*POSSIBLE SETS OF EXPERIMENTS FOR SIMULTANEOUS ANALYSIS OF DATA ON NUCLEON-NUCLEON SCATTERING AND POLARIZATION IN p-n COLLISIONS AT 635 Mev**

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It is suggested that n-p and p-p scattering data be analyzed simultaneously to reduce the number of experiments required to reconstruct the scattering amplitudes. Sets of experiments are presented which should yield sufficient information for such analysis. The angular dependence of polarization in p-n collisions at 635 Mev was measured. A difference was detected in the energy and angular dependences of the polarization for states of a nucleon-nucleon system possessing different isotopic spins ($T = 0$ and $T = 1$).

INTRODUCTION

It is known that the results of all experiments on elastic nucleon scattering can be described by different combinations of five complex coefficients of the scattering amplitude. In the general case nine independent experiments[1] are required for the determination of these coefficients (except for a common phase factor). Because of the unitarity of the scattering matrix the number of necessary experiments can be reduced to five[2] at nucleon energies below the meson-production threshold. At energies above 300 Mev reconstruction of the amplitude from elastic scattering alone requires nine experiments, while eighteen experiments are required to determine the two amplitudes A_{np} and A_{pp} through separate analyses of the data. However, it can be shown that by using the concept of the charge invariance of nuclear forces and by performing the simultaneous analysis of data on p-p and n-p scattering, one can in general reduce to thirteen the number of independent experiments required to reconstruct the p-p and n-p scattering amplitudes except for a common phase factor of the two systems, i.e., to determine nineteen real quantities. The basis for this lies in the fact that, subject to isotopic invariance, scattering in p-p and n-p systems can be described by ten complex functions defined in the angular range $0 \leq \vartheta \leq \pi/2$, which determine nucleon scattering in states with total isotopic spins $T = 0$ and $T = 1$. Therefore the performance of each pair of experiments to determine the same characteristics of the p-p system for $0 \leq \vartheta \leq \pi/2$ and of the n-p system for $0 \leq \vartheta \leq \pi$ provides information concerning three real functions that describe scattering. Two of these functions are determined by nucleon interactions in states with $T = 0$ and $T = 1$, while the third function corresponds to interference between these states. Thus for the purpose of determining all ten complex coefficients of the amplitudes (except for a common phase factor) we must perform six pairs of identical experiments on n-p and p-p scattering, giving eighteen independent equations, and an additional single experiment for the p-p or n-p system. In Appendix 1 we give the analytical expressions relating the quantities to be determined in separate experiments with the coefficients of the scattering amplitude. In Appendix 2 we present sets of experiments which will provide the required information from a minimum number of common experiments (or minimum number of experiments with the n-p system), and we indicate the relative difficulty of such experiments.

POSSIBILITY OF USING P-D SCATTERING DATA

Because of the difficulties encountered in experiments on neutron scattering by free protons some investigators have studied proton scattering by neutrons in p-d collisions. This procedure permits the use of more intense and almost monoenergetic proton beams with a considerably higher degree of polarization than for neutrons. But we must determine whether it is legitimate to use such data instead of data on free n-p scattering. In an earlier paper[3] we considered the conditions under which

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n-d scattering data can be used to obtain cross sections for elastic neutron-neutron scattering. We attempted to determine nucleon polarization in the nonrelativistic impulse approximation in the different types of nucleon-deuteron collisions and to establish their relation to polarization in free n-p scattering; our method was similar to that of Tamor.[4]

When the incident nucleon is scattered at angle θ in the laboratory system while the states of the other two nucleons are undetermined, we have the following expression for the polarized cross section $(PQ)(\theta)$:

$$(PQ)_{pd} = (PQ)_{pp} + (PQ)_{pn} + (PQ)_{\text{interf}} I,$$

where $I = I(\theta)$ is a function that is equal to unity for $\theta = 0°$ and decreases rapidly with increasing scattering angle. It is evident from this equation that in the angular region where the integral of $I(\theta)$ is small the polarized cross section for p-d collisions coincides with the sum of the polarized cross sections for p-p and p-n collisions.*

We note finally, that for approximate reconstruction of nucleon-nucleon scattering amplitudes other data on nucleon-deuteron collisions may be useful, since the expressions involved contain combinations of amplitude coefficients which are encountered only in the most complex experiments with free nucleons. Specifically, the expressions that we obtained for the polarized cross section of elastic p-d scattering contain, in addition to the usual Re ae* terms, additional terms of the type Re be*, which are contained only in expressions describing polarization correlations in the scattering of polarized beams.

EXPERIMENTAL PROCEDURE

Figure 1 is a diagram of our experiments on polarization in p-n collisions, using a beam of polarized (635 ± 15)-Mev protons from the synchrocyclotron of the Joint Institute for Nuclear Research. The beam intensity at the target was 4×10^5 sec^{-1} and its degree of polarization was $58 \pm 3\%$.[5] Proton scattering by neutrons was determined from the different counting rates obtained with targets of heavy and light water in thin-walled plexiglas vessels. Elastic p-n scattering events in the range $45° \leq \vartheta \leq 145.7°$ were detected by registering protons and neutrons through two coincidence-connected telescopes that were set up at angles corresponding to nucleon flight in elastic collisions. The angular resolution of the system was 7°. Protons were registered by a three-counter telescope with FEU-33 photomultipliers and plastic scintillators. Neutrons were registered by a highly efficient multilayer counter with a liquid scintillator.[6] To obviate the registration of charged particles by this counter it was preceded by a scintillation counter in anticoincidence with the neutron counter. Pulses from all of the scintillation counters were fed to coincidence circuits with 2×10^{-8} sec resolving time. The coincidence and anticoincidence circuits provided for the simultaneous registration of both p-p and p-n scattering. The efficiency of the anticoincidence circuit was frequently checked and found to be at least 99.8%.

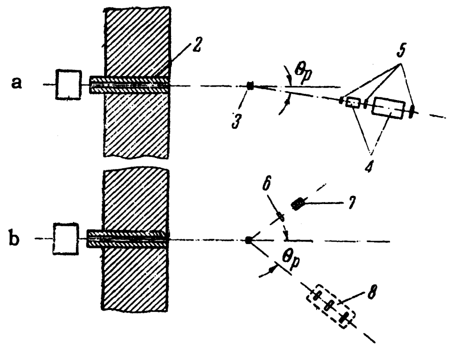

FIG. 1. Arrangement for measurements: a – $\vartheta_p < 57°$ (c.m.s.); b – $45° < \vartheta_p < 145.7°$. 1 – monitor, 2 – collimator, 3 – D_2O or H_2O scatterer, 4 – copper absorbers, 5 – scintillation counters, 6 – anticoincidence counter, 7 – neutron counter, 8 – proton telescope.

The registering procedure described above was unsuitable for the angular region in which one of the scattered particles had low energy; therefore for $18 \leq \vartheta \leq 57$ only elastically scattered protons were registered. Suitable absorbers between the counters prevented registration of inelastically scattered nucleons and mesons. In this case the angular resolution of the telescope was about 4°.

ADJUSTMENT OF APPARATUS

A special effort was made to provide an experimental setup which would obviate false asymmetries arising from (a) inaccurate adjustment of the apparatus with respect to the axis of the beam, (b) bending of the beam in stray magnetic fields, (c) beam inhomogeneities at the targets etc. The possible effect of a magnetic field on the beam was estimated to be small giving a deflection of the beam axis in a stray magnetic field which was less than 10′. The multiplier tubes were placed in iron and permalloy shields to prevent stray magnetic field effects. The efficiency of the shielding was

*Our measurements show that at 635 Mev the integral of $-I(\theta)$ becomes insignificant for angles $\theta \geq 8°$.

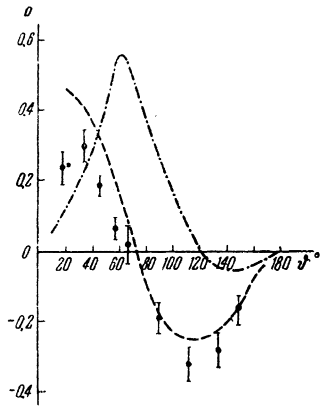

FIG. 2. Angular dependence of polarization in n – p scattering at different energies: dot-dash curve – 95 Mev;[12] dashed curve – 315 Mev;[13] dots – 635 Mev (present work).

considered adequate when no change was observed in the pulse count from a photomultiplier within a magnetic field during gamma irradiation.

Beam uniformity was checked by blackness densities of photographic films exposed at the target position and showed practically no variation across the beam, so far as could be determined from photomicrograms. The position of the center line of the beam was also determined photographically and all of the measuring apparatus was carefully adjusted with respect to this axis. An additional control of correct adjustment was the absence of asymmetry in elastic p-p scattering at c.m. angle 90°, as well as the agreement of p-p asymmetry that we observed at other angles with the asymmetry data given in reference 5 at the same energy.

MEASUREMENTS AND ANALYSIS OF RESULTS

In the range $45° \leq \vartheta \leq 145.7°$ we obtained the anticoincidence counting rates from D_2O and H_2O scatterers to the left and right of the beam axis (looking along the beam). The coincidence setup simultaneously recorded elastic and quasi-elastic p-p scattering events.

The asymmetry of quasi-elastic n-p scattering was determined from the expression

$$\varepsilon_{pn} = \frac{(N_{D_2O} - N_{H_2O})_L - (N_{D_2O} - N_{H_2O})_R}{(N_{D_2O} - N_{H_2O})_L + (N_{D_2O} - N_{H_2O})_R},$$

where N denotes an anticoincidence counting rate (after subtracting the background) with a given scatterer placed in the path of the beam and with the proton telescope located to the left (L) or to the right (R) of the beam. The experiments revealed a symmetrical background on both sides of the beam, and scattering asymmetry with and without correction for the background was identical within the limits of error.

As we have already stated, in the range $18° \leq \vartheta \leq 57°$ only a single (proton) telescope was used. In this case the proton yield from p-n collisions was also determined through the different counting rates from D_2O and H_2O targets. Absorbers of suitable thickness prevented the registration of mesons instead of quasi-elastically scattered protons. In this case N in the expression for ϵ_{pn} is the coincidence counting rate (after subtraction of the background) for a given target and relative telescope position. Control experiments indicated that in these measurements the background resulted mainly from proton scattering at the ends of the collimator and that it was also symmetrical to the left and right of the beam.

An argon-filled ionization chamber served as the monitor in all experiments.

EXPERIMENTAL RESULTS AND DISCUSSION

Some of the quantities involved in the sets of experiments considered in Appendix 2 were previously measured in a number of experiments performed on the synchrocyclotron of the Joint Institute for Nuclear Research. Thus at about 600 Mev we measured the total n-p[7] and p-p[8] cross sections, the differential cross sections for elastic n-p[9] and p-p[10] scattering and the polarization in elastic p-p scattering.[5] During the past year we studied the angular dependence of polarization in p-n scattering in p-d collisions ($E_p = 635$ Mev) and have measured the differential cross sections for elastic neutron scattering by free protons at small angles ($E_n \approx 600$ Mev).[11]

1. The results obtained for the angular dependence of polarization in p-n scattering are given in detail in Fig. 2 together with the statistical errors. This figure also gives the results obtained by other investigators at lower energies. The curve changes markedly from 100 to 300 Mev but much less from 300 to 635 Mev.

A more detailed analysis of the results is planned for a future date; we have considered it useful in the present paper to distinguish the polarized cross sections associated with interactions between nucleons in states with different isotopic spins. The

Asymmetry and polarization of p-n scattering at 635 Mev

$\vartheta°$ (c.m. system)	$(\varepsilon \pm \Delta\varepsilon)$, %	$(P \pm \Delta P)$, %
18.5	14.1±2.7	24.3±4.8
34.5	17.6±2.6	30.3±4.7
45.7	11.0±1.7	19.0±3.1
56.7	3.8±1.8	6.6±3.1
67.3	1.0±3.0	1.7±5.1
90.0	—11.2±2.6	—19.3±4.6
112.5	—18.6±2.8	—32.1±5.1
134.3	—16.2±2.9	—27.9±5.2
145.7	—10.2±3.3	—17.6±5.8

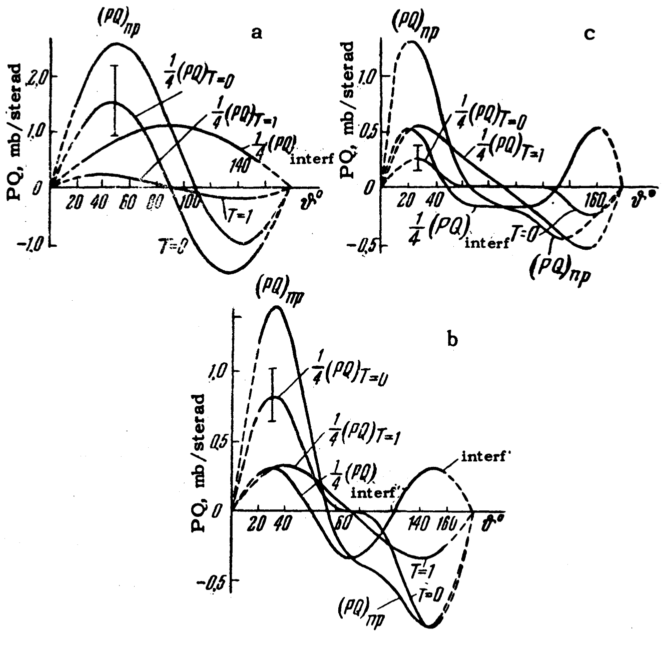

FIG. 3. Angular dependences of polarized n – p scattering cross sections for different isotopic spins at the following energies: a – 95 Mev, b – 315 Mev, c – 635 Mev.

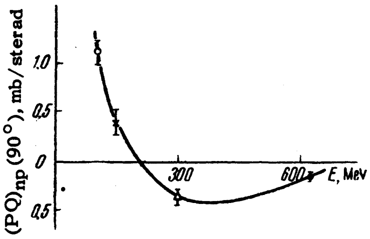

FIG. 4. Energy dependence of the polarized n – p scattering cross section at $\vartheta = 90°$. o – from reference 12, × – from reference 17, Δ – from reference 13, • – the present work.

results for three different nucleon energies are given in Fig. 3. All polarized partial cross sections are given with the same weights with which they are included in the polarized n-p scattering cross sections. The relative contributions of polarized partial cross sections in $(PQ)_{np}$ vary markedly with energy; these cross sections for $T = 0$ and $T = 1$ have different energy dependences, $(PQ)_{T=1}$ increasing with energy while $(PQ)_{T=0}$ decreases considerably. The relatively large polarized cross section for $T = 0$ at 635 Mev indicates a significant contribution of noncentral interactions in these states at 600 – 700 Mev just as at 100 – 300 Mev.[14] The observed reduction of polarized cross sections $(PQ)_{T=0}$ with increasing energy, together with previously obtained data on the angular dependence of the elastic scattering cross section for $T = 0$ and on the total cross sections for nucleon interactions in these states, which decrease with energy,[15] provides additional support for Smorodinskiĭ's[16] hypothesis that nucleon interactions for $T = 0$ can be described qualitatively by means of the Born approximation.

2. Polarization results obtained at different energies sometimes provide information about certain characteristics of nucleon interactions, even when a complete analysis is not made.

Let us consider the energy dependence of the polarized cross section $(PQ)_{np}$ at $\vartheta = 90°$ (c.m.), where only one term of $(PQ)_{np}$ remains, which is given by the interference between states with different isotopic spins. Figure 4 gives the data now available on $(PQ)_{np}$ (90°) for different nucleon energies. The sign is seen to change around 200 Mev. The calculations of Signell and Marshak[17] show that at energies up to 150 Mev both S phases diminish while all other phases increase with energy. A comparison of these calculations with a phase analysis of p-p scattering at 300 Mev[18] shows a reversal of the sign of the ^1S phase in the 100 – 300 Mev interval.

On the other hand, according to Wolfenstein[19] the principal role in polarization for n-p scattering near 100 Mev is played by $^3S - {}^3D$ interference. If it is assumed that at high energies an important part is played by the interference of different waves with the ^3S wave, we can conclude that the sign of the phase of the latter wave is reversed near 200 Mev, thus indicating that both S waves behave alike in the given energy region.

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APPENDIX 1

When nucleon-nucleon scattering amplitudes are written in the form

$$A_{pp} = \alpha_1 + \beta_1 (\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + \gamma (\sigma_1 + \sigma_2) \cdot \mathbf{n} + \delta (\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + \varepsilon (\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}), \quad (1)$$

$$A_{np} = {}^1/_2 (\alpha_1 + \alpha_0) + {}^1/_2 (\beta_1 + \beta_0)(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + {}^1/_2 (\gamma_1 + \gamma_0)(\sigma_1 + \sigma_2) \cdot \mathbf{n} + {}^1/_2 (\delta_1 + \delta_0)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + {}^1/_2 (\varepsilon_1 + \varepsilon_0)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}), \quad (2)$$

where the subscripts 1 and 0 denote the total isotopic spin of the system, and we introduce the functions

$$a_{1,0} = (\alpha + \beta)_{1,0}; \quad b_{1,0} = (\alpha - \beta)_{1,0}; \quad c_{1,0} = (\delta + \varepsilon)_{1,0}; \quad d_{1,0} = (\delta - \varepsilon)_{1,0}; \quad e_{1,0} = 2\gamma_{1,0} \quad (3)$$

which possess the following symmetry properties under the substitution $\vartheta \longleftrightarrow \pi - \vartheta$:

$$a_1(\pi - \vartheta) = -a_1(\vartheta);$$
$$b_1(\pi - \vartheta) = -c_1(\vartheta); \quad c_1(\pi - \vartheta) = -b_1(\vartheta);$$
$$d_1(\pi - \vartheta) = d_1(\vartheta); \quad e_1(\pi - \vartheta) = e_1(\vartheta);$$
$$a_0(\pi - \vartheta) = a_1(\vartheta); \quad b_0(\pi - \vartheta) = c_0(\vartheta); \quad c_0(\pi - \vartheta) = b_0(\vartheta);$$
$$d_0(\pi - \vartheta) = -d_0(\vartheta); \quad e_0(\pi - \vartheta) = -e_0(\vartheta), \quad (4)$$

then, as has already been mentioned, from two experiments where the same quantity for the n-p and p-p systems is measured we can obtain three independent combinations of scattering amplitude coefficients. This enables us to propose a set of thirteen experiments for the purpose of reconstructing the n-p and p-p scattering amplitudes except for a common phase factor of the two amplitudes. The expressions which relate scattering amplitudes to the results of certain basic experiments on elastic nucleon-nucleon scattering are the following:*

1. Elastic scattering cross section:

$$Q_{T=1}(\vartheta) = Q_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_1|^2 + |b_1|^2 + |c_1|^2 + |d_1|^2 + |e_1|^2\}. \quad (5.1)$$
$$Q_{T=0}(\vartheta) = 2[Q_{np}(\vartheta) + Q_{np}(\pi - \vartheta)] - Q_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_0|^2 + |b_0|^2 + |c_0|^2 + |d_0|^2 + |e_0|^2\}, \quad (5.2)$$
$$Q_{\text{interf.}}(\vartheta) = 2[Q_{np}(\vartheta) - Q_{np}(\pi - \vartheta)]$$
$$= \mathrm{Re}\,[a_1 a_0^* + b_1 b_0^* + c_1 c_0^* + d_1 d_0^* + e_1 e_0^*]. \quad (5.3)$$

2. Polarization of angular scattering:

$$(PQ)_{T=1}(\vartheta) = (PQ)_{pp}(\vartheta) = \mathrm{Re}\,a_1 e_1^*, \quad (6.1)$$
$$(PQ)_{T=0}(\vartheta) = 2[(PQ)_{np}(\vartheta) - (PQ)_{np}(\pi - \vartheta)]$$
$$- (PQ)_{pp}(\vartheta) = \mathrm{Re}\,a_0 e_0^*, \quad (6.2)$$
$$(PQ)_{\text{interf}}(\vartheta) = 2[(PQ)_{np}(\vartheta) + (PQ)_{np}(\pi - \vartheta)]$$
$$= \mathrm{Re}\,(a_1 e_0^* + a_0 e_1^*). \quad (6.3)$$

3. Normal component of polarization correlation:

$$(P_{nn}Q)_{T=1}(\vartheta) = (P_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_1|^2 - |b_1|^2 - |c_1|^2 + |d_1|^2 + |e_1|^2\}, \quad (7.1)$$
$$(P_{nn}Q)_{T=0}(\vartheta) = 2[(P_{nn}Q)_{np}(\vartheta)$$
$$+ (P_{nn}Q)_{np}(\pi - \vartheta)] - (P_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_0|^2 - |b_0|^2 - |c_0|^2 + |d_0|^2 + |e_0|^2\}, \quad (7.2)$$
$$(P_{nn}Q)_{\text{interf}}(\vartheta) = 2[(P_{nn}Q)_{np}(\vartheta) - (P_{nn}Q)_{np}(\pi - \vartheta)]$$
$$= \mathrm{Re}\,[a_1 a_0^* - b_1 b_0^* - c_1 c_0^* + d_1 d_0^* + e_1 e_0^*]. \quad (7.3)$$

4. Triple scattering in parallel planes (scattered particle):

$$(D_{nn}Q)_{T=1}(\vartheta) = (D_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_1|^2 + |b_1|^2 - |c_1|^2 - |d_1|^2 + |e_1|^2\}, \quad (8.1)$$
$$(D_{nn}Q)_{T=0}(\vartheta) = 2[(D_{nn}Q)_{np}(\vartheta)$$
$$+ (K_{nn}Q)_{np}(\pi - \vartheta)] - (D_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_0|^2 + |b_0|^2 - |c_0|^2 - |d_0|^2 + |e_0|^2\}, \quad (8.2)$$
$$(D_{nn}Q)_{\text{interf}}(\vartheta) = 2[(D_{nn}Q)_{np}(\vartheta) - (K_{nn}Q)_{np}(\pi - \vartheta)]$$
$$= \mathrm{Re}\,[a_1 a_0^* + b_1 b_0^* - c_1 c_0^* - d_1 d_0^* + e_1 e_0^*]. \quad (8.3)$$

*All expressions are given in nonrelativistic form. The relativistic forms whenever required can be obtained on the basis of Stapp's results,[20] and do not affect the conclusions regarding the reduction of the number of experiments required to reconstruct amplitudes in a simultaneous analysis of n-p and p-p scattering data.

5. Triple scattering in parallel planes (recoil particle):

$$(K_{nn}Q)_{T=1}(\vartheta) = (K_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_1|^2 - |b_1|^2 + |c_1|^2 - |d_1|^2 + |e_1|^2\}, \quad (9.1)$$
$$(K_{nn}Q)_{T=0}(\vartheta) = 2[(K_{nn}Q)_{np}(\vartheta)$$
$$+ (D_{nn}Q)_{np}(\pi - \vartheta)] - (K_{nn}Q)_{pp}(\vartheta)$$
$$= {}^1/_2\{|a_0|^2 - |b_0|^2 + |c_0|^2 - |d_0|^2 + |e_0|^2\}, \quad (9.2)$$
$$(K_{nn}Q)_{\text{interf.}}(\vartheta) = 2[(K_{nn}Q)_{np}(\vartheta) - (D_{nn}Q)_{np}(\pi - \vartheta)]$$
$$= \mathrm{Re}\,[a_1 a_0^* - b_1 b_0^* + c_1 c_0^* - d_1 d_0^* + e_1 e_0^*]. \quad (9.3)$$

6. Polarization correlation for scattering in mutually perpendicular planes:

$$(P_{ml}Q)_{T=1}(\vartheta) = (P_{ml}Q)_{pp}(\vartheta) = \mathrm{Im}\,d_1 e_1^*, \quad (10.1)$$
$$(P_{ml}Q)_{T=0}(\vartheta) = 2[(P_{ml}Q)_{np}(\vartheta) + (P_{ml}Q)_{np}(\pi - \vartheta)]$$
$$- (P_{ml}Q)_{pp}(\vartheta) = \mathrm{Im}\,d_0 e_0^*, \quad (10.2)$$
$$(P_{ml}Q)_{\text{interf.}}(\vartheta) = 2[(P_{ml}Q)_{np}(\vartheta) - (P_{ml}Q)_{np}(\pi - \vartheta)]$$
$$= \mathrm{Im}\,(d_1 e_0^* + d_0 e_1^*). \quad (10.3)$$

7. Rotation of polarization vector (scattered particle):

$$(D_{xm}Q)_{T=1}(\vartheta) = (D_{xm}Q)_{pp}(\vartheta)$$
$$= -\cos(\vartheta/2)\,\mathrm{Re}\,(a_1^* b_1 + c_1^* d_1) + \sin(\vartheta/2)\,\mathrm{Im}\,b_1^* e_1, \quad (11.1)$$
$$(D_{xm}Q)_{T=0}(\vartheta) = 2[(D_{xm}Q)_{np}(\vartheta)$$
$$- (K_{xl}Q)_{np}(\pi - \vartheta)] - (D_{xm}Q)_{pp}(\vartheta)$$
$$= -\cos(\vartheta/2)\,\mathrm{Re}\,(a_0^* b_0 + c_0^* d_0) + \sin(\vartheta/2)\,\mathrm{Im}\,b_0^* e_0, \quad (11.2)$$
$$(D_{xm}Q)_{\text{interf}}(\vartheta) = 2[(D_{xm}Q)_{np}(\vartheta) + (K_{xl}Q)_{np}(\pi - \vartheta)]$$
$$= -\cos(\vartheta/2)\,[\mathrm{Re}\,(a_1^* b_0 + a_0^* b_1) + \mathrm{Re}\,(c_1^* d_0 + c_0^* d_1)]$$
$$+ \sin(\vartheta/2)\,\mathrm{Im}\,(b_1^* e_0 + b_0^* e_1). \quad (11.3)$$

8. Rotation of polarization vector (recoil particle):

$$(K_{xl}Q)_{T=1}(\vartheta) = (K_{xl}Q)_{pp}(\vartheta) = \sin(\vartheta/2)\,\mathrm{Re}\,(a_1^* c_1 - b_1^* d_1)$$
$$+ \cos(\vartheta/2)\,\mathrm{Im}\,c_1^* e_1, \quad (12.1)$$
$$(K_{xl}Q)_{T=0}(\vartheta) = 2[(K_{xl}Q)_{np}(\vartheta)$$
$$- (D_{xm}Q)_{np}(\pi - \vartheta)] - (K_{xl}Q)_{pp}(\vartheta)$$
$$= \sin(\vartheta/2)\,\mathrm{Re}\,(a_0^* c_0 - b_0^* d_0) + \cos(\vartheta/2)\,\mathrm{Im}\,c_0^* e_0, \quad (12.2)$$
$$(K_{xl}Q)_{\text{interf.}}(\vartheta) = 2[(K_{xl}Q)_{np}(\vartheta) + (D_{xm}Q)_{np}(\pi - \vartheta)]$$
$$= \sin(\vartheta/2)\,[\mathrm{Re}\,(a_1^* c_0 + a_0^* c_1) - \mathrm{Re}\,(b_1^* d_0 + b_0^* d_1)]$$
$$+ \cos(\vartheta/2)\,\mathrm{Im}\,(c_1^* e_0 + c_0^* e_1). \quad (12.3)$$

9. Influence of longitudinal polarization component of incident beam on transverse component of scattered beam (scattered particle):

$$(D_{zm}Q)_{T=1}(\vartheta) = (D_{zm}Q)_{pp}(\vartheta)$$
$$= \sin(\vartheta/2)\,\mathrm{Re}\,(a_1^* b_1 + c_1^* d_1) + \cos(\vartheta/2)\,\mathrm{Im}\,b_1^* e_1, \quad (13.1)$$
$$(D_{zm}Q)_{T=0}(\vartheta) = 2[(D_{zm}Q)_{np}(\vartheta)$$
$$+ (K_{zl}Q)_{np}(\pi - \vartheta)] - (D_{zm}Q)_{pp}(\vartheta)$$
$$= \sin(\vartheta/2)\,\mathrm{Re}\,(a_0^* b_0 + c_0^* d_0) + \cos(\vartheta/2)\,\mathrm{Im}\,b_0^* e_0, \quad (13.2)$$
$$(D_{zm}Q)_{\text{interf}}(\vartheta) = 2[(D_{zm}Q)_{np}(\vartheta) - (K_{zl}Q)_{np}(\pi - \vartheta)]$$
$$= \sin(\vartheta/2)\,[\mathrm{Re}\,(a_1^* b_0 + a_0^* b_1) + \mathrm{Re}\,(c_1^* d_0 + c_0^* d_1)]$$
$$+ \cos(\vartheta/2)\,\mathrm{Im}\,(b_1^* e_0 + b_0^* e_1). \quad (13.3)$$

10. Influence of longitudinal polarization component of incident beam on transverse component of scattered beam (recoil particle):

$$(K_{zl}Q)_{T=1}(\vartheta) = (K_{zl}Q)_{pp}(\vartheta) = \cos(\vartheta/2)\,\mathrm{Re}\,(a_1^* c_1 - b_1^* d_1) - \sin(\vartheta/2)\,\mathrm{Im}\,c_1^* e_1, \quad (14.1)$$

$$(K_{zl}Q)_{T=0}(\vartheta) = 2\,[(K_{zl}Q)_{np}(\vartheta) + (D_{zm}Q)_{np}(\pi-\vartheta)] - (K_{zl}Q)_{pp}(\vartheta) = \cos(\vartheta/2)\,\mathrm{Re}\,(a_0^* c_0 - b_0^* d_0) - \sin(\vartheta/2)\,\mathrm{Im}\,c_0^* e_0. \quad (14.2)$$

$$(K_{zl}Q)_{\mathrm{interf}}(\vartheta) = 2\,[(K_{zl}Q)_{np}(\vartheta) - (D_{zm}Q)_{np}(\pi-\vartheta)] = \cos(\vartheta/2)\,[\mathrm{Re}\,(a_1^* c_0 + a_0^* c_1) - \mathrm{Re}\,(b_1^* d_0 + b_0^* d_1)] - \sin(\vartheta/2)\,\mathrm{Im}\,(c_1^* e_0 + c_0^* e_1). \quad (14.3)$$

APPENDIX 2

Let us now consider a possible set of 13 experiments which will permit reconstruction of the scattering amplitude.

Set of experiments No. 1

No.	Experiment	Formulas for analysis	Particle system used in experiment: pp	np
1	Polarization of scattering	6.(1–3)	+	+
2	Rotation of polarization vector (scattered particle)	11.(1–3)	+	+
3	The same for recoil particle	12.(1–3)	+	+
4	Polarization correlation for scattering in mutually perpendicular planes	10.(1–3)	+	+
5	Differential cross sections for elastic scattering	5.(1–3)	+	+
6	Correlation of normal polarization components	7.(1–3)	+	+
7	Triple scattering in parallel planes (scattered particle)	8.(1)	+	–

The proposed 13 experiments enable us to obtain a system of 19 equations in the 19 unknowns:

$$|a_1|, |b_1|, |c_1|, |d_1|, |e_1|, \varphi_{e_1}^{a_1}, \varphi_{e_1}^{b_1}, \varphi_{e_1}^{c_1}, \varphi_{e_1}^{d_1};$$

$$|a_0|, |b_0|, |c_0|, |d_0|, |e_0|, \varphi_{e_1}^{a_0}, \varphi_{e_1}^{b_0}, \varphi_{e_1}^{c_0}, \varphi_{e_1}^{d_0}, \varphi_{e_1}^{e_0},$$

$\varphi_{e_1}^{i}$ being the phase difference between i and e_1.

Because of the difficulties involved in performing experiments with an n-p system and the possibility of making fuller use of Eqs. (5), (7), (8), and (9) with a simpler structure, another set of 14 experiments will be of interest, consisting of 9 experiments on a p-p system and only 5 experiments on a n-p system. An example of such a set is the following:

Set of experiments No. 2

No.	Experiment	Formulas for analysis	Particle system used in experiment: pp	np
1	Differential cross sections	5.(1–3)	+	+
2	Triple scattering in parallel planes (scattered particle)	8.(1–3)	+	+
3	The same for recoil particle	9.(1–3)	+	+
4	Correlation of normal polarization components	7.(1–3)	+	+
5	Polarization of elastic scattering	6.(1–3)	+	+
6	Polarization correlation for scattering in mutually perpendicular planes	10.(1)	+	–
7	Rotation of polarization vector (scattered particle)	11.(1)	+	–
8	The same for recoil particle	12.(1)	+	–
9	Influence of longitudinal polarization component of incident beam on transverse component of scattered beam (scattered particle)	13.(1)	+	–

In this case the first five pairs of experiments determine the moduli

$$|a_1+e_1|, |a_1-e_1|, |b_1|, |c_1|, |d_1|,$$

$$|a_0+e_0|, |a_0-e_0|, |b_0|, |c_0|, |d_0|$$

and the angles between them: $\varphi_{a_1+e_1}^{a_0+e_0}$, $\varphi_{a_1-e_1}^{a_0-e_0}$, $\varphi_{b_1}^{b_0}$, $\varphi_{e_1}^{e_0}$, $\varphi_{d_1}^{d_0}$.

The four other experiments can be used to determine the angles $\varphi_{e_1}^{a_1+e_1}$, $\varphi_{e_1}^{a_1-e_1}$, $\varphi_{1}^{b_1}$, $\varphi_{e_1}^{c_1}$, $\varphi_{e_1}^{d_1}$. In other words, nine experiments comprise a full set for a p-p system, and the simultaneous analysis of five experiments on n-p scattering, together with the corresponding experiments for the p-p system, can be used to determine the moduli of five functions for a system with T = 0 and the phase differences between the corresponding functions for T = 0 and T = 1.

It must be noted, however, that since the given equations are bilinear with respect to a, b, c, d, and e the solutions obtained from these sets of equations may not be unique. We must therefore obtain additional relations between the quantities to be determined. We have considered two other sets of 14 equations that enable us to obtain such relations. The first of these sets (No. 3) agrees with set No. 2 except that Experiment No. 9 with the p-p system is replaced by Experiment No. 6 with the n-p system. This set of experiments leads to 20 independent equations for the calculation of the 19 unknowns given above.

Set No. 4 (below) consists of seven pairs of identical experiments with a p-p and a n-p system. The number of n-p experiments is here greater than in the previous set, but the very difficult experiment No. 6 is replaced by two experiments which are much easier from an experimental point of view.

Set of experiments No. 4

No.	Experiment	Formulas for analysis	Particle system used in experiment	
			pp	np
1	Differential cross sections	5.(1–3)	+	+
2	Triple scattering in parallel planes (scattered particles)	8.(1–3)	+	+
3	The same for recoil particle	9.(1–3)	+	+
4	Correlation of normal polarization components	7.(1–3)	+	+
5	Polarization of scattering	6.(1–3)	+	+
6	Rotation of polarization vector (scattered particle)	11.(1–3)	+	+
7	The same for recoil particle	12.(1–3)	+	+

This set of experiments leads to 21 independent equations for 19 unknowns and can thus be very useful for the unique reconstruction of p-p and n-p scattering amplitudes. The indeterminacy of the results can also be reduced by comparing solutions for different scattering angles and energies.

It should be noted that as information concerning scattering amplitudes is accumulated in some instances it may become unnecessary to perform certain experiments of the foregoing sets either over the entire angular range or for individual angles where the form of the amplitude is simpler.

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74