CONDITIONS FOR SUPERFLUIDITY OF THE ATOMIC NUCLEUS AND THE TEMPERATURE OF THE PHASE TRANSITION

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The conditions for the appearance of a superfluid state in the atomic nucleus are obtained with the aid of a variational principle proposed by N. N. Bogolyubov. The superfluidity conditions reduce to the requirement that attractive forces predominate in the interactions of protons located on the same shell at the energy of the Fermi surface. If the nucleus is in the superfluid state at zero temperature, a transition from the superfluid into the normal phase will take place as the temperature is increased. The critical temperature for this phase transition is found.

In a preceding paper[1] the author, starting with a shell model for a heavy nucleus, considered the weak interactions of protons (or neutrons) situated on the same shell, and showed that the interactions of protons having equal and opposite projections of their momenta upon the nuclear axis of symmetry lead to the appearance of a superfluid state of the nucleus. In the present paper, using variational principle proposed by Bogolyubov[2] and representing his extension of the statistical variational principle,[3] we shall derive the conditions for the appearance of the superfluid state of the atomic nucleus, as well as the temperature of the phase transition from the superfluid to the normal state.

On the basis of the variational principle we find that the energy of the normal state will not be minimal (and, consequently, that a superfluid state will appear) in the case for which the equation

$$2|E(s_0, m) - E_F|\psi_m(s_0) + \frac{1}{N}\sum_{m'} J(s_0 | m, m')\psi_{m'}(s_0) = E\psi_m(s_0) \tag{1}$$

has solutions with negative eigenvalues $E < 0$ (the symbols used are the same as in reference 1). With the object of determining the limits imposed upon $J(s_0 | m, m')$, we investigate the asymptotic solutions of Eq. (1) as J tends toward zero while E also tends to zero and remains negative. As a result of these calculations we find that, as for the Fermi system of a metal[4] or of nuclear matter,[5] the superfluidity conditions reduce to the requirement that attractive forces predominate at the energy of the Fermi surface. We thus obtain the conditions for superfluidity of the nucleus in the following form:

$$J(s_0 | m_0, m_0) < 0, \tag{2}$$

i.e., attractive forces must predominate between protons situated on an inner shell. On the basis of the statistical variational principle we find that for a temperature Θ different from zero the superfluid state of the atomic nucleus exists in the case for which the equation

$$2|E(s_0, m) - E_F|\coth\frac{|E(s_0, m) - E_F|}{2\Theta}\cdot\psi_m(s_0) + \frac{1}{N}{\sum_{m'}}' J(s_0 | m, m')\psi_{m'}(s_0) = E\psi_m(s_0) \tag{3}$$

has solutions with negative eigenvalues $E < 0$.

It is readily seen that for $\Theta = 0$ Eq. (3) reduces to (2). Let the superfluid state exist for $\Theta = 0$; as the temperature Θ increases E will also increase. At the transition from the superfluid into the normal phase, at the critical temperature Θ_0, the eigenvalue E must become zero. Taking into account the fact that $|E(s_0, m) - E_F|$ is relatively small, and going from the summation to the integral, we obtain the following approximate expression for determination of the critical temperature:

$$\cdot\psi_m(s_0) + \frac{1}{4\Theta_0}\int_{m_1}^{m_2} dm'\rho_0(m') J(s_0 | m, m')\psi_{m'}(s_0) = 0. \tag{4}$$

Here $\rho_0(m')$ is the level density. In order for (4) to have a non-zero solution, it is necessary that the determinant D be zero. From the equation $D = 0$, in view of the weakness of the interaction, we obtain the following expression for the temperature of the phase transition:

$$\Theta_0 = -\frac{1}{4}\int_{m_1}^{m_2} dm' \rho_0(m') J(s_0 \mid m', m'), \tag{5}$$

or

$$\Theta_0 \approx -\bar{\rho}_0 \bar{J}/4. \tag{5'}$$

In conclusion, I express my deep gratitude to Academician N. N. Bogolyubov for his constant interest in this work and his always valuable comments.

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[2] N. N. Bogolyubov, Dokl. Akad. Nauk SSSR **119**, 244 (1958); Soviet Phys. "Doklady" **3**, 292 (1958).

[3] I. A. Kvasnikov and V. V. Tolmachev, Dokl. Akad. Nauk SSSR **120**, 273 (1958); Soviet Phys. "Doklady" **3**, 553 (1958).

[4] Bogolyubov, Tolmachev, and Shirkov, Новый метод в теории сверхпроводимости (New Method in the Theory of Superconductivity), Academy of Sciences Press, Moscow, 1958.

[5] V. G. Solov'ev, Dokl. Akad. Nauk SSSR **123**, 3 (1958); Soviet Phys. "Doklady" (in press).

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