LIFETIME OF THE FIRST EXCITED STATE IN Be^{10}

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A method is described for measuring the lifetime of nuclear excited states, using the recoil nuclei, together with an experiment to find the lifetime τ of the 3.37-Mev excited state in Be^{10}. An upper limit for τ of 8×10^{-14} sec is obtained.

1. INTRODUCTION

EXPERIMENTAL investigation of γ-ray transition probabilities are an important source of information about nuclear structure. For light nuclei, the excitation energies of even the lowest states are rather large and the lifetimes of these states usually less than 10^{-10}sec, so that direct methods of obtaining τ, based on decay schemes, are not applicable. To measure short lifetimes, indirect methods must be used, such as measuring widths Γ in resonance reactions, resonance scattering of γ-rays, and the Doppler shift method.

Unfortunately, none of these methods can be applied over a wide range of conditions. Thus, in the first of those mentioned above the width Γ can be measured only for levels close to the dissociation energy of the nucleus. Resonance scattering of γ rays can be used only with stable isotopes occurring naturally with a reasonable abundance. The Doppler-shift method can be more generally applied, but technical difficulties limit its usefulness.

A new method has recently been proposed[1] for measuring the lifetimes of excited states in light nuclei. It has about the same range of applicability as does the Doppler shift method, but the technical difficulties involved are less.

The present paper is devoted to a theoretical exposition of this method and a description of an experiment to measure the lifetime of the first excited state in Be^{10}.

2. DESCRIPTION OF THE METHOD

The distinguishing characteristic of reactions involving light nuclei is that the kinetic energy of the recoil nucleus is comparable with the kinetic energy of the lighter component. This circumstance, together with the fact that light nuclei undergo little multiple scattering in thin targets, has the consequence that in a reaction both the direction in which the light component is emitted and the direction in which the nucleus recoils can be fixed.

If the light particle is observed to be emitted in a certain direction, there is a discrete spectrum of angles in which the nucleus could have recoiled, each angle corresponding to a definite energy level of this nucleus.[2] For the ground state, the direction of recoil is uniquely determined. Nuclei recoiling in an excited state had a momentum p_0 given to them during the reaction, and then get some more momentum p_γ when the γ ray is emitted. Usually, the condition $p_\gamma \ll p_0$ holds. Nuclei recoiling in a definite excited state will then be emitted in a cone with angle $\Phi_0 = p_\gamma/p_0$. The fact that Φ_0 depends on the speed of the nucleus when the γ ray was emitted can be used to find the lifetime of the excited state in question.

Let us assume that just after passing through a thin target the recoil nuclei must pass through an absorbing layer of material where they slow down and lose energy. Two cases must be considered: (1) The lifetime τ is small, so that decay occurs before the nuclei slow down; then the size of the cone will be determined by the angle Φ_0. (2) The lifetime τ is large,* so that decay occurs after the nuclei have been slowed down. In this case the angle of the cone Φ_d will be determined by the relation $\Phi_d = p_\gamma/p_d$, where p_d is the momentum of the recoil nucleus upon emerging from the absorbing layer.

It is clear that if the γ ray is emitted while the nucleus is slowing down, the angle of the cone and the angular distribution of recoil nuclei within the cone will be connected in some unique way with the value of τ.

The value of τ can be determined experimen-

*Actually we consider only values of τ which are small compared to the time of flight of the nucleus from the target to the detector (about 20 cm.).

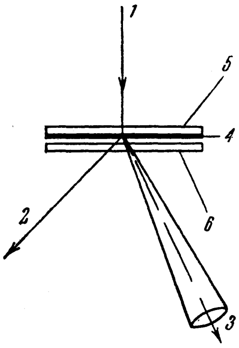


FIG. 1. Schematic of the experiment: 1 – incident beam; 2 – light particle; 3 – recoil nucleus; 4 – target; 5 – backing; 6 – compensating layer.

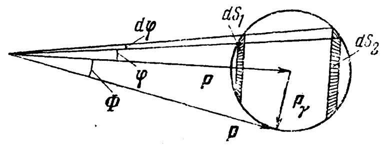
tally as follows: We count the number of nuclei passing through a small slit on the axis of the cone both when the decelerating layer is present and when it is absent. Since the ratio δ of these two counting rates is a function of τ, the value of the latter can then be calculated. As a decelerating layer we can use the target backing. Then δ is the ratio of the counting rates observed in two target positions differing by 180°.

The considerations above are valid only when we can neglect multiple scattering and the energy straggling in the nuclei due to the backing. If these effects cannot be neglected, the experimental setup must be changed so that these effects are the same in the two positions of the target. This can be achieved by putting a second layer of material, of the same thickness and composition as the target backing, a short distance from the target and parallel to it (see Fig. 1).

Let the target position shown in Fig. 1 be A, and let position B be obtained from A by rotation through 180°. For lifetimes $\tau \geq 10^{-10}$ sec the recoil nuclei travel a distance ~ 0.01 mm. Hence all the decays occur before the compensating layer is reached and the number of nuclei counted in position A will remain independent of lifetime. In this case, δ, the ratio of the counting rate in position B to that in position A, is still a suitable quantity to measure for finding τ.

From the qualitative considerations above, it is clear that the range of lifetimes τ which can be measured is determined by how long the nuclei are decelerated in matter ($10^{-12} - 10^{-14}$ sec). As is well known, the range of lifetimes covered by the Doppler-shift method is about the same. There is, however, a difference between the cases to which the two methods can be applied. Thus, the Doppler-shift technique can be applied to transitions between excited states. The method described here can be applied only to transitions to the ground state. On the other hand, if we differentiate between the various energy particles produced in the reaction, we can investigate transition probabilities for levels lying close together, which is very difficult to do with the Doppler-shift method.

FIG. 2. Angular distribution of recoil nuclei due to γ emission.



The purpose of the following argument is to obtain the functional relation between δ and τ. Suppose that just before the γ ray was emitted the recoil nucleus had momentum p and that the γ ray gave it an extra momentum p_γ. The distribution function of the nuclei within the cone can then be found from Fig. 2. From this drawing it is clear that the ends of the vectors $\mathbf{P} = \mathbf{p} + \mathbf{p}_\gamma$ will lie on the surface of a sphere of radius p_γ. Since the γ rays are emitted more or less isotropically, the fraction of nuclei scattered through an angle φ into the solid angle $d\varphi$ will be given by the ratio of the total area of the shaded spherical surface in the figure to the total area of the sphere:

$$dN / N = (ds_1 + ds_2) / 4\pi p_\gamma^2. \tag{1}$$

Remembering that $p_\gamma \ll p_0$, the distribution function $\psi(\varphi)$ of the recoil nuclei in the cone can easily be found, Normalizing $\psi(\varphi)$ so that

$$\int_0^\Phi 2\pi\psi(\varphi)\sin\varphi d\varphi = 1,$$

where $\Phi = p_\gamma/p$, we find that the distribution function has the form

$$\psi(\varphi) = 1 / 2\pi\Phi\sqrt{\Phi^2 - \varphi^2}. \tag{2}$$

Let the lifetime τ be comparable with the time of deceleration, and let the target be in position B. Then the distribution function will have two components, one due to nuclei decaying in the target backing, and the second due to nuclei that emit γ rays after leaving the backing. Neglecting multiple scattering for the time being, let us find these two distribution functions, which we denote by f_1 and f_2.

Assume that the total range R of a particle in matter is proportional to its speed, $R = \alpha v$.[3] Then from the exponential character of the decay it is easy to show that the probability that a nucleus decays in a layer of the backing having thickness dx and distant x from the target is given by the formula

$$U(x)\,dx = (\lambda\alpha / R)(1 - x/R)^{\lambda\alpha - 1} dx, \tag{3}$$

where $\lambda = \ln 2/\underline{\tau}$. Since $p = p_0(1 - x/R)$, formula (2) gives the distribution of recoil nuclei decaying in the layer dx:

$$\psi(x, \varphi) = (1 - x/R)^2 / 2\pi\Phi_0\sqrt{\Phi_0^2 - \varphi^2(1 - x/R)^2}. \tag{4}$$

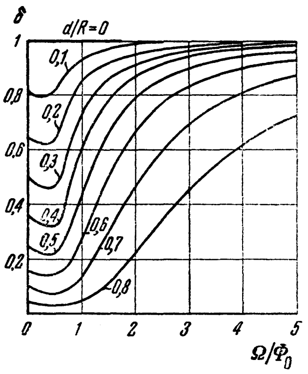


FIG. 3. Dependence of δ on Ω/Φ_0 for various values of the parameter d/R.

The function f_1 is then given by the following:

$$f_1=\begin{cases}\int\limits_0^d \psi(x,\ \varphi)\,U(x)\,dx & \text{for } 0<\varphi\leqslant\Phi_0,\\ \int\limits_{R(1-\Phi_0/\varphi)}^{d} \psi(x,\ \varphi)\,U(x)\,dx & \text{for } \Phi_0<\varphi\leqslant\dfrac{\Phi_0}{1-d/R}.\end{cases} \qquad (5)$$

From (3) it is easy to show that the probability a nucleus decays after passing through a backing of thickness d is $(1-d/R)^{\lambda\alpha}$; hence

$$f_2=\psi(d,\ \varphi)\,(1-d/R)^{\lambda\alpha}. \qquad (6)$$

Denote the total distribution by $f_B = f_1 + f_2$. If the target is in position A, the distribution function f_A can easily be found from f_B by going to the limit $\tau\to\infty$. It has the form:

$$f_A=1/2\pi\Phi_0\sqrt{\Phi_0^2-\varphi^2}. \qquad (7)$$

Now let us take into account multiple scattering in the backing. The distribution function for multiple scattering with dispersion Ω^2 can be written

$$w=(1/2\pi\Omega^2)\exp\{-\varphi^2/2\Omega^2\}. \qquad (8)$$

The functions f_B and f_A should be replaced by F_B and F_A, which take on the following values at $\varphi = 0$:

$$F_B(0)=2\pi\int\limits_0^{\Phi_0/(1-d/R)} f_B(\varphi)\,w(\varphi)\,\varphi d\varphi, \qquad (9)$$

$$F_A(0)=2\pi\int\limits_0^{\Phi_0} f_A(\varphi)\,w(\varphi)\,\varphi d\varphi. \qquad (10)$$

The slit in front of the counter has previously been assumed small enough that we can take $\delta = F_B(0)/F_A(0)$. τ enters the expression for δ through the decay constant λ appearing in (5) and (6). The integral in (9) cannot be expressed in terms of elementary functions and must be found by numerical integration.

To get an idea how sensitive the method is, we can consider the values of δ in two extreme cases:

(1) The lifetime τ is small compared with the deceleration time. Evidently, for this case $\delta = 1$.

(2) The lifetime τ is large compared with the deceleration time. In this case, taking $\lambda\to 0$, we obtain

$$\delta=\left(1-\frac{d}{R}\right)e^{-z_d^2}\int\limits_0^{z_d} e^{t^2}dt\Big/ e^{-z_0^2}\int\limits_0^{z_0} e^{t^2}dt, \qquad (11)$$

where

$$z_0=\Phi_0/\sqrt{2}\Omega,\ z_d=\Phi_0/\sqrt{2}\Omega\,(1-d/R).$$

(the integrals occurring in (11) are tabulated[4]).

Figure 3 shows δ as a function of Ω/Φ_0 calculated from formula (11) for various values of d/R. From this figure it is clear that δ is most sensitive to changes in τ for large values of d/R and small values of Ω/Φ_0. These quantities are not independent. At present, the relationship between them cannot be written down in the general case. This can be done, however, for light nuclei when the recoil nuclei and the nuclei in the decelerating layer have masses about 20 and the energy of the recoil nucleus is of order 1 Mev.

With an accuracy sufficient for our purposes, the theory of multiple scattering[5] gives the relation

$$\frac{\Omega}{\Phi_0}=2\sqrt{\pi N\rho Z_T v\alpha}\,\frac{Z_{nuc}^{1/3}c\hbar}{E_\gamma}\left(\frac{d/R}{1-d/R}\right)^{1/2}. \qquad (12)$$

In this formula, N is Avogadro's number, ρ is the density of the decelerating material, Z_T is its atomic number, v and Z_{nuc} are the speed and charge of the recoil nuclei, E_γ is the energy of the γ ray, c is the velocity of light, and h is Planck's constant. From (12) we see that the relation between Ω/Φ_0 and d/R depends critically on the value of E_γ. It is the value of E_γ which gives the fundamental limitation on the applicability of this method. Substituting values into (12) typical for light nuclei it is easy to see that in order for δ to be less than 0.8 it is necessary that E_γ be more than 1.2 Mev. This is not a serious limitation.

The analysis carried out above shows that for sufficiently large excitation energies, δ varies over a fairly wide range.

3. APPARATUS AND MEASUREMENTS

We measured the lifetime of the first excited state in Be^{10}, as formed in the reaction $Be^9(d,p)Be^{10}$. To our knowledge, the lifetime of this state had not been measured before. It is possible to calculate the wave functions of the ground and first excited states of the Be^{10} nucleus in various nuclear models, and from these wave functions the matrix element for the transition between them, so that an experimental measure of the lifetime would have theoretical interest.

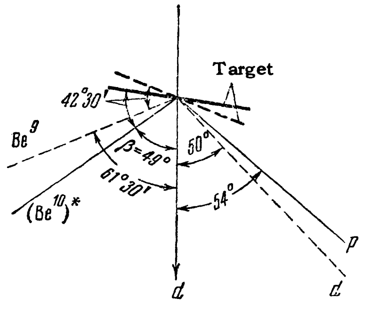


FIG. 4. Angles used in measuring Ω.

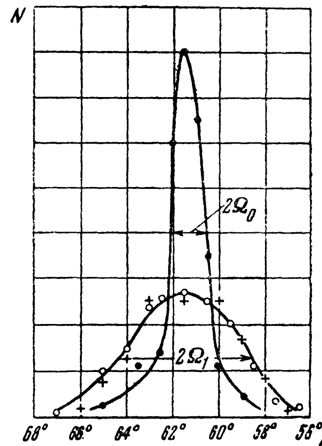


FIG. 5 Angular distribution of recoil nuclei in the elastic scattering of deuterons on Be^9 nuclei.

The incident beam of 4 Mev deuterons was obtained from the cyclotron of the Scientific Research Institute for Nuclear Physics, Moscow State University. Near the target the beam had a diameter of 6 mm and intensity about 10^{-7} amp. The target was a layer of beryllium on an aluminum backing situated at the center of the scattering chamber. The positions of the detectors used to count the protons and recoil nuclei could be varied continuously without breaking the vacuum.[6] The recoil nuclei were detected by an electron multiplier. The protons were registered by a proportional counter.

Pulses from both counters were fed into a coincidence circuit. There was a large unwanted counting rate in the multiplier, due to slow particles coming from the surface of the target[7] and to recoil nuclei from deuteron scattering on aluminum. To minimize the number of accidental coincidences, thin organic films were placed before the entrance to the multiplier. The thickness of the films was chosen so that aluminum recoil nuclei were absorbed while Be^{10} nuclei were transmitted.

The backing and absorbing layers were of aluminum stretched on suitable frames. The frames were mounted so as to make the aluminum foils accurately parallel. The mounting was also such that either foil could be removed at will. The distance between the foils was 1 mm. The angle between the target and the beam could be adjusted to a fraction of a degree. The aluminum layers were prepared by evaporating aluminum in vacuum onto an organic film glued to a thin rubber ring.[8] After the aluminum had been deposited, half of its surface was covered by a shield and beryllium deposited on the other half. The organic film was then dissolved in amyl acetate. The aluminum foil was glued onto two frames so that one carried a layer of aluminum and beryllium while the other carried only aluminum, the aluminum layers being of about the same thickness. The thickness of the compensating layer was measured by comparing the number of deuterons elastically scattered from it with the number scattered from a thicker foil of known thickness. The thickness of the beryllium target was determined by comparing it with a control sample which could be weighed. Particular attention was paid to the uniformity of the layers. Only films with mirror-like surfaces were used.

The experiment was carried out in two steps. First we measured the mean angle of multiple scattering Ω, which is required for the calculations, and compared the thicknesses of the backing and compensating layers. To do this we used the elastic scattering of deuterons from Be^9 nuclei. The second stage was the measurement of δ. This used the reaction $Be^9(d, p)Be^{10}$. In order that the quantity Ω, which was measured for Be^9 nuclei, could be used with Be^{10} it was necessary to choose the angles at which deuterons, protons and recoil nuclei were emitted so that the energies of the Be^9 and Be^{10} nuclei were the same. The geometry used in the experiments is shown in Fig. 4. The Be^9 and Be^{10} nuclei had energy 550 kev. In both cases the angle between the plane of the target and the direction of the recoil nuclei was 42°30′.

Figure 5 shows the measured angular distribution of Be^9 nuclei after elastic scattering. The sharp peak corresponds to target position A without the compensating layer. The wide peak was obtained with the compensating plate in place, the crosses referring to position A and the circles to position B. The thickness of the compensating layer was $(55 \pm 2)\ \mu g/cm^2$. The thickness of the beryllium target was $10\ \mu g/cm^2$. Since the target was at an angle 42°30′ to the direction of the recoil nuclei, we conclude that the recoil nuclei passed through a compensating layer of effective thickness $82\ \mu g/cm^2$. The slit in front of the counter had dimensions 4×4 mm., which corresponded to a solid angle (1×1)°. The slit in front of the multiplier had dimensions 2×36 mm (0.5×9)°. The width of the high peak was determined essentially by the size of the slits and the width of the beam, i.e., by factors that are unaffected by the presence of

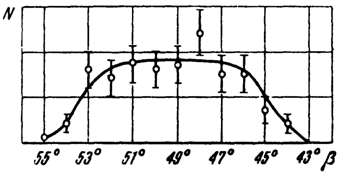


FIG. 6. Angular distribution of the recoil nuclei in the reaction $Be^9(d, p)Be^{10*}$.

	1	2	3	4
A	75×4	112×4	61×4	71×4
B	74×4	116×4	54×4	74×4

the scattering aluminum layer. The mean angle of multiple scattering can be obtained approximately from the relation

$$\Omega = \sqrt{\Omega_1^2 - \Omega_0^2}.$$

In our case $2\Omega_0 = (1.5 \pm 0.2)°$; $2\Omega_1 = (6 \pm 0.5)°$, so that $\Omega = (2.9 \pm 0.3)°$. To compare the thicknesses of the backing and compensating layer, we measured the coincidence counting rate at the peak (61°30′) for the two target positions A and B. The results of several reversals of the target showed that the difference was less than 5%.

The angular distribution of recoil nuclei from the reaction $Be^9(d, p)Be^{10*}$ with target position B is shown in Fig. 6. The slit in front of the counter was 8×8 mm, that in front of the multiplier was 4×20 mm. From the diagram it is evicent that the peak has a full width of about 8°, which is approximately what was expected. The distribution shown corresponds to nuclei recoiling in the first excited state of Be^{10}; nuclei recoiling in the ground state travel in directions around $\beta = 62°$. There could have been little admixture of nuclei in the higher excited states because of the absorber in front of the counter for slow protons.

The angle $\beta = 49°$ was chosen to measure δ. The slits in front of the counter and multiplier were of the same size, 8×8 mm (2×2°). The number of coincidence counts in positions A and B were measured for the same number of counts in the beam integrator. The results are shown in column 1 of the table. This was repeated at the angles used to compare the thicknesses of the aluminum layers. The results are shown in column 2. The whole procedure was then repeated with the results shown in columns 3 and 4. The number of counts in the integrator for each column was arbitrary.

From the table we can see that sithin the statistical errors the thicknesses of the layers are the same. From columns 1 and 3 we get

$$\delta = 0.93 \pm 0.08.$$

Figure 7 shows the functional dependence of δ

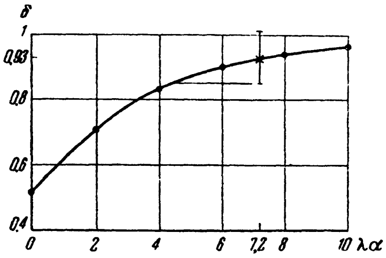


FIG. 7. Determination of limiting value of of $\lambda\alpha$.

on $\lambda\alpha$, as calculated from formulas (9) and (10) with the parameters $\Omega = 2.9°$; $\Phi_0 = 2°$; $d/R = 0.6$. The range R of the recoil nuclei was found from the formula $R = \alpha v$, where α was taken to be 4×10^{-13} sec.[9] The value $\delta = 0.93$ corresponds to $\lambda\alpha = 7.2$, but the statistical errors in measuring δ are such that we can only say $\lambda\alpha > 5$. This gives an upper limit to the lifetime $\tau < 8 \times 10^{-14}$ sec.

4. DISCUSSION OF THE RESULTS

Inglis[10] and Kurath[11] have shown that for the lightest nuclei (up to O^{16}) the shell model gives a satisfactory account of the level schemes, magnetic dipole moments of the ground states, and the probabilities of magnetic dipole γ ray transitions. The constant in the spin-orbit interaction increases monotonically with increasing number the nucleons in the nucleus. For Be^{10} we have $a/K \approx 5$ where a is the constant in the spin-orbit coupling and K is the exchange integral for the central interaction between two nucleons. The first excited state in Be^{10} (3.37 Mev) has spin $J = 2$, isotopic spin $T = 1$, while the ground state has $J = 0$ and $T = 1$.[12] Both states have positive parity so the transition is pure E2.

To calculate τ we use the relation $\tau = 6.58 \times 10^{-16}/\Gamma$, where Γ is the width of the excited state in electron volts and τ is in seconds. For an E2 transition the formula for Γ can be put in the form[13]

$$\Gamma = 8.1 \cdot 10^{-5} E^5 \,|\langle JT \,\|\, H^{(2)} \,\|\, J'T' \rangle|^2,$$

where E is the energy of the transition in mev and $\langle JT \| H^{(2)} \| J'T' \rangle$ is the reduced matrix element for an electric quadrupole transition from a state J, T to a state J′, T′. Denote the wave functions of the J, T and J′, T′ states in the L-S representation by $\{c_i\}$ and $\{b_i\}$ respectively, where i and j are indices labelling the supermultiplet state.[14] Then the expression for Γ has the form

$$\Gamma = 8.1 \cdot 10^{-5} E^5$$

$$\Big|\sum_{ij} c_i b_j \langle (L_i S_i) JT \,\|\, H^{(2)} \,\|\, (L_j' S_j') J'T' \rangle \Big|^2. \quad (13)$$

To calculate the reduced matrix elements in (13) we use the relations given in reference,[13] and $\langle r^2 \rangle = 10^{-25}$ cm^2. The wave functions were calculated for a centrally symmetric interaction consisting of 80% Majorana and 20% Bartlett forces, the ratio of the integrals for the central interaction being $L/K = 6.8$. a/K was taken to be 4.75. For these values of the parameters, $\tau = 2.1 \times 10^{-13}$ sec. Decreasing the strength of the spin orbit interaction decreases the value of τ, which reaches a value 1.4×10^{-13} sec in the extreme case of L-S coupling. For the limiting case of j-j coupling the corresponding matrix element is zero.

In view of the difference between the measured and computed values of τ for all possible values of a/K, it becomes interesting to look at the agreement between experimental and theoretical values for other E2 transitions.

Experimental values of τ for pure E2 transitions in the P-shell are known for C^{12} (4.43 mev $\rightarrow$ 0), C^{12} (16.1 mev $\rightarrow$ 0) and B^{10} (0.72 mev $\rightarrow$ 0). The experimental value τ_{exp} for the 4.43 mev $\rightarrow$ 0 transition in C^{12} is 5.25×10^{-14} sec.[15] With the reasonable value for the strength of the spin-orbit coupling $a/K \approx 6$ and with $\langle r^2 \rangle = 5.7 \times 10^{-26}$ cm^2 [16] the shell model gives the value $\tau_{theor} \approx 2.5 \times 10^{-13}$ sec. For the transition 16.1 Mev $\rightarrow$ 0 in C^{12}, $\tau_{exp} = 9 \times 10^{-16}$ sec[15] while $\tau_{theor} = 1.2 \times 10^{-15}$ sec. For the case B^{10}, $\tau_{exp} = 1.05 \times 10^{-9}$ sec[15] while for $a/K > 4$ and the larger value $\langle r^2 \rangle = 10^{-25}$ cm^2, $\tau_{theor} = 2 - 3 \times 10^{-9}$ sec.[11]

Thus in the third and first cases τ_{exp} is substantially smaller than τ_{theor}, while in the second case the two values agree.

The suggestion has been made[11] that the smallness of τ_{exp} compared to τ_{theor} is due to collective motions in the nucleus. In reference 17 it is pointed out that collective motions should have an effect on τ only for transitions where the isotopic spin does not change. The transitions C^{12} (4.43 Mev $\rightarrow$ 0) and B^{10} (0.72 Mev $\rightarrow$ 0) are just of this type. The isotopic spin changes in the transition C^{12} (16.1 Mev $\rightarrow$ 0) and accordingly the values τ_{exp} and τ_{theor} agree. In our case, the isotopic spin does not change in the Be^{10} (3.37 Mev $\rightarrow$ 0) transition ($T = T' = 1$), so the discrepancy between the experimental and theoretical values of the lifetime is not surprising and supports the conjecture that collective effects exist in light nuclei.

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