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POLARIZATION OF BETA-ELECTRONS FROM RaE

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Effects due to parity nonconservation in the β decay of RaE are studied. A formula is derived for the longitudinal polarization of the β electrons. It is found that the magnitude of the longitudinal polarization does not equal v/c. Throughout the calculation the possibility of violation of time reversal invariance is allowed for. The experimental data on the magnitude of the polarization of RaE β electrons severely restrict the region of possible violation of time reversal invariance.

EVER since the discovery of parity nonconservation in weak interactions the β decay of RaE has aroused a lot of interest.

First, Lewis[1] and Fujita et al.[2] pointed out the sensitivity of the β spectrum of RaE to a possible violation of time-reversal invariance. Secondly Alikhanov made the suggestion, confirmed by experiment,[3] that the longitudinal polarization of β electrons from RaE should differ from v/c and the amount of this difference should, generally speaking, depend on the possible violation of time reversal invariance.

Below we derive an expression for the longitudinal polarization of β electrons from RaE taking into account possible violation of time reversal invariance.

1. BETA SPECTRUM OF RaE

The β spectrum of RaE (a $1^- \rightarrow 0^+$ transition) is the only known example among first forbidden transitions of this type which does not have an allowed shape. This fact was successfully explained within the framework of modern β decay theory which takes into account the variation of the electron wave function within the nucleus.[4-7] In these calculations β decay was assumed to be invariant under time reversal.

Below we follow the procedure of Takebe, Nakamura and Taketani,[7] but without assuming that β decay is invariant under the parity (or, possibly, time reversal) operation. Using the two-component theory of the neutrino and the methods of Berestetskiĭ, Ioffe, Rudik, and Ter-Martirosyan[8] we find the following expression for the correction factor of the RaE spectrum shape (see Appendix I):

$$C(Z, R_0, W) = C_0(Z, R_0, W) + \Delta C.$$

S and T Interaction

$$C_0(Z, R_0, W) = \overline{M}_1(1+x)^2 + \overline{L}_1[{}^1/_3 q^2x^2 + y^2 + {}^1/_6 q^2 + {}^2/_3 qy(x-1)] + \overline{N}_1[{}^2/_3 q(x^2-1) + 2y(1+x)] + {}^1/_2\overline{L}_2(2x-1)^2, \quad (1')$$

$$\Delta C = [\overline{M}_1 + 2\overline{L}_2 + {}^1/_3 q^2\overline{L}_1 + {}^2/_3 q\overline{N}_1]\, x^2F^2. \quad (1'')$$

Here $\overline{M}_1$, $\overline{L}_1$, $\overline{L}_2$, and $\overline{N}_1$ are tabulated[7] functions of the total electron energy W (in the following W is given in units of mc^2). If variation of the electron wave function in the nucleus is ignored we have $\overline{M}_1 = M_0$, $\overline{L}_1 = L_0$, $\overline{L}_2 = L_1$ and $\overline{N}_1 = N_0$, where M_0, L_0, L_1, and N_0 are given in references 9 and 10. The two arbitrary parameters x and y denote ratios of matrix elements:

$$x = ig_S\int\beta\mathbf{r}\,/\,g_T\int\beta[\sigma\times\mathbf{r}]; \qquad y = \int\beta\boldsymbol{\alpha}\,/\int\beta[\sigma\times\mathbf{r}].$$

It is assumed that violation of time reversal invariance manifests itself in a complex value of the scalar interaction coupling constant: $G_S = g_S(1 + iF)$, $G_T = g_T$, where g_S, g_T and F are real.

V and A Interaction

$$C_0(Z, R_0, W) = \overline{M}_1(1+x)^2 + \overline{L}_1[{}^1/_3 q^2x^2 + y^2 + {}^1/_6 q^2 - {}^2/_3 qy(x-1)] + \overline{N}_1[-{}^2/_3 q(x^2-1) + 2y(1+x)] + {}^1/_2\overline{L}_2(2x-1)^2 \quad (2')$$

$$\Delta C = [\overline{M}_1 + {}^1/_2\overline{L}_2 + {}^1/_6 q^2\overline{L}_1 + {}^2/_3 q\overline{N}_1]\, F^2. \quad (2'')$$

Now x and y are given by

$$x = ig_V\int\mathbf{r}\,/\,g_A\int[\sigma\times\mathbf{r}]; \qquad y = g_V\int\boldsymbol{\alpha}\,/\,g_A\int[\sigma\times\mathbf{r}].$$

and it is assumed that violation of time reversal invariance manifests itself in a complex value of the axial vector interaction coupling constant; $G_V = g_V$, $G_A = g_A(1 + iF)$ (g_V, g_A, F — real).

2. POLARIZATION OF BETA ELECTRONS FROM RaE

We write the polarization of the β electrons from RaE in the form

$$\langle\sigma\rangle = (D_0(Z, R_0, W) + \Delta D) / (C_0(Z, R_0, W) + \Delta C), \quad (3)$$

where C_0 and ΔC are given by Eqs. (1′), (1″), (2′), and (2″) while D_0 and ΔD are given respectively by (see Appendix 1):

For S and T Interaction

$$D_0(Z, R_0, W) = \sin[\delta_1 - \delta_{-1}]\{(\overline{M}_1^2 - \overline{M}_{-1}^2)^{1/2}(1 + x)^2$$
$$+ (\overline{L}_1^2 - \overline{L}_{-1}^2)^{1/2}({}^1/_3 q^2x^2 + y^2 + {}^1/_6 q^2 + {}^2/_3 qy(x - 1))$$
$$- {}^1/_2[(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$+ (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}]({}^2/_3 q(x^2 - 1)$$
$$+ 2y(1 + x))\} + \sin[\delta_2 - \delta_{-2}](\overline{L}_2^2 - \overline{L}_{-2}^2)^{1/2}(2x - 1)^2/2, \quad (4')$$

$$\Delta D = \sin[\delta_1 - \delta_{-1}]\{(\overline{M}_1^2 - \overline{M}_{-1}^2)^{1/2} + {}^1/_3 q^2(\overline{L}_1^2 - \overline{L}_{-1}^2)^{1/2}$$
$$- {}^1/_3 q[(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$+ (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}]\}x^2F^2$$
$$+ 2\sin[\delta_2 - \delta_{-2}](\overline{L}_2^2 - \overline{L}_{-2}^2)^{1/2}x^2F^2$$
$$+ \cos[\delta_1 - \delta_{-1}]\{(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$- (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}\}(y - {}^2/_3 q)xF; \quad (4'')$$

For V and A Interaction

$$D_0(Z, R_0, W) = \sin[\delta_1 - \delta_{-1}]\{(\overline{M}_1^2 - \overline{M}_{-1}^2)^{1/2}(1 + x^2)$$
$$+ (\overline{L}_1^2 - \overline{L}_{-1}^2)^{1/2}[{}^1/_3 q^2x^2 + y^2 + {}^1/_6 q^2 - {}^2/_3 qy(x - 1)]$$
$$- {}^1/_2[(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2} + (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}]$$
$$\times[-{}^2/_3 q(x^2 - 1) + 2y(1 + x)]\}$$
$$+ \sin(\delta_2 - \delta_{-2})(\overline{L}_2^2 - \overline{L}_{-2}^2)^{1/2}(2x - 1)^2/2, \quad (5')$$

$$\Delta D = \sin[\delta_1 - \delta_{-1}]\{(\overline{M}_1^2 - \overline{M}_{-1}^2)^{1/2} + {}^1/_6 q^2(\overline{L}_1^2 - \overline{L}_{-1}^2)$$
$$- {}^1/_3 q[(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$+ (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}]\}F^2$$
$$+ {}^1/_2\sin[\delta_2 - \delta_{-2}](\overline{L}_2^2 - \overline{L}_{-2}^2)^{1/2}F^2$$
$$+ \cos[\delta_1 - \delta_{-1}][(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$- (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}](y - {}^2/_3 qx)F. \quad (5'')$$

All roots in Eqs. (4′), (4″), (5′), and (5″) are to be taken as positive. The functions $\overline{M}_{-1}$, $\overline{L}_{-1}$ and $\overline{L}_{-2}$ are given in Appendix II.

The symmetry between the functions C_0 and D_0 should be noted. Indeed, wherever C_0 contains the quantities $\overline{M}_1$, $\overline{L}_1$, $\overline{L}_2$ and $\overline{N}_1$, D_0 contains instead

$$\overline{M}_1' = [\overline{M}_1^2 - \overline{M}_{-1}^2]^{1/2}\sin(\delta_1 - \delta_{-1});$$
$$\overline{L}_1' = [\overline{L}_1^2 - \overline{L}_{-1}^2]^{1/2}\sin(\delta_1 - \delta_{-1});$$
$$\overline{L}_2' = [\overline{L}_2^2 - \overline{L}_{-2}^2]^{1/2}\sin(\delta_2 - \delta_{-2});$$
$$\overline{N}_1' = -{}^1/_2[(\overline{L}_1 + \overline{L}_{-1})^{1/2}(\overline{M}_1 + \overline{M}_{-1})^{1/2}$$
$$+ (\overline{L}_1 - \overline{L}_{-1})^{1/2}(\overline{M}_1 - \overline{M}_{-1})^{1/2}]\sin(\delta_1 - \delta_{-1});$$

If in the expansion of $\overline{M}_1$, $\overline{L}_1$, etc. and $\overline{M}_{-1}$, $\overline{L}_{-1}$, etc. only terms up to first order in pR_0 are kept (and further if the nuclear charge distribution is approximated by a point charge and the electron wave function is assumed to be constant within the nuclear volume) then, as was shown by Lee-Whiting,[11]

$$\overline{M}_1'/\overline{M}_1 \approx \overline{L}_1'/\overline{L}_1 \approx \overline{L}_2'/\overline{L}_2 \approx \overline{N}_1'/\overline{N}_1 \approx v/c.$$

This leads to a longitudinal polarization of the β electrons equal to v/c. In the case of RaE, however, the above approximations are not valid and the longitudinal polarization of the β electrons differs from v/c.

3. RESULTS OF CALCULATIONS AND DISCUSSION

We consider first the case when time reversal invariance is valid.

Using the values of x and y determined by Takebe, Nakamura, and Taketani[7] from requiring the best possible agreement between the theoretical and experimental C_0, we obtain in the case of S and T interaction the values listed in Table I for the deviation of the polarization from v/c.

TABLE I. Values of $\langle\sigma\rangle/(v/c)$ for S, T interaction ($r_0 = 1.17 \times 10^{-13}$)

x	y	W 1.2	1.8	2.4	3.0
0.2	18.8	0.53	0.43	0.28	0.13
0.4	22.4	0.67	0.64	0.57	0.44
0.6	25.7	0.71	0.69	0.63	0.50
1.0	32.35	0.76	0.75	0.71	0.62
1.2	35.6	0.77	0.76	0.72	0.65
3.0	64.8	0.80	0.80	0.78	0.75
5.5	104	0.80	0.80	0.77	0.76

For the V and A interaction a region of values of x and y was determined by requiring a satisfactory agreement between the theoretical and experimental values of C_0. As was to be expected, this region turned out to be somewhat different from the corresponding region for the S and T interaction. The deviation of the polarization from v/c in the case of V and A inter-

TABLE II. Values of $\langle\sigma\rangle/(v/c)$ for V, A interaction ($r_0 = 1.17 \times 10^{-13}$)

x	y	W: 1.2	1.8	2.4	3.0
0.2	19.85	0.83	0.83	0.81	0.76
0.5	24.6	0.80	0.80	0.77	0.70
1.0	32.35	0.76	0.75	0.71	0.62
1.3	37	0.74	0.73	0.68	0.60
1.7	42.95	0.71	0.68	0.62	0.56
2.0	47.3	0.69	0.65	0.58	0.54

action is listed in Table II (it is clear that for for $x = 1$ there is no difference between the S, T, and V, A interactions).

The dependence of the polarization of the β electrons on x allows, in principle, a unique determination of x. The experimental data[3] available at this time do not contradict the theoretical calculations for $x \approx 1.7$ (V, A interaction) or $x \approx 0.6$ (S, T interaction).

The different x-dependence of $\langle\sigma\rangle/(v/c)$ should be noted: for the S, T interaction $\langle\sigma\rangle/(v/c)$ increases, and for the V, A interaction it decreases as x increases. However, for all interactions and for any x and y which permit the fitting of the experimental spectrum shape of RaE by the theoretical one, we have $(-\langle\sigma\rangle/(v/c))_{max} \lesssim 0.84$.

Let us next take into account the possibility of violation of time-reversal invariance in β decay. Before quoting the results of the calculations for various admissible values of F let us explain why it is impossible to reconcile theory and experiment for large F ($F \sim 1$) in the β decay of RaE. We make use of the well-known fact that the experimentally required energy dependence of $C = C(Z, R_0, W)$ can only be obtained provided there are strong interference effects in $C(Z, R_0, W)$ (with a simultaneous decrease of $C(Z, R_0, W)$ by about two orders of magnitude).

Let us solve for y in the equation $C_{exp} = C(Z, R_0, W)$, for some fixed x. From Eqs. (2′) and (2″) we have for the V, A interaction:

$$y = (1/\overline{L}_1)\{\varepsilon_0 \pm (\varepsilon_0^2 - \varepsilon_1\overline{L}_1 + C_{exp}\overline{L}_1 - F^2\varepsilon_2\overline{L}_1)^{1/2}\}, \quad (6)$$

where

$$\varepsilon_0 = {}^1/_3 q(x-1)\overline{L}_1 - (1+x)\overline{N}_1,$$

$$\varepsilon_1 = x^2(\overline{M}_1 + {}^1/_3 q^2\overline{L}_1 + 2\overline{L}_2 - {}^2/_3 q\overline{N}_1) + 2x(\overline{M}_1 - \overline{L}_2) + \varepsilon_2,$$

$$\varepsilon_2 = \overline{M}_1 + {}^1/_2\overline{L}_2 + {}^1/_6 q^2\overline{L}_1 + {}^2/_3 q\overline{N}_1.$$

We give below values of $\epsilon_0^2 - \epsilon_1\overline{L}_1$ and $\epsilon_2\overline{L}_1$ for various x and W.

As was mentioned above, $\overline{L}_1 C_{exp} \sim \epsilon_0^2 - \epsilon_1\overline{L}_1$. On the other hand, we have from Tables III and IV that $\epsilon_0^2 - \epsilon_1\overline{L}_1 \ll \epsilon_2\overline{L}_1$. It then follows from Eq. (6) that F cannot be of the order of magnitude of unity (since y is to be real). It is important to note that the above considerations remain valid for slight changes in the functions $\overline{L}_1$, $\overline{L}_2$, $\overline{M}_1$, and $\overline{N}_1$ (such changes could be due to insufficiently accurate knowledge of the nuclear charge distribution, to corrections arising from third forbidden matrix elements, etc.).

TABLE III. Values of $(\epsilon_0^2 - \epsilon_1\overline{L}_1)$

x	W: 1.2	1.8	2.4	3.0
0.2	0.37	0.26	0.20	0.20
1.0	1.56	0.98	0.65	0.56
2.0	4.33	2.93	2.36	2.62

TABLE IV. Values of $\epsilon_2\overline{L}_1$

W	1.2	1.8	2.4	3.0
$\varepsilon_2\overline{L}_1$	121.70	126.77	130.80	134.38

However, for small values of F ($F < 10^{-2}$) one finds from numerical calculations that a satisfactory theoretical fit to the experimental spectrum shape of RaE is possible. The longitudinal polarization of β electrons in the allowed region for x and y and for $F^2 = 6 \times 10^{-3}$ and $F^2 = 3 \times 10^{-3}$ is listed in Tables V and VI for V, A interaction. As can be seen from Eq. (5″) the polarization depends not only on the magnitude of F^2 but also on the sign of F.

TABLE V. Values of $\langle\sigma\rangle/(v/c)$ for V, A interaction with $F^2 = 6 \times 10^{-3}$

		W: 1.2		1.8		2.4		3.0	
x	y	F<0	F>0	F<0	F>0	F<0	F>0	F<0	F>0
0.2	19	0.94	0.67	0.90	0.72	0.89	0.70	0.87	0.68
1.3	36.25	0.86	0.59	0.80	0.61	0.74	0.55	0.69	0.50
1.7	42.4	0.84	0.58	0.77	0.59	0.71	0.53	0.67	0.51

TABLE VI. Values of $\langle\sigma\rangle/(v/c)$ for V, A interaction with $F^2 = 3 \times 10^{-3}$

		W							
		1.2		1.8		2.4		3.0	
x	y	$F<0$	$F>0$	$F<0$	$F>0$	$F<0$	$F>0$	$F<0$	$F>0$
0.2	19.4	0.89	0.73	0.87	0.76	0.85	0.73	0.81	0.68
1.3	36.55	0.81	0.64	0.76	0.65	0.71	0.59	0.64	0.51
1.9	45.76	0.79	0.62	0.73	0.61	0.67	0.55	0.63	0.53

TABLE VII. Values of $\langle\sigma\rangle/(v/c)$ for S, T interaction with various F

			W							
			1.2		1.8		2.4		3.0	
F^2	x	y	$F<0$	$F>0$	$F<0$	$F>0$	$F<0$	$F>0$	$F<0$	$F>9$
$10\cdot10^{-3}$	0.4	21.94	0.82	0.47	0.72	0.47	0.62	0.36	0.49	0.23
	0.8	27.96	0.92	0.51	0.84	0.54	0.78	0.49	0.73	0.45
$6\cdot10^{-3}$	1.2	34.66	0.89	0.61	0.84	0.64	0.80	0.60	0.75	0.55
	2.0	47.20	0.92	0.65	0.88	0.69	0.86	0.67	0.84	0.67
$3\cdot10^{-3}$	1.2	35.00	0.84	0.66	0.80	0.67	0.75	0.62	0.68	0.54
	3.0	62.65	0.89	0.67	0.85	0.69	0.83	0.66	0.82	0.67

In Table VII the deviation of the polarization from v/c is listed in the case of S, T interaction for various values of F^2 (in the S, T case values of F^2 up to $F^2 = 10^{-2}$ are allowed). The irregular variation of $\langle\sigma\rangle/(v/c)$ as a function of F^2 is apparently due to the fact that for various values of x and F the theoretical and experimental spectrum shapes do not agree to the same degree of accuracy. (This irregularity is also present in the V, A case but only at high electron energies and to a much smaller extent.)

The following statements can be made based on a comparison of Tables V, VI, and VII with reference 3:

1. For V, A interaction: assuming that time reversal invariance is violated in β decay the experimental data exclude $F < 0$ for $F^2 = 6 \times 10^{-3}$ and $F^2 = 3 \times 10^{-3}$. For $F > 0$ theory and experiment can be made to agree for $x \approx 0.2$ ($F^2 = 6 \times 10^{-3}$) or $x \approx 0.7$ ($F^2 = 3 \times 10^{-3}$).

2. For S, T interaction: the value of $F^2 = 10^{-2}$, which is admissible as far as comparison of experimental and theoretical spectrum shapes is concerned, is excluded by the experimental data on electron polarization.[3] For $F^2 = 6 \times 10^{-3}$ and $F^2 = 3 \times 10^{-3}$ the case $F < 0$ is excluded, and the case $F > 0$ with either value for F^2 is allowed if $x \approx 1.7$.

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APPENDIX I

We shall use the methods of reference 8 (referred to hereafter as I). We write the electron wave function, with finite nuclear size effects taken into account, as

$$\Psi_{\mathbf{p},\xi}(r) = \begin{pmatrix} \varphi_p V_\xi \\ \chi_p V_\xi \end{pmatrix}, \qquad \text{(A1.1)}$$

where V_ξ is the two-component unit spinor determining the electron polarization,

$$\varphi_{\mathbf{p}} = \alpha_0 + i\mathbf{p}\cdot\mathbf{r}\,\alpha_1 + i\,(\sigma\cdot\mathbf{r})\,(\sigma\cdot\mathbf{n})\,\beta_c,$$

$$\chi_p = [\beta_0 + i\,\mathbf{p}\cdot\mathbf{r}\,\beta_1 + i\,(\sigma\cdot\mathbf{r})\,(\sigma\cdot\mathbf{n})\,\alpha_c]\,(\sigma\cdot\mathbf{n}), \qquad \text{(A1.2)}$$

$\mathbf{p}$ is the electron momentum, and $\mathbf{n} = \mathbf{p}/p$.

The quantities α_i and β_i are chosen as follows:

$$\alpha_0 = aie^{-i\delta_{-1}}\,g_{-1}; \qquad \alpha_1 = a\frac{3}{pr}\,e^{-i\delta_{-2}}g_{-2};$$

$$\beta_c = a\frac{1}{r}\,(e^{-i\delta_1}\,g_1 - e^{-i\delta_{-2}}g_{-2});$$

$$\beta_0 = ae^{-i\delta_1}\,f_1; \qquad \beta_1 = a\frac{3}{ipr}\,e^{-i\delta_2}\,f_2;$$

$$\alpha_c = a\frac{1}{ir}\,(e^{-i\delta_{-1}}f_1 - e^{-i\delta_2}\,f_2), \quad a = \sqrt{\pi/2pW}\cdot \qquad \text{(A1.3)}$$

In the actual calculations we used the phase shifts given in reference 12. The relation of these phase shifts to the ones in Eq. (A1.3) is as follows:

$$\delta_{-1} = \delta_{1/2,\,-1/2} + \pi/2; \qquad \delta_1 = \delta_{1/2,\,1/2};$$

$$\delta_{-2} = \delta_{3/2,\,-1/2}; \qquad \delta_2 = \delta_{3/2,\,1/2} - \pi/2$$

The values of the phase shifts are tabulated below.

Values of phase shifts

W	1.2	1.8	2.4	3.0
$\delta^{1/2}{}_{1/2}$	0.0208	0.2618	0.3228	0.3604
$\delta^{1/2}{}_{-1/2}$	0.7647	0.6646	0.6101	0.5713
$\delta^{3/2}{}_{1/2}$	−0.8175	−0.4438	−0.3643	−0.3204
$\delta^{3/2}{}_{-1/2}$	−0.3847	−0.2323	−0.2161	−0.2124

The functions $C(Z, R_0, W)$ and $D(Z, R_0, W)$ according to Eq. (I.35), are given by

$$C(Z, R_0, W) = \lambda_{ik}A_{ik} - \chi_{ik}b_{ik},$$

$$D(Z, R_0, W) = \lambda_{ik}b_{ik} - \chi_{ik}A_{ik},$$

$$D(Z, R_0, W) = D(Z, R_0, W) + \Delta D; \quad \mathbf{B}_{ik} = \mathbf{n}\, b_{ik}. \tag{A1.4}$$

After averaging over the direction of emission of the neutrino, summing over the final and averaging over the initial nuclear spin states, according to Eqs. (I.32), (I.38), and (I.39) we have for the V, A interaction (where the fact that the RaE β decay is a $1^- \rightarrow 0^+$ transition has been taken into account)*

$$A_{0c} = -[C_\tau - {}^1/_3 qC_b]\, C_a^*; \quad b_{0c} = 0;$$

$$A_{1c} = 0; \quad b_{1c} = -{}^1/_3 pC_a C_a^*;$$

$$A_{01} = 0; \quad b_{01} = {}^1/_3 p\,[C_\tau - {}^1/_3 qC_b]\, C_a^*;$$

$$A_{cc} = C_a C_a^*; \quad b_{cc} = 0;$$

$$A_{11} = {}^1/_3 p^2 |C_R|^2 + {}^1/_6 p^2 |C_T|^2; \quad b_{11} = 0;$$

$$A_{00} = {}^1/_3 q^2 |C_R|^2 + |C_\tau|^2 + {}^1/_6 q^2 |C_T|^2$$

$$- {}^2/_3 q\,[\operatorname{Re} C_R C_\tau^* - \operatorname{Im} C_T C_\tau^*]; \quad b_{00} = 0, \tag{A1.5}$$

where

$$\mathbf{a} = G_V \int \mathbf{r} - iG_A \int [\sigma \times \mathbf{r}]; \quad \mathbf{b} = G_V \int \mathbf{r} + iG_A \int [\sigma \times \mathbf{r}];$$

$$\mathbf{R} = G_V \int \mathbf{r}; \quad \boldsymbol{\tau} = -iG_V \int \boldsymbol{\alpha}; \quad \mathbf{T} = G_A \int [\sigma \times \mathbf{r}].$$

Assuming that violation of time reversal invariance is connected with the A-interaction coupling constant and introducing the ratios x and y of matrix elements we obtain (here in all A_{ik} and b_{ik} the common factor $g_A^2 \,|\int \sigma \times \mathbf{r}|^2$ is omitted; $\int \sigma \times \mathbf{r}$ and $\int \alpha$ are purely imaginary, $\int \mathbf{r}$ is purely real):

$$A_{0c} = -[y(1 + x) - {}^1/_3 q(x^2 - 1 - F^2)$$

$$- iF(y - {}^2/_3 qx)]; \quad b_{0c} = 0;$$

$$A_{1c} = 0; \quad b_{1c} = -{}^1/_3 pA_{cc}; \quad A_{01} = 0; \quad b_{01} = -{}^1/_3 pA_{0c};$$

$$A_{cc} = (1 + x)^2 + F^2; \quad b_{cc} = 0;$$

$$A_{11} = {}^1/_3 p^2 x^2 + {}^1/_6 p^2 (1 + F^2); \quad b_{11} = 0;$$

$$A_{00} = {}^1/_3 q^2 x^2 + y^2 + {}^1/_6 q^2 (1 + F^2)$$

$$- {}^2/_3 q(x - 1)\, y; \quad b_{00} = 0. \tag{A1.6}$$

Noting that for the S and T interactions q goes into $-q$ and assuming that violation of time reversal invariance is connected with the S-interaction coupling constant we get analogous expressions for A_{ik} and b_{ik} in the S, T case.

In the determination of λ_{ik} and χ_{ik} we note the connection between the functions $g_{\pm i}$ and $f_{\pm i}$ and the functions L_0, P_0, M_0, Q_0, L_1, P_1, and N_1.[9,10] Then, omitting factors entering into the phase space factor and the Fermi function, we find

$$\lambda_{00} = L_0; \quad \chi_{00} = (L_0^2 - P_0^2)^{1/2} \sin[\delta_{-1} - \delta_1];$$

$$\lambda_{11} = (9/p^2)\, L_1; \quad \chi_{11} = (9/p^2)(L_1^2 - P_1^2)^{1/2} \sin[\delta_{-2} - \delta_2];$$

$$\lambda_{cc} = M_0 + L_1 + (M_0 - Q_0)^{1/2}(L_1 - P_1)^{1/2} \cos(\delta_{-1} - \delta_2)$$

$$- (M_0 + Q_0)^{1/2}(L_1 + P_1)^{1/2} \cos(\delta_1 - \delta_{-2});$$

$$\chi_{cc} = (M_0^2 - Q_0^2)^{1/2} \sin(\delta_{-1} - \delta_1) + (L_1^2 - P_1^2)^{1/2} \sin(\delta_{-2} - \delta_2)$$

$$+ (L_1 - P_1)^{1/2}(M_0 + Q_0)^{1/2} \sin(\delta_2 - \delta_1)$$

$$+ (L_1 + P_1)^{1/2}(M_0 - Q_0)^{1/2} \sin(\delta_{-2} - \delta_{-1});$$

$$\lambda_{01} = -(3i/2p)\, \{(L_0 + P_0)^{1/2}(L_1 + P_1)^{1/2} e^{i(\delta_{-1} - \delta_{-2})}$$

$$+ (L_0 - P_0)^{1/2}(L_1 - P_1)^{1/2} e^{i(\delta_1 - \delta_2)}\};$$

$$\chi_{01} = -(3/2p)\, \{(L_0 + P_0)^{1/2}(L_1 - P_1)^{1/2} e^{i(\delta_{-1} - \delta_2)}$$

$$- (L_0 - P_0)^{1/2}(L_1 + P_1)^{1/2} e^{i(\delta_1 - \delta_{-2})};$$

$$\lambda_{0c} = -{}^1/_2\, \{2N_0 - (L_0 + P_0)^{1/2}(L_1 - P_1)^{1/2} e^{i(\delta_{-1} - \delta_2)}$$

$$+ (L_0 - P_0)^{1/2}(L_1 + P_1)^{1/2} e^{i(\delta_1 - \delta_{-2})}\};$$

$$\chi_{0c} = {}^1/_2\, i\, \{-(L_0 + P_0)^{1/2}(M_0 + Q_0)^{1/2} e^{i(\delta_{-1} - \delta_1)}$$

$$+ (L_0 - P_0)^{1/2}(M_0 - Q_0)^{1/2}\, e^{-i(\delta_{-1} - \delta_1)}$$

$$+ (L_0 + P_0)^{1/2}(L_1 + P_1)^{1/2} e^{i(\delta_{-1} - \delta_{-2})}$$

$$+ (L_0 - P_0)^{1/2}(L_1 - P_1)^{1/2} e^{i(\delta_1 - \delta_2)}\};$$

$$\lambda_{1c} = (3/2p)\, \{-2(L_1^2 - P_1^2)^{1/2} \sin(\delta_{-2} - \delta_2) + i(L_1 + P_1)^{1/2}$$

$$\times (M_0 - Q_0)^{1/2} e^{i(\delta_{-2} - \delta_{-1})} + i(L_1 - P_1)^{1/2}(M_0 + Q_0)^{1/2} e^{i(\delta_2 - \delta_1)}\};$$

$$\chi_{1c} = (3/2p)\, \{(M_0 + Q_0)^{1/2}(L_1 + P_1)^{1/2} e^{i(\delta_{-2} - \delta_1)}$$

$$- (M_0 - Q_0)^{1/2}(L_1 - P_1)^{1/2} e^{i(\delta_2 - \delta_1)} - 2L_1\}. \tag{A1.7}$$

Using the above expressions for A_{ik}, b_{ik}, λ_{ik} and χ_{ik} we obtain, after some simple manipulations, the values of $C(Z, R_0, W)$ and $D(Z, R_0, W)$. In the case of RaE which is of interest here one must make the replacements $M_0 \rightarrow \overline{M}_1$, $L_0 \rightarrow \overline{L}_1$, $L_1 \rightarrow \overline{L}_2$, $N_0 \rightarrow \overline{N}_1$, $Q_0 \rightarrow -\overline{M}_{-1}$, $P_0 \rightarrow -\overline{L}_{-1}$ and $P_1 \rightarrow -\overline{L}_{-2}$.

*We take this opportunity to point out an error in reference 8 in the definition of **a**: the correct definition is **a** = **m** − i**t** (and not **a** = **m** + i**t**).

APPENDIX II

From the definitions of the functions $\overline{M}_1$, $\overline{L}_1$, and $\overline{L}_2$[7] it is easy to obtain the following expressions for the functions $\overline{M}_{-1}$, $\overline{L}_{-1}$, and $\overline{L}_{-2}$:

$$2\overline{M}_{-1} = (X_{-1}^2 + Y_1^2)\, M_1^{(S)} + (X_{-1}^2 - Y_1^2)\, M_1^{(L)},$$

$$2\overline{L}_{-1} = (X_1^2 + Y_{-1}^2)\, L_1^{(S)} + (X_1^2 - Y_{-1}^2)\, L_1^{(L)},$$

$$2\overline{L}_{-2} = (X_2^2 + Y_{-2}^2)\, L_2^{(S)} + (X_2^2 - Y_{-2}^2)\, L_2^{(L)}. \quad \text{(A2.1)}$$

We give below values of $\overline{M}_{-1}$, $\overline{L}_{-1}$, and $\overline{L}_{-2}$ obtained using the functions tabulated in reference 7.

Values of $\overline{M}_{-1}$, $\overline{L}_{-1}$, and $\overline{L}_{-2}$
($r_0 = 1.17 \times 10^{-13}$)

W	$\overline{M}_{-1}$	$\overline{L}_{-1}$	$\overline{L}_{-2}$
1.2	104,270	—0.52366	—0.02967
1.8	67,211	—0.34027	—0.07549
2.4	48.663	—0.24878	—0.12031
3.0	37.527	—0.19404	—0.16538

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87