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RADIATION FROM A SPIN-2 PARTICLE MOVING UNIFORMLY IN A MEDIUM

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THE energy radiated from a particle with spin 2, moving uniformly in a medium with a velocity higher than the phase velocity of light in that medium (Cerenkov effect), can be determined in analogy to the phenomenological theory of this effect for the electron (see reference 1, §32).

The free field operator for a particle with spin 2 has, according to the general, relativistically-covariant equations of first order,[2] the form

$$D = i\hbar\gamma_4\partial / \partial t - \hbar c(\gamma\nabla) - mc^2, \tag{1}$$

where the matrices γ, of dimension thirty, are known from references 3 and 4. The interaction of charged particles with the electromagnetic field is obtained by changing the operator $\partial/\partial x_k$ to the operator $\partial/\partial x_k - (ie/\hbar c)A_k$. The field of virtual photons interacting with the particle moving in a dielectric with a refraction coefficient $n = c/c'$ is obtained from the fundamental formula (1) in analogy to the theory for the electron.[1] The result is

$$w^+(\mathbf{r}, t) = -ieL^{-3/2}\sum_{\mathrm{K}}\sqrt{2\pi c'\hbar / K}\,(\boldsymbol{\gamma}\cdot\mathbf{a}^+) \times \exp(ic'K_0t - i\mathbf{K}\cdot\mathbf{r}), \tag{2}$$

where e is the charge of the particle, K_1, K_2, K_3, and $K_0 = K = \sqrt{K_1^2 + K_2^2 + K_3^2}$ form the four-dimensional wave vector of the photon, $\mathbf{a}^+(a_1^+, a_2^+, a_3^+)$ are the amplitudes of the photon field acting on a function of the number of photons included in the wave function. The a_n^+ satisfy the commutation relations

$$a_{n'}^+ a_n = 0, \quad a_n a_{n'}^+ = \delta_{nn'} - K_n K_{n'} / K^2 \tag{3}$$

and the condition of transversality $(\mathbf{K}\cdot\mathbf{a}^+) = 0$.

The probability for the radiation process can be determined by considerations similar to those of the electron case. The energy W radiated from the particle per unit time is

$$W = e^2c\int_0^{\omega_m}\frac{k_0'K^2}{nk}G\,dK, \quad G = \frac{1}{5Kn}\sum_{s,s'}\frac{b^+(\boldsymbol{\gamma}^+\mathbf{a})\gamma_4^+b'\cdot b'^+\gamma_4(\boldsymbol{\gamma}\mathbf{a}^+)b}{b'^+\gamma_4^+b'\cdot b'^+\gamma_4 b'}, \tag{4}$$

where b and b′ are thirty-component functions of the wave vectors k_e and k_e'; the primes refer to the state of the particle after radiation; ω_m is the maximal radiation frequency, which depends on the momentum of the particle according to the formula

$$\omega_m = \frac{2pc}{n\hbar}\,\frac{1 - 1/n\beta}{1 - n^{-2}}, \tag{5}$$

the direction of the radiation is determined by

$$\cos\theta = 1/n\beta + (K/2k)(1 - n^{-2}), \tag{6}$$

$\beta = v/c$, where v is the velocity of the particle. The conservation laws are fulfilled: $\mathbf{k}' = \mathbf{k} - \mathbf{K}$, $k_0' = k_0 - K/n$. The frequency of the light radiated from the particle is equal to $\omega = cK/n$.

The classification of the wave functions according to the projection of the spin on the momentum of the particle and the normalization with respect to the charge $\psi^*A\gamma_4 = 1$ in the calculation of the quantity G were carried out with the help of the covariant method proposed by Fedorov.[5]

In the general case, as well as in the nonrelativistic approximation, we were faced with exceedingly complex calculations, which we were unable to master. The comparatively simple calculation in the extreme relativistic case leads, with (4), to

$$\begin{aligned} W = \frac{e^2E^2}{5120\,cE_0^4}\int_0^{\omega_m}\omega\Big\{&32\frac{(E - n\hbar\omega\cos\theta)^4}{(E - \hbar\omega)^2} + 32(E - n\hbar\omega\cos\theta)^2 \\ &+ \frac{(\hbar\omega)^2\sin^2\theta}{E - \hbar\omega}(E - n\hbar\omega\cos\theta) + (E - \hbar\omega)(E - n\hbar\omega\cos\theta) \\ &+ 4\cos^2\theta\left[\frac{n^6(\hbar\omega)^6\sin^6\theta}{(E - \hbar\omega)^4} + 4\frac{n^4(\hbar\omega)^4\sin^4\theta}{(E - \hbar\omega)^2} + 3n^2(\hbar\omega)^2\sin^2\theta\right] \\ &+ 8\cos\theta\sin\theta\left[\frac{n^5(\hbar\omega)^5\sin^5\theta}{(E - \hbar\omega)^4}(E - n\hbar\omega\cos\theta) + 8\frac{n^3(\hbar\omega)^3\sin^3\theta}{E - \hbar\omega}\right. \\ &\left.+ 3\frac{n^3(\hbar\omega)^3\sin^3\theta}{(E - \hbar\omega)^2}(E - n\hbar\omega\cos\theta)\right]\Big\}\,d\omega, \end{aligned} \tag{7}$$

where

$$\cos\theta = \frac{1}{n} + \frac{\hbar\omega}{2E}\left(n - \frac{1}{n}\right), \quad \omega_m = \frac{2pc}{(n+1)\hbar},$$

E is the initial energy of the particle, and $E_0 = mc^2$ is the rest energy.

For a medium with a refraction index close to unity ($\cos\theta \approx 1$, $\sin\theta \approx 0$), we obtain from (7):

$$W = \frac{13\, e^2 E^2}{1024\, c E_0^4} \int_0^{\omega_m} \omega (E - \hbar\omega)^2 d\omega, \quad \omega_m = \frac{pc}{\hbar}. \tag{8}$$

If the energy of the emitted quantum is significantly smaller than the energy of the particle ($\hbar\omega \ll E$), then $\cos\theta = 1/n$, and we have for the radiation energy, from (7),

$$W = \frac{13}{1024} \frac{e^2}{c} \left(\frac{E}{E_0}\right)^4 \int_0^{\omega_m} \omega d\omega, \tag{9}$$

where only $\omega_m = 2pc/(n+1)\hbar\,(n \gg 1)$ depends on the index of refraction.

A specific feature of these results is the unlimited increase of the radiation energy with growing initial energy of the particle. This is in agreement with the latest experimental data on the intensity of the Cerenkov radiation caused by the particles of the cosmic radiation. Jelley notes[6] that Bassi and his co-workers have established, in a series of experiments on the changes in the intensity of the Cerenkov radiation of cosmic-ray particles, that the yield of light increases with increasing energy for very high particle energies. Jelley's view[6] is that this increase of radiation for high energies cannot be explained by knock-on electrons, nor by any other cause, and that the results of these experiments confirm the conclusions of Budini, who predicted a logarithmic increase of the radiation of the particles at extremely relativistic energies in dense media, in analogy to the increase of the ionization losses at such energies.

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DISAPPEARANCE OF THE ISOTHERMAL JUMP AT LARGE RADIATION DENSITY

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THE theory of finite disturbances in gas, based on the differential equations of hydrodynamics without supplementary assumptions, leads, in the complete absence of dissipative factors and continuous initial conditions, to absurd triple-valued flow parameters.[1] To eliminate this difficulty, Stokes and Riemann postulated a flow discontinuity that satisfies the boundary conditions.[2] Rankin and Rayleigh,[3] staying within the framework of a continuum, examined a heat-conduction mechanism by which "jumps" can be realized, and considered the results of Stokes, Riemann, and Hugoniot as a trivial neglect of the dissipation mechanism. Rayleigh observed here that the heat-conduction structure of a compression wave with a profile that remains unchanged in time can be continuous only if

$$p_{+\infty} / p_{-\infty} \leqslant (\varkappa + 1)/(3 - \varkappa) \equiv (i + 1)/(i - 1). \tag{1}$$

Waves of greater amplitude experience an "overturn" similar to the Riemannian overturn of adiabatic wave. The "isothermal jump" (see reference 4) resolves the Rayleigh paradox, although the meaning of such a solution again reduces somehow to ignoring, within the scope of the stated problem, dissipative factors other than heat conduction (such as viscosity and diffusion). In this solution one has in mind a radiant heat conduction, but the radiation density is neglected, and the conclusion is that the structure of a wave of amplitude (1), with the inequality sign reversed, includes the isothermal jump.

Considering, as before, the gas to be a continuous heat-conducting medium that retains local ther-