ENERGY AND ANGULAR DISTRIBUTION IN DIFFRACTION DISINTEGRATION PROCESSES

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Submitted to JETP editor June 27, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 499-504 (February, 1959)

The energy and angular distributions have been obtained for particles obtained in diffraction disintegration of a weakly-bound quantum-mechanical system (deuteron etc.). The energy distributions are practically identical with those observed in stripping, whereas the angular distributions are appreciably different. A simple physical explanation of this difference can be proposed, and it may be of importance in interpreting the experimental data.

1. Diffraction disintegration of the deuteron was investigated by Feinberg,[1] Glauber,[2] and Akhiezer and Sitenko,[3] all of whom used a computation method analogous to the Kirchhoff method in the diffraction of light. It yields good results only in the vicinity of the geometric shadow. The most reliable result in the theory of diffraction disintegration is therefore the differential cross section of the process for small angles.

Diffraction disintegration of a deuteron results in simultaneous liberation of two particles, a proton and a neutron. It is possible in principle to set up correlation experiments by measuring the momenta of both liberated particles. Since the neutrons are difficult to observe, the distribution of only one particle (proton) is determined in practice. This raises the question of calculating the angular and energy distributions for one particle.

It was shown by Serber[4] (cf. also reference 5) that in the case of fast particles, to find the distributions over the energies E of one particle it is necessary to calculate the distribution over p_z for this particle (p_z is the projection of the particle momentum along the direction of the initial beam). Then $p_z = (E - E_x/2)/\sqrt{E_d/M}$.

Akhiezer and Sitenko[3] derive a formula [Eq. (16) of reference 3] for the energy distribution. Actually, however, this formula gives the distribution over the modulus of the vector of the relative momentum $f = \sqrt{f_x^2 + f_y^2 + f_z^2}$; the angular distribution of the vector **f** is already integrated in this formula, so that the required energy distribution cannot be obtained from it.

2. We proceed to calculate the energy distribution. Consider the wave function of the deuteron in the presence of an absolutely black nucleus. It will have approximately the form of the so-called "modified" function, introduced in reference 3:

$$\psi_0(\boldsymbol{\rho}, \mathbf{r}) = \varphi_0(r)\,\Omega(\rho_n)\,\Omega(\rho_p). \tag{1}$$

Here $\boldsymbol{\rho}$ is the radius vector of the deuteron center of mass in the plane perpendicular to the axis of the incident beam; **r** is the radius vector of the relative distance between the proton and neutrons; $\varphi_0(r) = \sqrt{\alpha/2\pi}\; e^{-\alpha r}/r$ is the wave function of the relative motion in the deuteron in the approximation where the nuclear forces radius is zero, and R is the nuclear radius;

$$\Omega(\rho) = \begin{cases} 0 & \text{for } \rho < R \\ 1 & \text{for } \rho > R \end{cases}.$$

The function (1) characterizes a state of the deuteron beam directly after the passage of the nucleus. It contains the deuterons scattered as a whole and deuterons that have experienced a diffraction breakup. It is convenient to separate from ψ_0 the portion orthogonal to $\varphi_0(\mathbf{r})$:

$$\psi_1(\boldsymbol{\rho}, \mathbf{r}) = \psi_0(\boldsymbol{\rho}, \mathbf{r}) - \varphi_0(r)\int d\mathbf{r}'\psi_0(\boldsymbol{\rho}, \mathbf{r}')\,\varphi_0(r'). \tag{2}$$

The function ψ_1 describes only disintegrated deuterons. Henceforth we shall assume $R_d \ll R$. We can then neglect, for deuterons passing near the nucleus at a distance $\sim R_d$, the curvature at the edge of the nucleus, and consider the nucleus to be a plane screen with straight edge, analogous to the procedure used in references 2 and 4.* Here ψ_1 becomes

$$\psi_1(\boldsymbol{\rho}, \mathbf{r}) = \varphi_0(r)\,[\Omega(\rho_n)\,\Omega(\rho_p) - 1] + \varphi_0(r)\,I(\rho), \tag{3}$$

where

*As shown in reference 8, the error introduced thereby is not greater than the error due to an inaccurately selected function $\varphi_0(r)$.

$$I(\rho) = \begin{cases} 1 & \text{for } \rho < R \\ \varepsilon_1 [4\alpha(\rho - R)] & \text{for } \rho > R \end{cases}.$$

The function $\epsilon_1(x)$ is the Gold integral (see reference 6)

$$\varepsilon_1(x) = \int_1^\infty \frac{e^{-xt}}{t^2} dt. \quad (4)$$

We note that

$$\psi_1(\rho, \mathbf{r}) \equiv 0 \quad \text{for } \rho < R. \quad (5)$$

At each point of the edge of the nucleus we introduce a local system of coordinates: The deuteron beam travels along the z axis, and the y and x axes lie in the plane of the screen, with the y axis being directed along its edge and the x axis perpendicular to the edge and directed outward from the nucleus. To find the distribution of one nucleon over the momenta, we must expand the function ψ_1 in plane waves of the motion of the center of mass and in the wave functions

$$\varphi_{\mathbf{f}}(\mathbf{r}) = e^{i\mathbf{f}\mathbf{r}} - e^{-ifr} / r(\alpha - if)$$

of relative motion, which correspond to the motion of the nucleons with relative momentum **f** at infinity, liberated as a result of the breakup. Since no momentum is transferred to the center of mass of the deuteron in the direction along the edge of the nucleus, it is not necessary to expand ψ_1 in plane waves in the Y direction, and we can calculate instead the cross section per element dY of the length of the edge of the nucleus. Thus

$$\psi_1(k, \mathbf{f}, Y) = (2\pi)^{-2} \int \psi_1(X, Y, \mathbf{r}) e^{-ikX} \varphi_{\mathbf{f}}^*(\mathbf{r})\, dX d\mathbf{r}. \quad (6)$$

Here

$$k = p_{px} + p_{nx} = \lambda + \mu, \quad f_x = (p_{px} - p_{nx})/2 = (\lambda - \mu)/2,$$
$$f_y = p_{py}, \quad f_z = p_{pz}. \quad (7)$$

Formulas (7) result from the fact that we consider here a screen with a straight edge and neglect the momentum transfer in the direction of the primary beam. For a screen with a straight edge we have

$$\psi_1(X, Y, \mathbf{r}) = \begin{cases} 0 & \text{for } X < 0 \\ -[\omega(x_n) + \omega(x_p)]\varphi_0(r) + \varepsilon_1(4\alpha X)\varphi_0(r) & \text{for } X > 0, \end{cases}$$

where

$$\omega(x) = 1 - \Omega(x) = \begin{cases} 0 & \text{for } x > 0 \\ 1 & \text{for } x < 0. \end{cases}$$

Using the Fourier expansion

$$\varphi_0(r) = (2\pi)^{-3} \int \frac{2\sqrt{2\pi\alpha}}{\alpha^2 + f^2} e^{-i\mathbf{f}\mathbf{r}} d\mathbf{f}$$

and Eq. (4), we get

$$\psi_1(\lambda, \mu, f_y, f_z) = \sqrt{\frac{\alpha}{(2\pi)^3}} \left\{ \frac{1}{P(\mu - \lambda - i2P)(\mu - iP)} + \frac{1}{P(\mu - \lambda + i2P)(iP - \lambda)} + \frac{1}{\alpha^2 + f^2} \int_1^\infty \frac{dt}{t^3 \left[2\alpha - 2if + i\dfrac{\lambda + \mu}{t} \right]} \right\}. \quad (8)$$

Here $P = \sqrt{\alpha^2 + f_y^2 + f_z^2}$; $|\psi_1|^2 d\lambda d\mu df_y df_z dY$ gives the effective cross section of process, at which the quantities p_{nx}, p_p, and Y are located in the corresponding intervals. Integration over Y yields $2\pi R$. Integrating over all the momenta, we obtain the total cross section, accurate to within terms of R_d^2:

$$2\pi R \int d\lambda d\mu df_y df_z |\psi_1|^2 = \frac{\pi R R_d}{3}(2\ln 2 - 1/2),$$

this agrees with the cross section calculated in references 2 and 3.

To obtain the energy distribution, it is necessary to integrate the expression

$$2\pi R |\psi_1(\lambda, \mu, f_y, f_z)|^2 \quad (9)$$

over the momentum of the neutron, $p_{nx} = \mu$ and also over $p_{px} = \lambda$ and $p_{py} = f_y$. This integration is difficult to perform in exact form, owing to the presence of two different radicals P and f. However, a good approximate expression can be found for the distribution over $p_z = f_z$. In the denominator of the last term of (8) we replace the expression $2\alpha - 2if + i(\lambda + \mu)/t$, which is slowly varying, by its value at $t = 1$. This does not introduce a great error, owing to the presence of a rapidly diminishing factor $1/t^3$. After this, (8) becomes

$$\psi_2 = \sqrt{\frac{\alpha}{(2\pi)^3}} \left\{ \frac{1}{P(\mu - \lambda - i2P)(\mu - iP)} + \frac{1}{P(\mu - \lambda + i2P)(iP - \lambda)} + \frac{1}{(\alpha^2 + f^2)(2\alpha - 2if + i(\lambda + \mu))} \right\}. \quad (10)$$

The use of ψ_2 instead of ψ_1 causes the total cross section to deviate 15% from the value $\pi R R_d(2\ln 2 - \frac{1}{2})/3$. Furthermore, a special estimate has shown that in the angular range $\lesssim \alpha$ the differential cross section changes merely by 10%. This indeed determines the accuracy of the formula obtained later on. After making the above simplification, the integration over λ, μ, and f_y proceeds without difficulty, and we obtain the following energy distribution:

$$\frac{d\sigma_E}{dq} = \frac{R R_d}{4} \frac{1}{(1+q^2)^{5/2}} \left\{ \frac{3}{2}\pi + \pi q^2 + \frac{2(1+q^2)^{3/2}}{q} \tan^{-1} q - \sqrt{1+q^2} \right\},$$
$$q = (E - E_d/2) / \sqrt{\varepsilon E_d}. \quad (11)$$

Here E is the proton energy, E_d is the energy

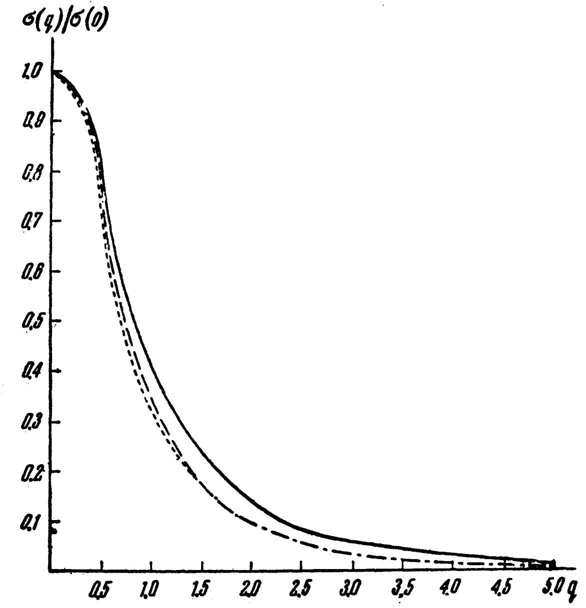

FIG. 1. Energy distributions: Solid line – diffraction disintegration, dotted line – stripping.

of the primary deuteron, and ϵ is the binding energy of the deuteron.

The distribution (11) is shown in Fig. 1 by a solid line. It is seen from (11) that the center of the energy distribution of the proton is $E_d/2$, and that the half-width is $\sqrt{\epsilon E_d}$.

It is interesting to call attention to the following fact. Were it possible to neglect the interaction between the proton and the neutron after the deuteron breakup, then the relative motion of these particles would be described by a plane wave $e^{i\mathbf{f}\cdot\mathbf{r}}$, and in order to find the momentum distribution it would be enough to expand $\psi_1(X, Y, \mathbf{r})$ in a Fourier integral over the coordinates X and $\mathbf{r}$. Calculations with a plane wave are simpler than those using the function $\varphi_{\mathbf{f}}(\mathbf{r})$, and the results are quite close to each other. The energy distribution obtained by replacing $\varphi_{\mathbf{f}}(\mathbf{r})$ with $e^{i\mathbf{f}\cdot\mathbf{r}}$ is shown dotted in Fig. 1. In addition, the same figure shows for comparison the energy distribution for stripping. Obviously, all distributions are practically the same.

3. Let us proceed to find the angular distribution of an individual nucleon. For this purpose it is necessary to integrate in Eq. (9) over μ, f_z, and φ, where $\lambda = p_\perp \cos\varphi$ and $f = p_{\parallel} \sin\varphi$ (see reference 4). The exact integration leads to very complicated expressions. It is convenient to use the following approximation. Since the integral in the last term of (8) is preceded by a factor $1/(\alpha^2 + f^2)$, which has a sharp maximum, we can put in the integrand $f = 0$ when integrating over f_z. Furthermore, we can replace $2\alpha + 2i\lambda/t$ by $2\alpha + 2i\lambda$, as was done earlier. These simplifications lead to an error on the order of 5% in the total cross section, and to an error of approximately 10% in the angular distribution for $p_\perp \sim \alpha$. We then obtain instead of ψ_1

$$\psi_3 = \sqrt{\frac{\alpha}{(2\pi)^3}}\left\{\frac{1}{P(\mu-\lambda-i2P)(\mu-iP)} + \frac{1}{P(\mu-\lambda+i2P)(iP-\lambda)} + \frac{1}{2(\alpha^2+f^2)(\alpha+i\lambda)}\right\}.$$

Substituting this value of ψ_3 for ψ_1 in Eq. (9) and integrating over μ, f_z, and φ, we get

$$\frac{d\sigma}{d\Omega_\zeta} = \frac{RR_d}{4\pi(1+\zeta^2)^{5/2}}\left\{\frac{4}{3} + 4\left(1-\frac{8}{3\sqrt{9+\zeta^2}}\right) - \frac{\pi}{2+\zeta^2} - \frac{1+\zeta^2}{1+\zeta^2/2}\left[\frac{3\pi}{4(1+\zeta^2)}\left(1-\frac{8}{3\sqrt{9+\zeta^2}}\right) + \frac{3\pi/4-2/3}{17+9\zeta^2}\left(1+\frac{8}{3\sqrt{9+\zeta^2}}\right) + \frac{2}{3}\left(1-\frac{8}{3\sqrt{9+\zeta^2}}\right) - \frac{1+\zeta^2/3}{1+\zeta^2}\right]\right\},$$
$$p_\perp = \alpha\zeta, \quad d\Omega_\zeta = 2\pi\zeta d\zeta. \tag{12}$$

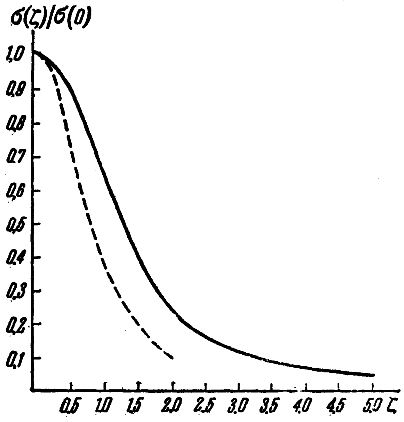

FIG. 2. Angular distribution. ——— diffraction disintegration, – – – – – stripping.

The distribution (12) is shown graphically in Fig. 2. The angular distribution in stripping, shown on the same diagram, is quite different, being "narrower." This can also be understood qualitatively. In stripping, the stripped nucleon does not acquire additional momentum in the transverse direction, and has the same momentum that it had inside the bound deuteron. To the contrary, diffraction disintegration cannot occur without one of the nucleons receiving a momentum $p_\perp \sim \alpha$, sufficient to destroy the weakly-bound deuteron, and this leads to a larger probability of momenta proportional to α.

4. So far we have considered diffraction disintegration of a deuteron, in which the proton and neutron have equal masses. However, there exist weakly bound systems which also can experience diffraction disintegration, but in which the masses of the component particles are quite unequal. An example is Be^9, in which the binding energy of the last nucleon is merely 0.6 Mev (see reference 7), i.e., even less than in a deuteron. Diffraction disintegration of Be^9 results in the production of a neutron and the nucleus Be^8. This raises the problem of finding the energy and angular distributions of the diffraction-disintegration products of a weakly-bound system, consisting of particles of unequal mass. This calculation is quite analogous to the previously analyzed case of equal masses, where instead of (8) we get

$$\psi_1(\lambda, \mu, f_y, f_z) = \sqrt{\frac{\alpha}{(2\pi)^3}}\left\{\frac{1}{P(\mu - b\lambda/a + iP/a)(\lambda - iP)} + \frac{1}{P(\lambda - a\mu/b + iP/b)(\mu - iP)} - \frac{1}{\alpha^2 + f^2}\int_1^\infty \frac{dt}{t^3[(\alpha - if)/a + ik/t]} - \frac{1}{\alpha^2 + f^2}\int_1^\infty \frac{dt}{t^3[(\alpha - if)/b + ik/t]}\right\}. \quad (13)$$

The symbols in (13) are the same as in (7), except that now $f_x = b\lambda - a\mu$, where

$$a = m_1/(m_1 + m_2), \quad b = m_2/(m_1 + m_2), \quad a + b = 1.$$

We note that the shapes of the curves are determined by the parameters a and b. But in cases of practical interest, whenever $a \neq b$, we also have $a \ll b$ (for example, for Be^9 we have $a = b/8$). We shall therefore calculate our curves for two asymptotic cases: when a light particle is observed ($a = 0$, $b = 1$), and when a heavy one is observed ($a = 1$, $b = 0$).

The shape of the energy distribution is the same for a light and heavy particle, and is given by

$$\frac{d\sigma_E}{dq} = \frac{R}{4\alpha(1 + q^2)^{5/2}}\left\{\frac{\pi}{2} - \frac{7}{4}\sqrt{1 + q^2} + \pi(1 + q^2) - \frac{2(1 + q^2)^{3/2}}{q}\tan^{-1} q\right\},$$

$$q = \left(E - \frac{m}{M}E_{nuc}\right)/\sqrt{2E_{nuc}/M}, \quad (14)$$

where E_{nuc} is the kinetic energy of the incident nucleus, E is the energy of the observed particle, m is its mass, and M is the sum of the masses of the formed particles.

The entire difference in the energy distributions of the light and heavy particles reduces to the fact that the center of the distribution for the light particles lies at mE_{nuc}/M, while that for the heavy nucleus is at $(M - m)E_{nuc}/M$. It can be shown that the distribution (14), expressed in terms of the variable q, hardly differs from the distribution for the case of equal masses, but the meaning of q becomes different.

Let us now proceed to the angular distributions. The angular distributions for the light and heavy particles respectively are given by

$$\frac{d\sigma_l}{d\Omega_\zeta} = \frac{R}{3\alpha}\left\{\left(1 + \frac{3\pi}{16}\right)\frac{1}{(1 + \zeta^2)^{3/2}} - \frac{1}{(1 + \zeta^2)^2}\right\}, \quad (15)$$

$$\frac{d\sigma_h}{d\Omega_\zeta} = \frac{R}{\alpha}\left\{\frac{1}{(1 + \zeta^2)^{3/2}}\left(1 - \frac{1}{3\sqrt{1 + \zeta^2}}\right) - \frac{1}{4 + 2\zeta^2}\left(1 - \frac{1}{3\sqrt{1 + \zeta^2}}\right) + \frac{1}{3(1 + \zeta^2)^{3/2}(2 + \zeta^2)}\left(1 + \frac{1}{5(1 + \zeta^2)}\right) - \frac{7\pi - 8}{32(2 + \zeta^2)\sqrt{1 + \zeta^2}}\right\}. \quad (16)$$

Here $\zeta = p_\perp/\alpha$, $d\Omega_\zeta = 2\pi\zeta\, d\zeta$.

In conclusion, the authors thank E. L. Feĭnberg for guidance and for a detailed evaluation of this work.

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Translated by J. G. Adashko
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