$ are Clebsch-Gordan coefficients,[6] W(abcd; ef) are Racah functions,[7] and $w(\mu_0)$ is the probability of a given value μ_0 for the angular-momentum component of the oriented nucleus. The values of $z^g_{LL'}$ and $y^m_{LL'}$ are given in Appendix 1. The values of $f_m(\chi, \omega, \vartheta)$ are given in Appendix 5, and $P_g(\cos\vartheta)$ is the Legendre polynomial. The values of $j_g(\mathbf{j}_0) \equiv j_0^{-g} h_g(\mathbf{j}_0)$ are given in reference 8.

The angular β-γ correlation for first-forbidden transitions in oriented nuclei has the form

$$W(\mathbf{j}_0, \mathbf{p}, \mathbf{k}) = \sum_{SJg} \sum_{L\leqslant L'} z^J_{LL'} U^{LL'}_{SgJ} h_g(\mathbf{j}_0)\, B_S F_{SgJ}(\mathbf{p}, \mathbf{k}), \tag{6}$$

$$U^{LL'}_{SgJ} = (-1)^{g+J}\sqrt{(2g+1)(2S+1)} \times X(j_1j_1S,\ j_0j_0g,\ L'LJ), \tag{7}$$

*The expression $W(\mathbf{j}_0\mathbf{p}\boldsymbol{\zeta})$ for the STP interactions and without inclusion of the effect of the finite dimensions of the nucleus has also been obtained by Berestetskii, Rudik, Ioffe, and Ter-Martirosyan.[2] We have learned that they have generalized their result, but have neglected in their calculations terms of the order $(pR)^2$ and v_{nuc}^2; these can however, have an important effect for certain values of the nuclear matrix elements.

$$B_S = \sqrt{(2j_1+1)(2I+1)}\,[1 - S(S+1)/2I(I+1)]$$

$$\times W(j_2 I j_1 S; j_1 I)\, C^{I0}_{I0S0}. \qquad (8)$$

Here L and L′ = 0, 1, 2; J = 0, 1, 2, 3; $0 \le g \le 2j_0$, with $|S - J| \le g \le S + J$. As before, only even values of g are possible for aligned nuclei. The quantities X(abc, def, ghi) are Fano functions; their properties and explicit form, together with a number of particular values, are given in reference 6. The $F_{SgJ}(\mathbf{p}, \mathbf{k})$ are given in Appendix 5.

If we are interested in the correlation between the direction of the β particle and the circular polarization of the subsequent γ quantum, we must insert in Eq. (6) or Eq. (14) instead of B_S the quantity

$$B_{SM} = \sqrt{(2I+1)(2j_1+1)}\, C^{IM}_{IMS0}\, W(j_2 I j_1 S; j_1 I), \qquad (9)$$

where M = 1 or −1, respectively, for right or left circular polarization of the γ quantum. In the particular case in which S = 1,

$$B_{1M} = M\,[j_1(j_1+1) - j_2(j_2+1)$$

$$+ I(I+1)]\,[2I(I+1)\sqrt{j_1(j_1+1)}]^{-1}. \qquad (10)$$

If the observed β-γ cascade has the form $j_0(\beta)\, j_1(\gamma_1)\, j_2(\gamma_2) \ldots j_{N-1}(\gamma)\, j_N$, and an experiment is made to study the angular distribution of the γ quantum of the $j_{N-1}(\gamma)\, j_N$ transition, then we must take j_{N-1} instead of j_1, j_N instead of j_2 in the expressions for B_S or B_{SM} and multiply these expressions by the product

$$\prod_{k=2}^{N-1} \sqrt{(2j_k+1)(2j_{k-1}+1)}\, W(j_k I_k S j_{k-1}; j_{k-1} j_k). \qquad (11)$$

If the γ transition is of mixed nature (for example, E2 and M1), then instead of B_S we must take the expression of Eq. (13) of reference 9.

If the nuclei are unoriented, g = 0, and the expression (2) determines the total polarization of the electrons. The degree of polarization can be characterized by the quantity

$$\langle\zeta\rangle = \frac{W(\chi=0) - W(\chi=\pi)}{W(\chi=0) + W(\chi=\pi)}\,\frac{\mathbf{p}}{p}. \qquad (12)$$

Using Eq. (2), we get

$$\langle\zeta\rangle = (\mathbf{p}/p)\,\sum_L y^0_{LL} \Big/ \sum_L z^0_{LL}. \qquad (13)$$

Setting g = 0 in Eq. (6), we get the expression for the β-γ correlation for unoriented nuclei:*

$$W(\mathbf{p}, \mathbf{k}) = \sum_{S=0,2} \sum_{L \le L'} (2S+1)\, z^S_{LL'}$$

$$\times W(j_0 j_1 L S; L' j_1)\, B_S P_S(\cos\theta_{pk}). \qquad (14)$$

In the case in which the nuclear charge $Z \gg 2A^{1/3}E$ the expression (2) can be considerably simplified:

$$W(j_0, \mathbf{p}, \zeta) = z^0_{00} + \sqrt{1/3}\, z^0_{11} + (y^0_{00} + \sqrt{1/3}\, y^0_{11}) \cos\chi$$

$$+ \{\sqrt{3}(z^1_{11} U - z^1_{01}) \cos\vartheta + 3\,[(y^1_{11} - \sqrt{2/5}\, y^3_{11}) U$$

$$- (y^1_{01} - \sqrt{2/5}\, y^3_{01})] \cos\chi \cos\vartheta - 3\,[(y^1_{11} + \sqrt{1/10}\, y^3_{11}) U$$

$$- (y^1_{01} + \sqrt{1/10}\, y^3_{01})] \sin\vartheta \sin\chi \cos\omega$$

$$- 3\sqrt{1/2}\,(y^2_{11} U - y^2_{01}) \sin\vartheta \sin\chi \sin\omega\}$$

$$\times [j_0(j_0+1)]^{-1/2} \sum_{\mu_0} \mu_0 w(\mu_0),$$

$$\sqrt{2}\, U = [j_0(j_0+1) - j_1(j_1+1) + 2]$$

$$\times [2\sqrt{j_0(j_0+1)}]^{-1}. \qquad (15)$$

The values of $z^g_{LL'}$ and $y^m_{LL'}$ as used in Eq. (15) are as follows:

$$\sqrt{3}\, z^0_{11} = \varkappa^2 \xi^-_1 (b_{11} + 2b_{66} + 2\sqrt{2}\, b_{16}),$$

$$3\sqrt{3}\, z^1_{11} = \sqrt{6}\, y^0_{11} = \sqrt{2}\, \varkappa^3 \xi^0_3 (c_{11} + 2c_{66} + 2\sqrt{2}\, c_{16}),$$

$$27 y^1_{11} = \sqrt{2}\, \varkappa^2 [\xi^-_1] (b_{11} + 2b_{66} + 2\sqrt{2}\, b_{16}),$$

$$27 y^3_{11} = 4\sqrt{5}\, \varkappa^2 \lambda^-_1 (b_{11} + 2b_{66} + 2\sqrt{2}\, b_{16}),$$

$$9 y^2_{11} = -2\varkappa^3 \varepsilon^0_3 (c_{11} + 2c_{66} + 2\sqrt{2}\, c_{16}),$$

$$3 z^1_{01} = 2\varkappa^3 \xi^0_3 (c_{12} + \sqrt{2}\, c_{26}),$$

$$9\sqrt{6}\, y^3_{01} = 8\sqrt{5}\, \varkappa^2 \lambda^-_1 (b_{12} + \sqrt{2}\, b_{26}),$$

$$9\sqrt{3}\, y^1_{01} = 2\varkappa^2 [\xi^-_1] (b_{12} + \sqrt{2}\, b_{26}),$$

$$3\sqrt{6}\, y^2_{01} = -4\varkappa^3 \varepsilon^0_3 (c_{12} + \sqrt{2}\, c_{26}),$$

$$z^0_{00} = \varkappa^2 \xi^-_1 b_{22}, \quad y^0_{00} = \varkappa^3 \xi^0_3 c_{22}.$$

Here

$$\varkappa^2 \xi^-_1 b_{ii'} = (W-1)^2 (E+\gamma)\, \mathrm{Re}\, b^+_{ii'} + (W+1)^2 (E-\gamma)\, \mathrm{Re}\, b^-_{ii'},$$

$$\varkappa^2 [\xi^-_1]\, b_{ii'} = -3(W-1)^2 (E+\gamma)\, \mathrm{Re}\, b^+_{ii'}$$

$$+ (W+1)^2 (E-\gamma)\, \mathrm{Re}\, b^-_{ii'},$$

$$\varkappa^2 \lambda^-_1 b_{ii'} = (W+1)^2 (E-\gamma)\, \mathrm{Re}\, b^-_{ii'},$$

$$E \xi^0_3 c_{ii'} = p\, \mathrm{Re}\,(c^+_{ii'} + c^-_{ii'}) - \alpha Z\, \mathrm{Im}\,(c^-_{ii'} - c^+_{ii'}).$$

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*W(p, k) agrees with Eq. (10) of reference 9 if we note that in the notations of reference 9 the quantities $z^S_{LL'}$ are $(2S+1)^{-1}(2L'+1)^{1/2} \sum_{i \le k} [\mathrm{Re} M_{Li} M^*_{L'k}]\, C^{SLL'}_{ik}$. Unlike those of reference 9, our expressions $z^S_{LL'}$ are given for the β interaction of general type with parity nonconservation.

We get the values of $\kappa^2\epsilon_1^- b_{ii'}$ and $E\epsilon_3^0 c_{ii'}$ from $\kappa^2\xi_1^- b_{ii'}$ and $E\xi_3^0 c_{ii'}$ if in the latter we replace $\mathrm{Re}\, x_{ii'}^{\pm}$ by $\mathrm{Im}\, x_{ii'}^{\pm}$ and $\mathrm{Im}\, x_{ii'}^{\pm}$ by $-\mathrm{Re}\, x_{ii'}^{\pm}$. The values of $b_{ii'}$ and $c_{ii'}$ are given in Appendix 1. The forbiddenness of the β transition that comes from the smallness of pR is removed for large values of Z, since instead of p the integrand contains the quantity $\kappa = [(E + V)^2 - 1]^{1/2}$, and $\kappa \gg p$. Since $\kappa \approx V$, where V is the effective depth of the Coulomb well in the region of the nucleus, the dependence on the energy drops out, and the β transition becomes similar to an allowed one. A comparison of Eq. (15) with the formula for the polarization of the β particles from allowed transitions of oriented nuclei[1] shows that the character of the polarization is the same in the two cases (the nuclear matrix elements are of course different).

For $Z \gg 2A^{1/3}E$ the expression (6) also becomes simpler.

$$W(\mathbf{j}_0, \mathbf{p}, \mathbf{k}) = \sum_{S,g} \sqrt{2S+1}\, \{[(2j_0+1)^{-1} z_{00}^0 + \sqrt{1/3}\, W(Sj_0j_11;\; j_0 j_1)\, z_{11}^0]\, \delta_{Sg} P_S(\cos\theta) - (-1)^g \sqrt{2g+1}\, [z_{01}^1 W(j_0Sj_01;\, j_0 g)\, \sqrt{1/3,(2j_0+1)}\, (-1)^g + z_{11}^1 X(j_1j_1S,\, j_0j_0g,\, 111]\, F_{Sg1}(\mathbf{p},\mathbf{k})\}\, h_g(j_0)\, B_S. \quad (16)$$

For the case of an even value of the quantity S + g + J the values of $z_{LL'}^J$ are the same as the corresponding values of $z_{LL'}^g$ in Eq. (16), and in the case of odd values they are obtained from them if we replace $\kappa^2\xi_1^- b_{ii'}$ by $\kappa^2\epsilon_1^- b_{ii'}$ and $E\xi_3^0 c_{ii'}$ by $E\epsilon_3^0 c_{ii'}$. In allowed transitions of unoriented nuclei there is no β-γ correlation. Thus it is to be expected that for $Z \gg 2A^{1/3}E$ the correlation will also be small for first-forbidden transitions. This result can be obtained directly from the explicit form of the $z_{LL'}^S$ appearing in Eq. (14). On the other hand, for β-γ transitions of oriented nuclei we must expect a large correlation in the case $Z \gg 2A^{1/3}E$, since there is such a correlation for allowed transitions. Comparison of Eq. (16) with the expression for the β-γ correlation in allowed transitions[1] shows that the character of the correlation is the same in the two cases.

For aligned nuclei with $Z \gg 2A^{1/3}E$ the correlation between the directions of $\mathbf{j}_0$, $\mathbf{p}$, and $\mathbf{k}$ can be large only if the invariance of the theory under time reversal is violated.

APPENDIX 1

Values of $z_{LL'}^g$ in Eqs. (4) and (5) and, for the case of even values of the quantity S + g + J, in Eqs. (6) and (14):

$$z_{00}^0 = \varkappa^2\varphi_1\xi_1^- b_{22} - 6\varkappa^2\varphi_2\xi_1^0 b_{23} + 9\varphi_0\xi_1^+ b_{33} - 2q\varkappa^2\varphi_2\xi_1^0 a_{22} + 6q\varphi_0\xi_1^+ a_{23} + q^2\varphi_0\rho_1^- b_{22},$$

$$\sqrt{3}\, z_{11}^0 = \varkappa^2\varphi_1 [(\xi_1^- + 2\xi_2^-) b_{11} + (2\xi_1^- + \xi_2^-) b_{66} + 2\sqrt{2}(\xi_1^- - \xi_2^-) b_{16}] + 6\sqrt{3}\varkappa^2\varphi_2\xi_1^0 (\sqrt{2} b_{46} + b_{14}) + 27\varphi_0\xi_1^+ b_{44} - 2q\varkappa^2\varphi_2\xi_1^0 [a_{11} - 2a_{66} + 4\sqrt{2}(a_{61} - a_{16})] - 6\sqrt{3} q\varphi_0\xi_1^+ (a_{14} - \sqrt{2} a_{46}) + 3q^2\varphi_0\rho_1^- (b_{11} + b_{66}),$$

$$\sqrt{15}\, z_{11}^2 = -\varkappa^2\varphi_1 [2\xi_5^- (\sqrt{2} b_{11} - \sqrt{2} b_{66} + 2b_{16} - b_{61}) + \xi_2^- (\sqrt{2} b_{11} + \sqrt{1/2}\, b_{66} - 2b_{16})] - 6\sqrt{3}\varkappa^2\varphi_2\xi_5^0 (\sqrt{2} b_{14} - b_{64}) + 2q\varkappa^2\varphi_2\rho_5^0 (\sqrt{2} a_{11} + \sqrt{2} a_{66} - 2a_{16} - a_{61}),$$

$$z_{22}^0 = \sqrt{5}(\varkappa^2\varphi_1\xi_2^- + q^2\varphi_0\rho_1^-) b_{55}, \quad 2\sqrt{5}\, z_{22}^2 = -\sqrt{14}\varkappa^2\varphi_1\xi_2^- b_{55}$$

$$\sqrt{5}\, z_{02}^2 = -2\sqrt{2}\{\varkappa^2\varphi_1\xi_5^- b_{52} - 3\varkappa^2\varphi_2\xi_5^0 b_{53} - q\varkappa^2\varphi_2\rho_5^0 a_{52}\},$$

$$\sqrt{5}\, z_{12}^2 = -\sqrt{2}\{\varkappa^2\varphi_1 [\xi_5^- (\sqrt{2} b_{51} + 2b_{56}) + \xi_2^- (b_{56} - \sqrt{2} b_{51})] + 3\sqrt{6}\varkappa^2\varphi_2\xi_5^0 b_{54} - q\varkappa^2\varphi_2\rho_5^0 (\sqrt{2} a_{51} - 2a_{56})\},$$

$$3\sqrt{3}\, z_{11}^1 = \sqrt{2}\{\varkappa^3\varphi_1 [\xi_3^0 (c_{11} + 2c_{66} + 2\sqrt{2} c_{16}) + 2\xi_4^0 (c_{66} - c_{11} - \sqrt{2} c_{16} + \sqrt{1/2}\, c_{61}) + \xi_6^0 (c_{11} + 1/2\, c_{66} - \sqrt{2} c_{16})] + 6\sqrt{3}\varkappa\varphi_2 [\xi_3^+ (c_{14} + \sqrt{2} c_{64}) - \xi_4^+ (c_{14} - \sqrt{1/2}\, c_{64})] + 27\varkappa\varphi_0\xi_3^0 c_{44} - 2q\varkappa\varphi_2 [\rho_3^- (d_{11} - 2d_{66} - \sqrt{2} d_{16} + \sqrt{2} d_{61}) - \rho_4^- (d_{66} + d_{11} - \sqrt{2} d_{16} - \sqrt{1/2}\, d_{61})] - 6\sqrt{3} q\varkappa\varphi_0\xi_3^0 (d_{14} - \sqrt{2} d_{64}) + 3q^2\varkappa\varphi_0\rho_3^0 (1/2\, c_{66} - \sqrt{2} c_{16})\},$$

$$\sqrt{35}\, z_{22}^3 = -3\sqrt{2}\varkappa^3\varphi_1\xi_6^0 c_{55},$$

$$\sqrt{30}\, z_{22}^1 = (3\varkappa^3\varphi_1\xi_6^0 + 5q^2\varkappa\varphi_0\rho_3^0) c_{55},$$

$$3z_{01}^1 = 2\{\varkappa^3\varphi_1 [\xi_3^0 (c_{12} + \sqrt{2} c_{26}) + \xi_4^0 (2c_{12} - \sqrt{2} c_{62})] + 3\varkappa\varphi_2 [\xi_3^+ (\sqrt{3} c_{24} - \sqrt{2} c_{63} - c_{13}) + \xi_4^+ (\sqrt{2} c_{63} - 2c_{13})] - 2q\varkappa\varphi_2 [\rho_3^- (d_{12} + \sqrt{1/2}\, d_{26} - \sqrt{1/2}\, d_{62}) + \rho_4^- (d_{12} - \sqrt{1/2}\, d_{62})] - 3q\varkappa\varphi_0\xi_3^0 (\sqrt{3} d_{42} - d_{31} + \sqrt{2} d_{36}) - 9\sqrt{3}\varkappa\varphi_0\xi_3^0 c_{34} + q^2\varkappa\varphi_0\rho_3^0 (c_{12} - \sqrt{2} c_{26})\},$$

$$3z_{12}^1 = -\sqrt{5}\{\varkappa^3\varphi_1\xi_4^0 (2c_{51} + \sqrt{2} c_{56}) - 6\sqrt{3}\varkappa\varphi_2\xi_4^+ c_{54} - q\varkappa\varphi_2\rho_4^- (2d_{51} - \sqrt{2} d_{56}) + 2q^2\varkappa\varphi_0\rho_3^0 (2c_{15} + \sqrt{2} c_{56})\},$$

$$\sqrt{35}\, z_{12}^3 = 6\varkappa^3\varphi_1\xi_6^0 (c_{65} - \sqrt{2} c_{15}).$$

Values of $y_{LL'}^m$ in Eq. (4):

$$y_{00}^0 = \varkappa^3\varphi_1\xi_3^0 c_{22} - 6\varkappa\varphi_2\xi_3^+ c_{23} + 9\varkappa\varphi_0\xi_3^0 c_{33} - 2q\varkappa\varphi_2\rho_3^- d_{22} + 6q\varkappa\varphi_0\xi_3^0 d_{23} + q^2\varkappa\varphi_0\rho_3^0 c_{22},$$

$$9\sqrt{6}\, y_{01}^3 = 4\sqrt{5}\{\varkappa^2\varphi_1 \left[2\xi_1^- (b_{12} + \sqrt{2} b_{26}) + \xi_5^- \left(b_{12} - \sqrt{1/2}\, b_{26}\right)\right] + 3\varkappa^2\varphi_2 [2\xi_1^0 (\sqrt{3} b_{24} - \sqrt{2} b_{36} - b_{13}) + \xi_5^0 (\sqrt{1/2}\, b_{36} - b_{13})] - q\varkappa^2\varphi_2 [2\rho_1^0 (a_{21} + a_{12} - \sqrt{2} a_{26} + \sqrt{2} a_{62}) + \rho_5^0 (a_{12} - \sqrt{1/2}\, a_{62})] - 6q\varphi_0\xi_1^+ (\sqrt{3} a_{24} - a_{13} + \sqrt{2} a_{63}) - 18\sqrt{3}\varphi_0\xi_1^+ b_{34} + 2q^2\varphi_0\rho_1^- (b_{12} - \sqrt{2} b_{26})\},$$

$$5\sqrt{5}\,y_{02}^{4} = 2\{\varkappa^3\varphi_1\xi_4^0 c_{52} - 3\varkappa\varphi_2\xi_4^+ c_{53} - q\varkappa\varphi_2\rho_4^- d_{52}\},$$

$$-15\sqrt{14}\,y_{22}^{4} = 30\,y_{22}^{5} = 7\sqrt{7}\,z_{22}^{3},$$

$$27\,y_{11}^{1} = \sqrt{6}\,m_{11}^{0} - 2\sqrt{15}\,m_{11}^{2} - 3\sqrt{2}\,Q_1,$$

$$27\,y_{11}^{3} = 5\sqrt{6}\,n_{11}^{2} + 4\sqrt{15}\,n_{11}^{0} - 12\sqrt{5}\,P_1,$$

$$\sqrt{2}\,y_{11}^{0} = 3m_{11}^{1} - \sqrt{3}\,Q_3,$$

$$3y_{11}^{2} = -\sqrt{6}\,s_{11}^{1},\quad 5\sqrt{10}\,y_{11}^{4} = 3t_{11}^{1},\quad 5\sqrt{5}\,y_{11}^{1} = -3\sqrt{7}\,n_{11}^{1},$$

$$3y_{22}^{7} = -2s_{22}^{3},\quad y_{22}^{0} = \sqrt{6}\,m_{22}^{1},\quad 3\sqrt{2}\,y_{22}^{1} = -m_{22}^{0},$$

$$\sqrt{3}\,y_{22}^{2} = -\sqrt{2}\,s_{22}^{1},\quad 3\sqrt{5}\,y_{22}^{3} = n_{22}^{0},\quad 7y_{22}^{8} = -2\sqrt{5}\,m_{22}^{2},$$

$$49y_{22}^{9} = 24\sqrt{3}\,n_{22}^{2},\quad \sqrt{6}\,y_{01}^{2} = -2s_{01}^{1},\quad \sqrt{2}\,y_{02}^{6} = s_{02}^{2},$$

$$\sqrt{6}\,y_{12}^{2} = 2s_{12}^{1},\quad \sqrt{2}\,y_{12}^{6} = s_{12}^{2},\quad 3y_{12}^{7} = -2s_{12}^{3},$$

$$7\sqrt{35}\,y_{12}^{8} = 5\sqrt{2}\,t_{12}^{2},\quad 9\sqrt{3}\,y_{12}^{1} = 5m_{12}^{2} - 2\sqrt{5}\,Q_2,$$

$$9\sqrt{6}\,y_{12}^{3} = 5(\sqrt{5}\,n_{12}^{2} - 8P_2),\quad 25\sqrt{2}\,y_{12}^{4} = 3n_{12}^{1} - {}^1/_3\sqrt{14}\,z_{12}^{3},$$

$$5y_{12}^{5} = {}^7/_3\sqrt{{}^1/_2}\,z_{12}^{3} + {}^6/_5\sqrt{7}\,m_{12}^{1}.$$

In the expressions for $z_{LL'}^{g}$ and $y_{LL'}^{m}$ the following notations are used: $\kappa^2 = W^2 - 1$, $W = E + V$, where E is the energy of the electron in units mc^2, including the rest energy ($\hbar = m = c = 1$). For the case of a surface distribution of charge $V = \alpha Z/R$, where R is the radius of the nucleus; for a uniform volume distribution $V = 3\alpha Z/2R$. The energy of the neutrino is denoted by q.

$$\xi_l^{\pm} x_{ii'} \equiv (W \pm 1)(\beta_l^+ \,\mathrm{Re} + \alpha_l^+ \,\mathrm{Im})\, x_{ii'}^{+}$$

$$+ (W \mp 1)(\beta_l^- \,\mathrm{Re} + \alpha_l^- \,\mathrm{Im})\, x_{ii'}^{-},$$

$$\xi_l^{0} x_{ii'} \equiv (\beta_l^+ \,\mathrm{Re} + \alpha_l^+ \,\mathrm{Im})\, x_{ii'}^{+} + (\beta_l^- \,\mathrm{Re} + \alpha_l^- \,\mathrm{Im})\, x_{ii'}^{-}.$$

The quantities $\beta_l^{\pm}$ and $\alpha_l^{\pm}$ are determined from the condition of smooth joining of the electron wave function at the surface of the nucleus. The explicit forms for these quantities are given in Appendix 3. The quantity $x_{ii'}$ is any one of the quantities $a_{ii'}$, $b_{ii'}$, $c_{ii'}$, or $d_{ii'}$.

For the STP or VA interactions $\rho_l^0 = \xi_l^0$ and $\rho_l^{\pm} = \xi_l^{\pm}$. The meanings of $\rho_l^{\pm} x_{ii'}$ and $\rho_l^0 x_{ii'}$ become the same as those of $\xi_l^{\pm} x_{ii'}$ and $\xi_l^0 x_{ii'}$, respectively, if one replaces all the β_l^+ by β_l^-, all the α_l^+ by α_l^-, and vice versa. The expressions for $z_{LL'}^{J}$ for odd $S + g + J$ can be obtained if in the values of $z_{LL'}^{g}$ given above one replaces $\mathrm{Re}\, x_{ii'}^{\pm}$ by $\mathrm{Im}\, x_{ii'}^{\pm}$ and $\mathrm{Im}\, x_{ii'}^{\pm}$ by $-\mathrm{Re}\, x_{ii'}^{\pm}$. In calculating y_{01}^{3} one must replace β_5^+ and α_5^+ by $3\beta_5^+$ and $3\alpha_5^+$ and set $\beta_1^+ = \alpha_1^+ = 0$. In the calculation of y_{02}^{4} one must put $-5\beta_4^+$ and $-5\alpha_4^+$ instead of β_4^+ and α_4^+. We get y_{01}^{1} from y_{01}^{3} if we multiply β_5^- and α_5^-, β_1^- and α_1^- by $-2(^2/_5)^{1/2}$, $(^2/_5)^{1/2}/4$, respectively, and set $\beta_1 = -3\beta_1^-$, $\alpha_1^+ = -3\alpha_1^-$, and $\beta_5^+ = \alpha_5^+ = 0$. We get y_{02}^{5} from y_{02}^{4} if we set $\beta_4^+ = \alpha_4^+ = 0$ and multiply β_4^- and α_4^- by $2(14)^{1/2}$. We get $m_{LL'}^{g}$, $n_{LL'}^{g}$, and $t_{LL'}^{g}$ in the corresponding $z_{LL'}^{g}$ we replace $\beta_l^{\pm}$ by $\gamma_l^{\pm}\beta_l^{\pm}$ and $\alpha_l^{\pm}$ by $\gamma_l^{\pm}\alpha_l^{\pm}$, using the values of $\gamma_l^{\pm}$ from the table. We get $s_{LL'}^{g}$ if in addition to this we replace $\mathrm{Re}\, x_{ii'}^{\pm}$ by $\mathrm{Im}\, x_{ii'}^{\pm}$ and $\mathrm{Im}\, x_{ii'}^{\pm}$ by $\mathrm{Re}\, x_{ii'}^{\pm}$.

Odd values of g

$\gamma_l^{\pm}$	m_{11}^1	s_{11}^1	n_{11}^1	t_{11}^1	m_{22}^1	s_{22}^1	s_{01}^1	n_{12}^1	m_{12}^1	s_{12}^1
$\gamma_3^{\pm}$	1	1	0	0	1	1	1	0	0	1
γ_4^{+}	0	—1/2	0	—5			—1/2	—5	0	1/2
γ_4^{-}	0	1/2	—2	1			1/2	1	1	—1/2
$\gamma_6^{\pm}$	2	2	1	2	5/3	2				

Even values of g

$\gamma_l^{\pm}$	m_{11}^0	m_{11}^2	n_{11}^2	n_{11}^0	m_{22}^0	m_{22}^2	n_{22}^0	n_{22}^2	s_{02}^2	m_{12}^2	n_{12}^2	s_{12}^2	t_{12}^2
γ_1^{+}	—3			0	1		0						
γ_1^{-}	1			1	—1/3		10/3						
γ_2^{+}	3/2	0	—3	0	—3/5	—1/7	3	1		—3/5	—6/5	0	1
γ_2^{-}	—5/2	0	1	0	1	1	—1	0		1	2/5	0	—7
γ_5^{+}		0	3/2						—1	0	3	—1	0
γ_5^{-}		1	1/2						1	—2	1	1	0

$$Q_1 = q^2\varphi_0[\rho_1^-](b_{11} + {}^1/_2 b_{66} + \sqrt{2}b_{16}),$$
$$Q_2 = q^2\varphi_0[\rho_1^-](b_{15} + \sqrt{{}^1/_2}b_{65}),$$
$$Q_3 = q^2\varkappa\varphi_0\rho_3^0(c_{11} + {}^1/_2 c_{66} + \sqrt{2}c_{16}).$$

We get $[\rho_1^-]$ if in the expressions for ρ_1^- we replace the quantities β_1^+ and α_1^+ by $-3\beta_1^+$ and $-3\alpha_1^+$. We get the values of P_1 and P_2 from Q_1 and Q_2 if we set $\beta_1^- = \alpha_1^- = 0$ and do not multiply β_1^+ and α_1^+ by -3. All the other $z_{LL'}^g$ and $y_{LL'}^m$ are zero. The quantities $x_{ii'}$ for the STP interaction have the form:

$$a_{ii'}^{\pm} = b_{ii'}^{\pm} = \eta_{ii'}K_iK_{i'}^*, \quad c_{ii'}^{\pm} = d_{ii'}^{\pm} = \eta'_{ii'}K_iK_{i'}^*,$$
$$\eta_{ii'} = C_iC_{i'}^* + C'_iC'^*_{i'}, \quad \eta'_{ii'} = C_iC'^*_{i'} + C'_iC^*_{i'},$$
$$\eta_{11} = |C_S|^2 + |C'_S|^2, \quad \eta_{33} = |C_P|^2 + |C'_P|^2,$$
$$\eta_{13} = C_SC_P^* + C'_SC'^*_P.$$
$$\eta_{1k} = C_SC_T^* + C'_SC'^*_T, \quad \eta_{3k} = C_PC_T^* + C'_PC'^*_T \text{ for } k \neq 1, 3.$$
$$\eta_{kk'} = |C_T|^2 + |C'_T|^2 \text{ for } k, k' = 2, 4, 5, 6,$$
$$\eta_{ii'} = \eta_{i'i}^*.$$

The quantities C_S, C_T, C_P, C_V, and C_A are the constants for the scalar, tensor, pseudoscalar, vector, and axial-vector β interactions; C'_S, C'_T, and so on are the analogous constants for the terms that can be admitted only since there is parity nonconservation (i.e., the terms containing an additional factor $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, $\gamma_4 = -\beta$); the K_i are the nuclear matrix elements for the β interaction. Their explicit form is given in Appendix 2. For the VA interaction

$$b_{ii'}^{\pm} = -a_{ii'}^{\pm} = \zeta_{ii'}L_iL_{i'}^*, \quad d_{ii'}^{\pm} = -c_{ii'}^{\pm} = \zeta'_{ii'}L_iL_{i'}^*,$$
$$\zeta_{ii'} = C_iC_{i'}^* + C'_iC'^*_{i'}, \quad \zeta'_{ii'} = C_iC'^*_{i'} + C'_iC^*_{i'},$$
$$\zeta_{11} = \zeta_{44} = \zeta_{14} = C_V{}^2 + C'_V{}^2,$$
$$\zeta_{kk'} = C_A{}^2 + C'_A{}^2 \quad \text{for } k, k' \neq 1, 4;$$
$$\zeta_{1k} = \zeta_{4k} = C_VC_A^* + C'_VC'^*_A \quad \text{for } k \neq 1, 4,$$
$$\zeta_{ii'} = \zeta_{i'i}^*.$$

The nuclear matrix elements L_i differ from the corresponding K_i by an additional matrix β in the integrand. The expressions for $x_{ii'}$ in the general case of STPVA interactions are given in Appendix 4. Equations (2) and (6) contain the imaginary part of $x_{ii'}$ because we have not assumed the invariance of the β interaction with respect to time reversal. If we make this assumption all terms containing $\mathrm{Im}\, x_{ii'}^{\pm}$ must be thrown out, and comparison with experiment becomes considerably simpler. To settle the question of the invariance of the theory with respect to time reversal it is better to study the β–γ correlation and the polarization of the β particles from allowed transitions of oriented nuclei,[1] since in that case the interpretation of experiments is simpler and more unambiguous.

The values of $z_{LL'}$ and $y_{LL'}$ given above are for the case of a surface distribution of the nuclear charge. If we assume a volume charge distribution, we must make the following replacements:

$$\varphi_0 \to \varphi_3, \quad \varphi_1\xi_1^- \to \varphi_4\xi_1^- - (3\alpha Z/5R)\,\xi_1^0 + (3\alpha Z/10\varkappa R)^2\,\xi_1^+,$$
$$\varphi_2\xi_1^0 \to \varphi_5\xi_1^0 - (3\alpha Z/10\varkappa^2 R)\,\xi_1^+, \quad \varphi_1\xi_2^- \to \varphi_6\xi_2^-,$$
$$\varphi_1\xi_5^- \to \varphi_7\xi_5^- - (3\alpha Z/10R)\,\xi_5^0, \quad \varphi_2\xi_5^0 \to \varphi_8\xi_5^0,$$
$$\varphi_1\xi_3^0 \to [\varphi_4 - 3\alpha ZW/5R\varkappa^2 + (3\alpha Z/10\varkappa R)^2]\,\xi_3^0,$$
$$\varphi_1\xi_4^0 \to \varphi_7\xi_4^0 - (3\alpha Z/10\varkappa^2 R)\,\xi_4^-, \quad \varphi_2\rho_5^0 \to \varphi_8\rho_5^0,$$
$$\varphi_2\xi_3^+ \to \varphi_5\xi_3^+ - (3\alpha Z/10R)\,\xi_3^0, \quad \varphi_1\xi_6^0 \to \varphi_6\xi_6^0,$$
$$\varphi_2\rho_3^- \to \varphi_5\rho_3^- - (3\alpha Z/10R)\,\rho_3^0, \quad \varphi_2\xi_4^+ \to \varphi_8\xi_4^+,$$
$$\varphi_2\rho_4^- \to \varphi_8\rho_4^-;$$
$$V = \alpha Z/R \to 3\alpha Z/2R.$$

The values of the φ_i are given in Appendix 3. Apart from terms of the order $(\alpha Z)^2/4$ one can set $\varphi_i = 1$.

To go over to the case of a point nucleus one must take the formulas for the surface charge distribution and set $V = 3\alpha Z/2R$.

APPENDIX 2

$$K_i = \sqrt{4\pi}\int \psi_{j_1\mu_1}^*\tau O_i\psi_{j_0\mu_0}d\mathbf{r}, \quad L_i = \sqrt{4\pi}\int \psi_{j_1\mu_1}^*\tau\beta O_i\psi_{j_0\mu_0}d\mathbf{r},$$
$$O_1 = C_1 r\beta Y_{1\lambda}, \quad O_2 = C_0 r\beta(\boldsymbol{\sigma}\cdot\mathbf{Y}'_{00}),$$
$$O_3 = -iC_0\beta\gamma_5 Y_{00}, \quad O_4 = iC_1\beta(\boldsymbol{\alpha}\cdot\mathbf{Y}_{1\lambda}^{-1}),$$
$$O_5 = C_2 r\beta(\boldsymbol{\sigma}\mathbf{Y}_{2\lambda}^{-1}), \quad O_6 = -C_1 r\beta(\boldsymbol{\sigma}\cdot\mathbf{Y}_{1\lambda}^0),$$
$$C_L \equiv [C_{L\lambda j_0\mu_0}^{j_1\mu_1}]^{-1},$$
$$K_1 = \sqrt{3}\int \beta\mathbf{r}, \quad K_2 = \int \beta\,\boldsymbol{\sigma}\cdot\mathbf{r}, \quad K_3 = -i\int \beta\gamma_5,$$
$$K_4 = \int i\beta\boldsymbol{\alpha}, \quad K_5 = \frac{\sqrt{3}}{2}\int B_{ij}^{\beta}, \quad K_6 = -\sqrt{\frac{3}{2}}\int \beta[\boldsymbol{\sigma}\times\mathbf{r}],$$
$$L_1 = \sqrt{3}\int \mathbf{r}, \quad L_2 = \int \boldsymbol{\sigma}\cdot\mathbf{r}, \text{ and so on.}$$

$\psi_{j_0\mu_0}$ and $\psi_{j_1\mu_1}$ are the wave functions of the initial and final states of the nucleus; τ is an operator that acts on the isotopic spin variables; β, γ_5, $\boldsymbol{\sigma}$, and $\boldsymbol{\alpha}$ are Dirac matrices; $\mathbf{Y}_{JM}$ is a vector spherical harmonic (cf. reference 1).

APPENDIX 3

$$\beta_1^{\pm} = a_{1/2\,\mp 1/2}^2[(W \pm 1)\varkappa]^{-1},$$
$$\beta_2^{\pm} = 9a_{3/2\,\pm 1/2}^2[(W \pm 1)\varkappa^3]^{-1},$$
$$\beta_3^{\pm} = a_{1/2\,1/2}a_{1/2\,-1/2}\varkappa^{-2}\cos(\delta_{1/2\,\mp 1/2} - \delta_{1/2\,\pm 1/2}),$$
$$\beta_4^{\pm} = 3a_{1/2\,\mp 1/2}a_{3/2\,\mp 1/2}[(W \pm 1)\varkappa^2]^{-1}\cos(\delta_{1/2\,\mp 1/2} - \delta_{3/2\,\mp 1/2}),$$
$$\beta_5^{\pm} = 3a_{1/2\,\mp 1/2}a_{3/2\,\pm 1/2}\varkappa^{-3}\cos(\delta_{1/2\,\mp 1/2} - \delta_{3/2\,\pm 1/2}),$$
$$\beta_6^{\pm} = 9a_{3/2\,1/2}a_{3/2\,-1/2}\varkappa^{-4}\cos(\delta_{3/2\,\pm 1/2} - \delta_{3/2\,\mp 1/2}).$$

Here the $a_{j\lambda}$ are the coefficients for joining-on the electron wave function at the edge of the nucleus;

j is the total angular momentum and $l = j + \lambda$ the orbital angular momentum of the electron, $\lambda = \pm\frac{1}{2}$. Tables of the numerical values are given in the book of Sliv and Volchek.[10] The $\delta_{j\lambda}$ are the phase shifts of the electron wave function in the Coulomb field of the nucleus. Tables of the $\delta_{j\lambda}$ are also given in reference 10.

The quantities $\alpha_l^\pm$ differ from the corresponding $\beta_l^\pm$ only by the fact that they contain $\sin(\delta_{j\lambda} - \delta_{j'\lambda'})$ instead of $\cos(\delta_{j\lambda} - \delta_{j'\lambda'})$, and $\alpha_1^\pm = \alpha_2^\pm = 0$. Neglecting terms smaller than the order of $(\alpha Z)^2$, we can obtain simple expressions for $\beta_l^\pm$ and $\alpha_l^\pm$:

$$\beta_1^\pm = (E \pm \gamma)/(W \pm 1), \quad \beta_2^\pm = [p^2 + (\alpha ZE)^2]\,\varkappa^{-2}\beta_1^\mp,$$

$$\beta_3^\pm = p^2/\varkappa\sqrt{p^2 + (\alpha Z)^2}, \quad \beta_4^\pm = \sqrt{p^2 + (\alpha ZE)^2}\,\varkappa^{-1}\beta_1^\pm \cos\omega_1^\pm,$$

$$\beta_5^\pm = \sqrt{p^2 + (\alpha ZE)^2}\,p\varkappa^{-2}\cos\omega_2^\pm,$$

$$\beta_6^\pm = 2p^2[p^2 + (\alpha ZE)^2]/\varkappa^3\sqrt{4p^2 + (\alpha Z)^2}, \quad \alpha_3^\pm = \pm\alpha Z\beta_3^\pm/p,$$

$$\alpha_4^\pm = \beta_4^\pm \tan\omega_1^\pm, \quad \alpha_5^\pm = \beta_5^\pm \tan\omega_2^\pm, \quad \alpha_6^\pm = \mp\frac{\alpha Z}{2p}\beta_6^\pm,$$

$$\tan\omega_1^\pm = \left[\frac{3}{4}\frac{\alpha ZE}{p} \pm \frac{1}{4}\frac{\alpha Z}{p}\right]$$

$$\times\left[1 + \frac{3}{8}\left(\frac{\alpha Z}{p}\right)^2 - (\alpha Z)^2 \mp \frac{1}{8}\left(\frac{\alpha Z}{p}\right)^2 E - \left(\frac{\alpha ZE}{p}\right)^2\right]^{-1},$$

$$\tan\omega_2^\pm = \left[\frac{3}{4}\frac{\alpha ZE}{p} \pm \frac{3}{4}\frac{\alpha Z}{p}\right]$$

$$\left[1 - \frac{1}{8}\left(\frac{\alpha Z}{p}\right)^2 - (\alpha Z)^2 \mp \frac{3}{8}\left(\frac{\alpha Z}{p}\right)^2 E + \left(\frac{\alpha ZE}{p}\right)^2\right]^{-1},$$

$$\gamma = \sqrt{1 - (\alpha Z)^2}.$$

$$\varphi_0 = 1 - (\varkappa R)^2/5, \quad \varphi_1 = 1 - 3(\varkappa R)^2/25, \quad \varphi_2 = 1 - 4(\varkappa R)^2/25,$$

$$\varphi_3 = 1 - 11(\varkappa R)^2/75, \quad \varphi_4 = 1 - 2(\varkappa R)^2/35, \quad \varphi_5 = 1 - 7(\varkappa R)^2/75,$$

$$\varphi_6 = 1 - 3(\varkappa R)^2/35, \quad \varphi_7 = 1 - 12(\varkappa R)^2/175,$$

$$\varphi_8 = 1 - 4(\varkappa R)^2/35.$$

APPENDIX 4

For the STPVA interactions the quantities $a_{ii'}$, $b_{ii'}$, and so on have the forms:

$$a_{ii'}^\pm = (\eta_{ii'}K_iK_{i'}^* - \zeta_{ii'}L_iL_{i'}^*) \pm (\chi_{ii'}K_iL_{i'}^* - \chi_{i'i}^*L_iK_{i'}^*),$$

$$b_{ii'}^\pm = (\eta_{ii'}K_iK_{i'}^* + \zeta_{ii'}L_iL_{i'}^*) \pm (\chi_{ii'}K_iL_{i'}^* + \chi_{i'i}^*L_iK_{i'}^*),$$

$$c_{ii'}^\pm = (\eta'_{ii'}K_iK_{i'}^* - \zeta'_{ii'}L_iL_{i'}^*) \pm (\chi'_{ii'}K_iL_{i'}^* - \chi'^*_{i'i}L_iK_{i'}^*),$$

$$d_{ii'}^\pm = (\eta'_{ii'}K_iK_{i'}^* + \zeta'_{ii'}L_iL_{i'}^*) \pm (\chi'_{ii'}K_iL_{i'}^* + \chi'^*_{i'i}L_iK_{i'}^*).$$

The values of $\eta_{ii'}$, $\eta'_{ii'}$, $\zeta_{ii'}$, and $\zeta'_{ii'}$ are the same as in Appendix 1. For $\chi_{ii'}$ we get:

$$\chi_{11} = \chi_{14} = C_SC_V^* + C'_SC'^*_V; \quad \chi_{31} = \chi_{34} = C_PC_V^* + C'_PC'^*_V;$$

$$\chi_{k1} = \chi_{k4} = C_TC_V^* + C'_TC'^*_V, \quad k \neq 1, 3, \quad \chi_{ii'} \neq \chi_{i'i}^*;$$

$$\chi_{1k} = C_SC_A^* + C'_SC'^*_A, \quad \chi_{3k} = C_PC_A^* + C'_PC'^*_A, \quad k \neq 1, 4;$$

$$\chi_{kk'} = C_TC_A^* + C'_TC'^*_A, \quad k \neq 1, 3, k' \neq 1, 4.$$

If $\chi_{ii'} = C_iC_{i'}^* + C'_iC'^*_{i'}$, then $\chi'_{ii'} = C_iC'^*_{i'} + C'_iC_{i'}^*$.

APPENDIX 5

Values of $f_m(\chi, \omega, \vartheta)$ in (4) and $F_{SgJ}(\mathbf{p}, \mathbf{k})$ in (6).

$$f_0 = \cos\chi, \quad f_1 = 3\{\cos\chi\cos\vartheta - \sin\chi\sin\theta\cos\omega\},$$

$$f_2 = 3\sqrt{1/2}\sin\chi\sin\vartheta\sin\omega,$$

$$f_3 = -3\sqrt{1/10}\{2\cos\chi\cos\vartheta + \sin\chi\sin\vartheta\cos\omega\},$$

$$f_4 = -\sqrt{5/2}\{\cos\chi(3\cos^2\vartheta - 1) - 3\sin\chi\sin\vartheta\cos\vartheta\cos\omega\},$$

$$f_5 = 3/2\sqrt{5/7}\{\cos\chi(3\cos^2\vartheta - 1) + 2\sin\chi\sin\vartheta\cos\vartheta\cos\omega\},$$

$$f_6 = -3\sqrt{5/2}\sin\chi\sin\vartheta\cos\vartheta\sin\omega,$$

$$f_7 = 3/4\sqrt{7}\sin\chi\sin\vartheta(5\cos^2\vartheta - 1)\sin\omega,$$

$$f_8 = 3/2\sqrt{7/5}\{\cos\chi(5\cos^3\vartheta - 3\cos\vartheta)$$

$$- \sin\chi\sin\vartheta(5\cos^2\vartheta - 1)\cos\omega\},$$

$$f_9 = -1/4\sqrt{7/3}\{4\cos\chi(5\cos^3\vartheta - 3\cos\vartheta)$$

$$+ 3\sin\chi\sin\vartheta(5\cos^2\vartheta - 1)\cos\omega\},$$

$$F_{SgJ}(\mathbf{p}, \mathbf{k}) = 4\pi i^\gamma \sum C_{g,S\sigma}^{J\sigma} Y_{S\sigma}(\theta\Phi) Y_{J\sigma}^*(\vartheta\varphi),$$

where $$2\gamma = 1 - (-1)^{S+g+J}.$$

Some particular values of $F_{SgJ}(p, k)$ can be expressed in terms of the values of $f_m(-\theta, \varphi - \Phi, \vartheta)$:

$$F_{110} = -f_0, \quad F_{101} = f_1,$$

$$F_{111} = -f_2, \quad F_{121} = f_3, \quad F_{112} = -f_4,$$

$$F_{132} = -f_5, \quad F_{122} = f_6, \quad F_{133} = -f_7, \quad F_{123} = f_8, \quad F_{143} = f_9.$$

[1] A. Z. Dolginov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1363 (1956), Soviet Phys. JETP **6**, 1047 (1958). Nucl. Phys. **5**, 512 (1958).

[2] Berestetsky, Ioffe, Rudik, and Ter-Martirosyan, Nucl. Phys. **5**, 464 (1958).

[3] B. Bleaney, Proc. Phys. Soc. **A64**, 315 (1951).

[4] R. V. Pound, Phys. Rev. **76**, 1410 (1949).

[5] Simon, Rose, and Jauch, Phys. Rev. **84**, 1155 (1951).

[6] E. Condon and G. Shortley, The Theory of Atomic Spectra (Macmillan, N. Y. 1935). H. Matsunobu and H. Takebe, Prog. Theor. Phys. **14**, 589 (1955).

[7] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953). H. Jahn, Proc. Roy. Soc. **205**, 192 (1951).

[8] J. A. M. Cox and H. A. Tolhoek, Physica **19**, 673 (1953).

[9] A. Z. Dolginov and I. N. Toptigin, Nucl. Phys. **2**, 147 (1956/57).

[10] L. A. Sliv and B. A. Volchek, Таблицы кулоновских фаз и амплитуд с учетом конечных размеров ядра (Tables of Coulomb Phases and Amplitudes, Including Effects of the Finite Dimensions of the Nucleus), AN SSSR Press 1956.

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89