BETA-GAMMA CORRELATION IN FIRST-FORBIDDEN BETA TRANSITIONS

Yu. V. GAPONOV

Nuclear Physics Institute, Moscow State University

Submitted to JETP editor June 30, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 193-203 (January, 1959)

A general formula is derived for the β-γ correlation of β-decay electrons and circularly polarized γ rays emitted by the excited daughter nucleus. First-forbidden β transitions are considered. The calculation is performed for an arbitrary mixture of β couplings, with the nuclear Coulomb field, in particular the so called "Coulomb terms," taken into account. Two special cases are considered, unique and Coulomb transitions.

ONE of the experimental consequences of parity nonconservation in weak interactions[1] is the appearance of spatial asymmetries in the angular distributions of β-decay electrons and circularly polarized γ rays emitted by the excited daughter nucleus. A theoretical calculation of the degree of asymmetry for first-forbidden β transitions was given by Alder, Stech, and Winther.[2] Assuming the β interaction to be a mixture of the s, t, and p covariants, they calculated the angular distribution of β electrons and of γ rays emitted immediately following the β decay. They included effects due to the nuclear Coulomb field but neglected the terms which, in heavy nuclei, give rise to so called "Coulomb" transitions.

The aim of the present work is to study the β-γ angular correlation of circularly polarized γ rays in first-forbidden β transitions, in particular in Coulomb transitions, for an arbitrary mixture of covariants and including Coulomb effects.

The probability that in a γ transition following β decay the γ ray is emitted at an angle θ with respect to the direction of the electron momentum is given by

$$W(\theta) = \sum_R \beta_R \gamma_R P_R(\cos\theta)(2R+1), \qquad (1)$$

where β_R is a coefficient which depends on the characteristics of the β-decay (interaction coupling constants, nuclear matrix elements, electron and neutrino energy and momentum, nuclear charge), and γ_R is a coefficient that depends on the characteristics of the γ transition (spin of the final nucleus, multipole order). The quantity R, which determines the order of the Legendre polynomial $P_R(\cos\theta)$, is related to the order of forbiddeness l by $0 \le R \le 2l+1$, and varies between 0 and 3 in the case of first forbidden β transitions.

The quantity γ_R can be calculated for an arbitrary γ transition — pure electric, or pure magnetic, or mixed. In the case of a pure transition γ_R does not depend on the transition type and is given by

$$\gamma_R = C_{L-\mu}^{L-\mu R0} \sqrt{(2j_2+1)(2L+1)}\, W(j_2 j_3 RL; L j_2), \qquad (2)$$

where j_2 is the spin of the excited nucleus formed after β-decay, j_3 is the spin of the final state to which the γ-transition proceeds, and L is the multipole order of the γ-radiation; $\mu = \pm 1$ corresponds to right- and left-circular polarization of the γ ray. $C_{L-\mu}^{L-\mu R_0}$ is a Clebsch-Gordan coefficient[3] and $W(j_2 j_3 RL; L j_2)$ is a Racah coefficient tabulated in reference 4.

Occasionally the nuclear γ radiation is a mixture of electric radiation of multipole order L + 1 and magnetic radiation of multipole order L. We denote the ratio of the amplitudes of electric and magnetic radiations in the mixture by a constant Δ, so that the ratio of the corresponding intensities is Δ^2. For such a radiation

$$\gamma_R = C_{L-\mu}^{L-\mu R0} \sqrt{(2j_2+1)(2L+1)}\, W(j_2 j_3 RL; L j_2) + 2\mu\Delta \sqrt{\frac{L+2}{2L+1}\frac{2L+1}{2L+3}}\, C_{L+1-\mu}^{L-\mu R0} \sqrt{(2j_2+1)(2L+1)}\, W \times (j_2 j_3 RL; L+1\, j_2) + \Delta^2 C_{L+1-\mu}^{L+1-\mu R0} \frac{L+2}{2L+3} \sqrt{(2j_2+1)(2L+3)} \times W(j_2 j_3 RL+1; L+1\, j_2). \qquad (3)$$

$W(\theta)$ is also given by formula (1) if angular correlation of a β electron with any γ ray from a cascade accompanying the β decay is considered. In that case γ_R will depend on the characteristics of that γ transition as well as on those of the γ transitions preceding it. Thus, if an excited nucleus of spin j_2 is formed following β decay, which emits successive γ rays of multipole order L_1, $L_2 \ldots L_n$ as it cascades through the levels of spin $j_3 \ldots j_{n+2}$, the angular correlation function $W(\theta)$

of the electron and n-th γ ray will contain for a pure radiation

$$\gamma_R = C^{L_n-\mu_n R0}_{L_n-\mu_n} \sqrt{(2j_{n+1}+1)(2L_n+1)}\, W(j_{n+1} j_{n+2} R L_n;\ L_n j_{n+1})$$
$$\times \prod_{i=1}^{n-1} \sqrt{(2j_{i+1}+1)(2j_{i+2}+1)}\, W(j_{i+1} L_i R j_{i+2};\ j_{i+2} j_{i+1}), \quad (4)$$

and for a mixed radiation

$$\gamma_R = [C^{L_n-\mu_n R0}_{L_n-\mu_n} \sqrt{(2j_{n+1}+1)(2L_n+1)}\, W(j_{n+1} j_{n+2} R L_n;\ L_n j_{n+1})$$
$$+ 2\mu_n \Delta_n \sqrt{\frac{L_n+2}{2L_n+1}\frac{2L_n+1}{2L_n+3}}\, C^{L_n-\mu_n R0}_{L_n+1-\mu_n} \sqrt{(2j_{n+1}+1)(2L_n+3)}$$
$$\times W(j_{n+1} j_{n+2} R L_n;\ L_n+1 j_{n+1}) + \Delta_n^2 C^{L_n+1-\mu_n R0}_{L_n+1-\mu_n} \frac{L_n+2}{2L_n+3}$$
$$\times \sqrt{(2j_{n+1}+1)(2L_n+3)}\, W(j_{n+1} j_{n+2} R L_n+1;\ L_n+1 j_{n+1})]$$
$$\times \prod_{i=1}^{n-1} [\sqrt{(2j_{i+1}+1)(2j_{i+2}+1)}\, W(j_{i+1} L_i R j_{i+2};\ j_{i+2} j_{i+1}) \quad (5)$$
$$+ \Delta_i^2 \sqrt{(2j_{i+1}+1)(2j_{i+2}+1)}\, W(j_{i+1} L_i+1 R j_{i+2};\ j_{i+2} j_{i+1})],$$

where $\Delta_1, \Delta_2 \ldots \Delta_n$ describe the relative amount of electric and magnetic radiations in the γ transitions $L_1, L_2 \ldots, L_n$, and μ_n gives the circular polarization of the n-th γ ray.

The quantity β_R is calculated in this work for first forbidden β transitions without taking into account finite nuclear size corrections. It has the following form:

$$\beta_R = \sum_{JJ'} f(J, J', R) \sqrt{2j_2+1}\, W(J j_1 R j_2;\ j_2 J'). \quad (6)$$

Here j_1 and j_2 are the spins of the nucleus before and after the β transition, and J and J′ represent the total angular momentum of the electron-neutrino pair and may be equal to l, $l \pm 1$ where l is the order of forbiddenness. $W(Jj_1Rj_2; j_2J')$ is a Racah coefficient.[4] $f(J, J', R)$ is a complicated expression which depends only on the interaction coupling constants and nuclear matrix elements. The form of $f(J, J', R)$ for a general first forbidden transition is given in the Appendix. Some special cases, easy to interpret, are discussed below. The calculations were performed with the β-interaction Hamiltonian as given by Lee and Yang.[1]

The electron wavefunction can be written as follows:[5]

$$\psi_e = \Big[A\cdot 1 + B\gamma_4 + Ci\mathbf{p}\cdot\mathbf{r}\cdot 1 + Di\mathbf{p}\cdot\mathbf{r}\gamma_4$$
$$+ iN\frac{\mathbf{r}}{r}\boldsymbol{\gamma} + iM\frac{\mathbf{r}}{r}\gamma_4\boldsymbol{\gamma}\Big] u_\xi, \quad A = \tfrac{1}{2}(\gamma_1+1) + i\frac{\alpha Z}{2p}E;$$
$$C = \zeta\frac{\gamma_2+2}{4} + i\frac{\alpha Z}{2p}E\zeta; \quad N = \frac{\gamma_1-1}{2p}; \quad B = -i\frac{\alpha Z}{2p};$$
$$D = -i\frac{\alpha Z}{4p}\zeta; \quad M = i\frac{\alpha Z}{2} + \frac{(\gamma_1-1)}{2p}E; \quad \gamma_1 = \sqrt{1-\alpha^2 Z^2};$$
$$\gamma_2 = \sqrt{4-\alpha^2 Z^2}; \quad \zeta = \frac{12\Gamma(2\gamma_1+1)}{\Gamma(2\gamma_2+1)}\frac{\Gamma(\gamma_2 + i\alpha ZE/p)}{\Gamma(\gamma_1 + i\alpha ZE/p)}; \quad (7)$$

where p is the electron momentum, E its total energy, Z is the charge of nucleus 1, γ_4, $\boldsymbol{\gamma}$ are four-by-four matrices, and u_ξ is the unit two-component spinor.

It is easy to see that this form of ψ_e will give rise to three types of terms in first forbidden β transitions: relativistic terms, whose matrix elements are of order v_n/c (v_n is the nucleon velocity in the nucleus), ordinary nonrelativistic terms of order pR_N (R_N is the average nuclear radius), and Coulomb terms of order αZ due to the nuclear Coulomb field. In heavy nuclei the Coulomb terms are dominant and all others may be neglected. Beta transitions of this type are called Coulomb transitions and they are in many respects similar to ordinary allowed transitions. However there is one important difference. Whereas in allowed β transitions terms testing time reversal invariance appear as small corrections, in the case of Coulomb transitions they can play a leading role. This circumstance makes a study of these terms particularly interesting.

Whereas the quantity γ_R can always be calculated from the formulas (2) to (5) if the spins of the excited states of the final nucleus and the multipole order of the γ rays are known, the computation of β_R is complicated by nuclear matrix elements which can be estimated only with great difficulties. This naturally leads to a consideration of various special cases of the general formula where the number of unknown nuclear matrix elements is at a minimum.

1. COULOMB TRANSITIONS

This type of transitions is found in heavy nuclei when the conditions $\alpha Z \gg pR_N$ and $\alpha Z \gg v_n/c$ are satisfied. In this case β_R depends on three kinds of nuclear matrix elements whose numerical value is unknown: $\langle r/r\rangle_1$, $\langle (r/r)\sigma\rangle_0$, $\langle (r/r)\sigma\rangle_1$. All three enter into $\Delta j = 0$ transitions, only $\langle r/r\rangle_1$ and $\langle (r/r)\sigma\rangle_1$ enter into $\Delta j = \pm 1$ transitions so that the asymmetry coefficient depends on two ratios of nuclear matrix elements in the former case and on one ratio in the latter. $\Delta j = \pm 2$ transitions are forbidden. The quantity β_R is given by

$$\beta_0 = [2\mathrm{Re}\, g_{sv} |\langle r/r\rangle_1|^2 - 2\sqrt{2}\,\mathrm{Re}(g_{vt} + g_{sa})\langle r/r\rangle_1 \langle (r/r)\sigma\rangle_1$$
$$+ 2\mathrm{Re}\, g_{ta}(3|\langle (r/r)\sigma\rangle_0|^2 + 2|\langle (r/r)\sigma\rangle_1|^2)]\Big(X_{21}\frac{1}{E} + X_{22}\Big)$$
$$- [(g_{ss} + g_{vv})|\langle r/r\rangle_1|^2 - 2\sqrt{2}\,\mathrm{Re}(g_{st} + g_{va})\langle r/r\rangle_1 \langle (r/r)\sigma\rangle_1 \quad (8)$$
$$+ (g_{tt} + g_{aa})(3|\langle (r/r)\sigma\rangle_0|^2 + 2|\langle (r/r)\sigma\rangle_1|^2]\Big(X_{21} + \frac{1}{E}X_{22}\Big).$$

$$\beta_1 = \frac{p}{E}\sum_{JJ'}\Big\{\frac{(\gamma_1-1)^2+\alpha^2Z^2}{4}\Big[(g'_{ss}-g'_{vv})\langle r/r\rangle_J\langle r/r\rangle_{J'}\frac{\sqrt{6}}{3}\delta_{J1}\delta_{J'1}$$
$$-2\,\mathrm{Re}\,(g'_{st}-g'_{va})\langle r/r\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\Big(\delta_{J'0}\delta_{J1}+\frac{2\sqrt{3}}{3}\delta_{J'1}\delta_{J1}\Big)$$
$$+2\,\mathrm{Re}\,(g'_{tt}-g'_{aa})\langle (r/r)\,\sigma\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\frac{2\sqrt{6}}{3}\delta_{J1}\delta_{J'1}\Big]$$
$$+\frac{\alpha Z}{2p}(1-\gamma_1)\Big[2\,\mathrm{Im}\,g'_{vs}\langle r/r\rangle_J\langle r/r\rangle_{J'}\frac{\sqrt{6}}{3}\delta_{J1}\delta_{J'1}$$
$$-2\,\mathrm{Im}\,(g'_{vt}-g'_{sa})\langle r/r\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\Big(\delta_{J'0}\delta_{J1}+\frac{2\sqrt{3}}{3}\delta_{J1}\delta_{J'1}\Big)$$
$$+2\,\mathrm{Im}\,g'_{at}\langle (r/r)\,\sigma\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\frac{2\sqrt{6}}{3}\delta_{J1}\delta_{J'1}\Big]\Big\}$$
$$\times\sqrt{2j_2+1}\,W(Jj_1 1j_2;\, j_2J').$$

Since R can only be either 0 or 1 it is convenient to write formula (1) as

$$W(\theta)=1+\lambda\frac{v}{c}\cos\theta, \tag{9}$$

where v is the electron velocity and the angular asymmetry coefficient λ is given in terms of β_R and γ_R by

$$\lambda=\frac{E}{p}\frac{\beta_1\gamma_1}{\beta_0\gamma_0}. \tag{10}$$

Coulomb transitions are of interest mainly in connection with the question of time reversal invariance. Should even one combination of the imaginary parts of the interaction coupling constants turn out to be different from zero, violation of time-reversal invariance would be proven. It is easy to see that the expressions multiplying the imaginary and the real parts of the coupling constants behave differently, namely the former contain an additional factor $\alpha Z/p$ which increases with decreasing electron energy and increasing nuclear charge. Consequently a study of the energy dependence of the asymmetry coefficient λ in heavy nuclei would give an estimate of the contribution of time reversal non-invariant terms in β decay.

Recently a number of hypotheses have been put forward regarding the covariants involved in β decay. One of them, proposed by Feynman and Gell-Mann[6] and consistent with the two-component neutrino theory developed by Landau[7] and Lee and Yang,[8] states that

$$C_s=C'_s=C_t=C'_t=0;\quad C'_v=C_v;\quad C'_a=C_a, \tag{11}$$

In that case formula (8) becomes

$$\beta_0=-[2|C_v|^2|\langle r/r\rangle_1|^2-4\sqrt{2}\,\mathrm{Re}\,C_v^*C_a\langle r/r\rangle_1\langle (r/r)\,\sigma\rangle_1+2|C_a|^2(3|\langle (r/r)\,\sigma\rangle_0|^2+2|\langle (r/r)\,\sigma\rangle_1|^2)]\Big(X_{21}+\frac{1}{E}X_{22}\Big).$$
$$\beta_1=-\frac{p}{E}\sum_{JJ'}\frac{(\gamma_1-1)^2+\alpha^2Z^2}{4}\Big[2|C_v|^2\langle r/r\rangle_J\langle r/r\rangle_{J'}\frac{\sqrt{6}}{3}\delta_{J1}\delta_{J'1}-4\,\mathrm{Re}\,C_vC_a^*\langle r/r\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\Big(\delta_{J'0}\delta_{J1}+\frac{2\sqrt{3}}{3}\delta_{J1}\delta_{J'1}\Big)$$
$$+2|C_a|^2\langle (r/r)\,\sigma\rangle_J\langle (r/r)\,\sigma\rangle_{J'}\frac{2\sqrt{6}}{3}\delta_{J1}\delta_{J'1}\Big]\sqrt{2j_2+1}\,W(Jj_1 1j_2;\, j_2J'). \tag{12}$$

Formula (12) permits the study of ratios of nuclear matrix elements and can be used to determine unknown spins of nuclear levels.

Examples of Coulomb β transitions are: Re^{186}, Ir^{194}, Au^{196}, An^{198}, Bi^{212}, Np^{238}.

2. UNIQUE TRANSITIONS $\Delta j = \pm 2$

In this case, as a consequence of selection rules for the nuclear matrix elements, most of the terms in expression (16) are to be omitted. β_R becomes independent of the nuclear matrix elements and is given by

$$\beta_0=2\,\mathrm{Re}\,g_{at}\Big[q^2\Big(X_1\frac{1}{E}+X_2\Big)+\frac{p^2}{3}\Big(X_{13}\frac{1}{E}+X_{14}\Big)\Big]$$
$$+(g_{tt}+g_{aa})\Big[q^2\Big(X_1+\frac{1}{E}X_2\Big)+\frac{p^2}{5}\Big(X_{13}+\frac{1}{E}X_{14}\Big)\Big];$$
$$\beta_1=-\frac{\sqrt{30}}{6}\frac{p}{E}\Big[(g'_{tt}-g'_{aa})\Big(q^2X_3+\frac{p^2}{5}X_{15}\Big)$$
$$+2\,\mathrm{Im}\,g'_{at}\Big(q^2X_4+\frac{p^2}{5}X_{16}\Big)\Big]\sqrt{2j_2+1}\,W(2j_1 1j_2;\, j_2 2);$$
$$\beta_2=-\frac{\sqrt{70}}{90}p^2\Big[2\,\mathrm{Re}\,g_{at}\Big(X_{13}\frac{1}{E}+X_{14}\Big)$$
$$+(g_{tt}+g_{aa})\Big(X_{13}+\frac{1}{E}X_{14}\Big)\Big]\sqrt{2j_2+1}\,W(2j_1 2j_2;\, j_2 2);$$
$$\beta_3=\frac{\sqrt{70}}{35}\frac{p^3}{E}[(g'_{tt}-g'_{aa})X_{15}$$
$$+2\,\mathrm{Im}\,g'_{at}X_{16}]\sqrt{2j_2+1}\,W(2j_1 3j_2;\, j_2 2). \tag{13}$$

The X_i are given in the Appendix, formulas (17).

As in the case of Coulomb β transitions the terms violating time reversal invariance have an additional factor $\alpha Z/p$. If time-reversal invariance holds and the hypothesis (11) is valid the quantities β_R may be calculated exactly since they are then independent of the interaction coupling constants. Omitting factors common to all β_R and irrelevant for $W(\theta)$ we obtain from (13)

$$\beta_0=q^2\Big(X_1+\frac{1}{E}X_2\Big)+\frac{p^2}{3}\Big(X_{13}+\frac{1}{E}X_{14}\Big);$$
$$\beta_1=\frac{\sqrt{30}}{6}\frac{v}{c}\Big(q^2X_3+\frac{p^2}{5}X_{15}\Big)\sqrt{2j_2+1}\,W(2j_1 1j_2;\, j_2 2);$$
$$\beta_2=-\frac{\sqrt{70}}{90}p^2\Big(X_{13}+\frac{1}{E}X_{14}\Big)\sqrt{2j_2+1}\,W(2j_1 2j_2;\, j_2 2);$$
$$\beta_3=-\frac{v}{c}p^2\frac{\sqrt{70}}{35}X_{15}\sqrt{2j_2+1}\,W(2j_1 3j_2;\, j_2 2). \tag{14}$$

$\langle A\rangle_J \langle B\rangle_{J'}$	$K(A, B, R)$
$\langle\gamma_5\rangle_0\langle\gamma_5\rangle_0$	$\left[(g_{aa}+g_{pp})\left(X_1+\frac{1}{E}X_2\right)-2\mathrm{Re}\,g_{ap}\left(X_1\frac{1}{E}+X_2\right)\right]3\delta_{R0}$
$\langle\gamma_5\rangle_0\langle\alpha\rangle_1$	$[2\mathrm{Re}\,(g'_{av}-g'_{pt})X_3+2\mathrm{Im}\,(g'_{at}-g'_{pv})X_4]\frac{p}{E}3\delta_{R1}$
$\langle\alpha\rangle_1\langle\alpha\rangle_1$	$\left[(g_{vv}+g_{tt})\left(X_1+\frac{1}{E}X_2\right)-2\mathrm{Re}\,g_{vt}\left(X_1\frac{1}{E}+X_2\right)\right]3\sqrt{3}\delta_{R0}+[(g'_{vv}-g'_{tt})X_3-2\mathrm{Im}\,g'_{tv}X_4]\frac{p}{E}\sqrt{6}\delta_{R1}$
$\langle r\rangle_1\langle r\rangle_1$	$\left[2\mathrm{Re}\,g_{sv}\left(X_1\frac{1}{E}+X_2\right)+(g_{ss}+g_{vv})\left(X_1+\frac{1}{E}X_2\right)\right]\left(q^2\sqrt{3}\delta_{R0}+\frac{p^2}{3}\sqrt{3}\delta_{R0}-p^2\frac{\sqrt{30}}{15}\delta_{R2}\right)$
	$+[2\mathrm{Re}\,(g_{ss}-g_{vv})X_7+2\mathrm{Im}\,(g_{vs}-g_{sv})X_8]\frac{p^2q}{E}\left(-\frac{\sqrt{3}}{3}\delta_{R0}+\frac{\sqrt{30}}{15}\delta_{R2}\right)$
$\langle r\rangle_1\langle r\sigma\rangle_0$	$[2\mathrm{Re}\,(g'_{st}-g'_{va})X_{15}+2\mathrm{Im}\,(g'_{vt}-g'_{sa})X_{16}]\frac{p^3}{E}\frac{1}{3}\delta_{R1}+[2\mathrm{Re}\,(g'_{st}-g'_{va})X_3+2\mathrm{Im}\,(g'_{vt}-g'_{sa})X_4]\frac{pq^2}{E}\frac{1}{3}\delta_{R1}$
	$+\left[4\mathrm{Re}\,(g'_{vt}+g'_{sa})\left(X_5\frac{1}{E}+X_6\right)+4\mathrm{Re}\,(g'_{st}+g'_{va})\left(X_5+\frac{1}{E}X_6\right)\right]\frac{pq}{3}\delta_{R1}$
$\langle r\rangle_1\langle r\sigma\rangle_1$	$[2\mathrm{Re}\,(g'_{va}-g'_{st})X_3+2\mathrm{Im}\,(g'_{sa}-g'_{vt})X_4]\frac{pq^2}{E}\frac{\sqrt{3}}{3}\delta_{R1}+[2\mathrm{Re}\,(g_{av}-g_{ts})X_7+2\mathrm{Im}\,(g_{as}-g_{tv})X_8]\frac{p^2q}{E}\left(\frac{\sqrt{6}}{3}\delta_{R0}-\frac{2\sqrt{15}}{15}\delta_{R2}\right)$
	$+[2\mathrm{Re}\,(g_{st}-g_{va})X_7+2\mathrm{Im}\,(g_{vt}-g_{sa})X_8]\frac{p^2q}{E}\left(\frac{\sqrt{6}}{3}\delta_{R0}+\frac{2\sqrt{15}}{15}\delta_{R2}\right)$
	$+\left[2\mathrm{Re}(g'_{vt}+g'_{sa})\left(X_5\frac{1}{E}+X_6\right)+2\mathrm{Re}(g'_{st}+g'_{va})\left(X_5+\frac{1}{E}X_6\right)\right]pq\frac{\sqrt{3}}{3}\delta_{R}$
$\langle r\rangle_1\langle r\sigma\rangle_2$	$[2\mathrm{Re}\,(g'_{st}-g'_{va})X_3+2\mathrm{Im}\,(g'_{vt}-g'_{sa})X_4]\frac{p}{E}\left(q^2\frac{\sqrt{5}}{3}\delta_{R1}+p^2\frac{2\sqrt{5}}{15}\delta_{R1}-p^2\frac{\sqrt{70}}{35}\delta_{R3}\right)+[2\mathrm{Re}(g_{st}-g_{va})X_7+2\mathrm{Im}(g_{vt}-g_{sa})X_8]$
	$\times\frac{p^2q}{E}\frac{\sqrt{5}}{5}\delta_{R2}+\left[2\mathrm{Re}\,(g'_{vt}+g'_{sa})\left(X_5\frac{1}{E}+X_6\right)+2\mathrm{Re}\,(g'_{st}+g'_{va})\left(X_5+\frac{1}{E}X_6\right)\right]pq\frac{\sqrt{5}}{3}\delta_{R1}$
$\langle r\sigma\rangle_0\langle r\sigma\rangle_0$	$\left[2\mathrm{Re}\,g_{at}\left(X_1\frac{1}{E}+X_2\right)+(g_{tt}+g_{aa})\left(X_1+\frac{1}{E}X_2\right)\right]q^2\delta_{R0}+\left[2\mathrm{Re}\,g_{at}\left(X_{13}\frac{1}{E}+X_{14}\right)+(g_{tt}+g_{aa})\left(X_{13}+\frac{1}{E}X_{14}\right)\right]\frac{p^2}{3}\delta_{R0}$
	$+[2\mathrm{Re}\,(g_{aa}-g_{tt})X_7+2\mathrm{Im}\,(g_{ta}-g_{at})X_8]\frac{p^2q}{E}\delta_{R0}$
$\langle r\sigma\rangle_1\langle r\sigma\rangle_0$	$\left[2\mathrm{Re}\,g'_{at}\left(X_5\frac{1}{E}+X_6\right)+2\mathrm{Re}\,(g'_{tt}+g'_{aa})\left(X_5+\frac{1}{E}X_6\right)\right]pq\frac{\sqrt{2}}{3}\delta_{R1}$
$\langle r\sigma\rangle_0\langle r\sigma\rangle_2$	$[2\mathrm{Re}\,(g_{tt}-g_{aa})X_7+2\mathrm{Im}\,(g_{at}-g_{ta})X_8]\frac{p^2q}{E}\frac{\sqrt{10}}{5}\delta_{R2}$
$\langle r\sigma\rangle_1\langle r\sigma\rangle_1$	$[2\mathrm{Re}\,(g_{tt}-g_{aa})X_7+2\mathrm{Im}\,(g_{at}-g_{ta})X_8]\frac{p^2q}{E}\left(\frac{2\sqrt{3}}{3}\delta_{R0}+\frac{\sqrt{30}}{15}\delta_{R2}\right)$
	$+\left[2\mathrm{Re}\,g_{at}\left(X_1\frac{1}{E}+X_2\right)+(g_{tt}+g_{aa})\left(X_1+\frac{1}{E}X_2\right)\right]q^2\sqrt{3}\delta_{R0}-[(g'_{tt}-g'_{aa})X_3+2\mathrm{Im}\,g'_{at}X_4]\frac{pq^2}{E}\frac{\sqrt{6}}{6}\delta_{R1}$
	$+\left[2\mathrm{Re}\,g_{at}\left(X_{13}\frac{1}{E}+X_{14}\right)+(g_{tt}+g_{aa})\left(X_{13}+\frac{1}{E}X_{14}\right)\right]\frac{p^2}{3}\left(\sqrt{3}\delta_{R0}+\frac{\sqrt{30}}{30}\delta_{R2}\right)-[(g'_{tt}-g'_{aa})X_{15}+2\mathrm{Im}\,g'_{at}X_{16}]\frac{p^3}{E}\frac{\sqrt{6}}{6}\delta_{R1}$
	$+\left[4\mathrm{Re}\,g'_{at}\left(X_5\frac{1}{E}+X_6\right)+2\mathrm{Re}\,(g'_{tt}+g'_{aa})\left(X_5+\frac{1}{E}X_6\right)\right]pq\frac{\sqrt{6}}{3}\delta_{R1}$
$\langle r\sigma\rangle_1\langle r\sigma\rangle_2$	$[2\mathrm{Re}\,(g_{tt}-g_{aa})X_7+2\mathrm{Im}\,(g_{at}-g_{ta})X_8]\frac{p^2q}{E}\frac{\sqrt{10}}{5}\delta_{R1}+\left[4\mathrm{Re}\,g'_{at}\left(X_5\frac{1}{E}+X_6\right)+2\mathrm{Re}\,(g'_{tt}+g'_{aa})\left(X_5+\frac{1}{E}X_6\right)\right]pq\frac{\sqrt{10}}{3}\delta_{R1}$
$\langle r\sigma\rangle_2\langle r\sigma\rangle_2$	$\left[2\mathrm{Re}\,g_{ta}\left(X_1\frac{1}{E}+X_2\right)+(g_{tt}+g_{aa})\left(X_1+\frac{1}{E}X_2\right)\right]q^2\sqrt{5}\delta_{R0}-[(g'_{tt}-g'_{aa})X_3+2\mathrm{Im}\,g'_{at}X_4]\frac{pq^2}{E}\frac{\sqrt{30}}{6}\delta_{R1}$
	$+\left[2\mathrm{Re}\,g_{ta}\left(X_{13}\frac{1}{E}+X_{14}\right)+(g_{tt}+g_{aa})\left(X_{13}+\frac{1}{E}X_{14}\right)\right]\frac{p^2}{3}\left(\sqrt{5}\delta_{R0}-\frac{\sqrt{70}}{30}\delta_{R2}\right)-[(g'_{tt}-g'_{aa})X_{15}+2\mathrm{Im}\,g'_{at}X_{16}]\frac{p^3}{E}\left(\frac{\sqrt{30}}{30}\delta_{R1}-\frac{\sqrt{70}}{35}\delta_{R3}\right)$
$\langle r/r\rangle_1\langle r\rangle_1$	$\left[2\mathrm{Re}\,(g_{sv}-g_{vs})\left(X_9+\frac{1}{E}X_{10}\right)+2(g_{vv}-g_{ss})X_{10}\right]p\sqrt{3}\delta_{R0}+[4\mathrm{Re}\,g'_{sv}X_{11}+2(g'_{ss}+g'_{vv})X_{12}]\frac{p^2}{E}\frac{\sqrt{6}}{3}\delta_{R1}$
	$+[4\mathrm{Re}\,g_{sv}X_{19}+2\mathrm{Re}\,(g_{ss}+g_{vv})X_{20}]\frac{p^2}{E}\left(-\frac{\sqrt{3}}{3}\delta_{R0}+\frac{\sqrt{30}}{15}\delta_{R2}\right)$
$\langle r/r\rangle_1\langle r\sigma\rangle_0$	$-[2\mathrm{Re}\,(g'_{tv}+g'_{as})X_{11}+2\mathrm{Re}\,(g'_{ts}+g'_{av})X_{12}]\frac{p^2}{E}\delta_{R1}+\left[2\mathrm{Re}\,(g'_{av}-g'_{ts})\left(X_{17}\frac{1}{E}+X_{18}\right)+2\mathrm{Re}(g'_{tv}-g'_{as})\left(X_{17}+\frac{1}{E}X_{18}\right)\right]\frac{p}{3}\delta_{R1}$

$\langle A\rangle_J \langle B\rangle_{J'}$	$K(A, B, R)$
$\langle r/r\rangle_1 \langle r\sigma\rangle_1$	$\left[2\mathrm{Re}\,(g_{av}-g_{ts})\left(X_9\frac{1}{E}+X_{10}\right)+2\mathrm{Re}(g_{tv}-g_{as})\left(X_9+\frac{1}{E}X_{10}\right)\right]p\sqrt{6}\,\delta_{R0}$
	$+\,[2\mathrm{Re}\,(g'_{tv}+g'_{as})\,X_{11}+2\mathrm{Re}\,(g'_{ts}+g'_{av})\,X_{12}]\,\frac{p^2}{E}\,\frac{2\sqrt{3}}{3}\,\delta_{R1}$
	$+[2\mathrm{Re}\,(g_{vt}+g_{sa})\,X_{19}+2\mathrm{Re}\,(g_{st}+g_{va})\,X_{20}]\,\frac{p^2}{E}\left(\frac{\sqrt{6}}{3}\,\delta_{R0}+\frac{\sqrt{15}}{15}\delta_{R2}\right)$
	$+\left[2\mathrm{Re}\,(g'_{av}-g'_{ts})\left(X_{17}\frac{1}{E}+X_{18}\right)+2\mathrm{Re}(g'_{tv}-g'_{as})\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\,\frac{\sqrt{3}}{3}\,\delta_{R1}$
$\langle r/r\rangle_1\langle r\sigma\rangle_2$	$[2\mathrm{Re}\,(g_{vt}+g_{sa})\,X_{19}+2\mathrm{Re}\,(g_{st}+g_{va})\,X_{20}]\,\frac{p^2}{E}\,\frac{\sqrt{5}}{5}\,\delta_{R2}+\left[2\mathrm{Re}\,(g'_{av}-g'_{ts})\left(X_{17}\frac{1}{E}+X_{18}\right)+2\mathrm{Re}(g'_{tv}-g'_{as})\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\,\frac{\sqrt{5}}{3}\,\delta_{R1}$
$\langle (r/r)\sigma\rangle_0\langle r\rangle_1$	$[2\mathrm{Re}\,(g'_{tv}+g'_{sa})\,X_{11}+2\mathrm{Re}\,(g'_{st}+g'_{va})\,X_{12}\,\frac{p^2}{E}\,\delta_{R1}+\left[2\mathrm{Re}\,(g'_{st}-g'_{va})\left(X_{17}\frac{1}{E}+X_{18}\right)+2\mathrm{Re}\,(g'_{vt}-g'_{sa})\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\delta_{R0}$
$\langle (r/r)\sigma\rangle_0\langle r\sigma\rangle_0$	$\left[2\mathrm{Re}\,(g_{aa}-g_{tt})\,X_{10}+2\mathrm{Re}\,(g_{ta}-g_{at})\left(X_9+\frac{1}{E}X_{10}\right)\right]p3\delta_{R0}-[4\mathrm{Re}\,g_{ta}X_{19}+2\mathrm{Re}\,(g_{tt}+g_{aa})\,X_{20}]\,\frac{p^2}{E}\,\delta_{R0}$
$\langle (r/r)\sigma\rangle_0\langle r\sigma\rangle_1$	$[4\mathrm{Re}\,g'_{at}X_{11}+2\,(g'_{tt}+g'_{aa})\,X_{12}]\,\frac{p^2}{E}\sqrt{2}\,\delta_{R1}$
$\langle (r/r)\sigma\rangle_0\langle r\sigma\rangle_2$	$[4\mathrm{Re}\,g_{ta}X_{19}+2\mathrm{Re}\,(g_{tt}+g_{aa})\,X_{20}]\,\frac{p^2}{E}\,\frac{\sqrt{10}}{5}\,\delta_{R2}$
$\langle (r/r)\sigma\rangle_1\langle r\rangle_1$	$\left[2\mathrm{Re}\,(g_{st}-g_{va})\left(X_9\frac{1}{E}+X_{10}\right)+2\mathrm{Re}\,(g_{vt}-g_{sa})\left(X_9+\frac{1}{E}X_{10}\right)\right]p\sqrt{6}\,\delta_{R0}$
	$-\,[2\mathrm{Re}\,(g'_{vt}+g'_{sa})\,X_{11}+2\mathrm{Re}\,(g'_{st}+g'_{va})\,X_{12}]\,\frac{p^2}{E}\,\frac{2\sqrt{3}}{3}\,\delta_{R1}$
	$+\,[2\mathrm{Re}\,(g_{tv}+g_{as})\,X_{19}+2\mathrm{Re}\,(g_{ts}+g_{av})\,X_{20}]\,\frac{p^2}{E}\left(\frac{\sqrt{6}}{3}\,\delta_{R0}-\frac{2\sqrt{15}}{15}\delta_{R2}\right)$
$\langle (r/r)\sigma\rangle_1\langle r\sigma\rangle_0$	$-\,[4\mathrm{Re}\,g'_{ta}X_{11}+2\,(g'_{tt}+g'_{aa})\,X_{12}]\,\frac{p^2}{E}\sqrt{2}\,\delta_{R1}+\left[2\mathrm{Re}\,(g'_{tt}-g'_{aa})\,X_{18}-2\mathrm{Im}\,g'_{at}\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\,\frac{\sqrt{2}}{3}\,\delta_{R1}$
$\langle (r/r)\sigma\rangle_1\langle r\sigma\rangle_1$	$[2\mathrm{Re}\,(g_{tt}-g_{aa})\,X_{10}+2\mathrm{Re}\,(g_{at}-g_{ta})\,X_9]\,p2\sqrt{3}\,\delta_{R0}+[4\mathrm{Re}\,g'_{ta}X_{11}+2(g'_{tt}+g'_{aa})X_{12}]\,\frac{p^2}{E}\,\delta_{R1}\left(-\frac{2\sqrt{6}}{3}\right)$
	$+\,[4\mathrm{Re}\,g_{ta}X_{19}+2\mathrm{Re}\,(g_{tt}+g_{aa})\,X_{20}]\,\frac{p^2}{E}\left(-\frac{2\sqrt{3}}{3}\,\delta_{R0}-\frac{\sqrt{30}}{15}\,\delta_{R2}\right)+\left[2\mathrm{Re}\,(g'_{tt}-g'_{aa})\,X_{18}-2\mathrm{Im}\,g'_{at}\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\,\frac{\sqrt{6}}{3}\,\delta_{R1}$
$\langle (r/r)\sigma\rangle_1\langle r\sigma\rangle_2$	$-\,[4\mathrm{Re}\,g_{ta}X_{19}+2\mathrm{Re}\,(g_{tt}+g_{aa})\,X_{20}]\,\frac{p^2}{E}\,\frac{\sqrt{10}}{5}\,\delta_{R2}+\left[2\mathrm{Re}\,(g'_{tt}-g'_{aa})\,X_{18}-2\mathrm{Im}\,g'_{at}\left(X_{17}+\frac{1}{E}X_{18}\right)\right]p\,\frac{\sqrt{10}}{3}\,\delta_{R1}$
$\langle r/r\rangle_1\langle r/r\rangle_1$	$\left[2\mathrm{Re}\,g_{sv}\left(X_{21}\frac{1}{E}+X_{22}\right)-(g_{ss}+g_{vv})\left(X_{21}+\frac{1}{E}X_{22}\right)\right]\sqrt{3}\,\delta_{R0}+[(g'_{ss}-g'_{vv})\,X_{23}+2\mathrm{Im}\,g'_{vs}X_{24}]\,\frac{p}{E}\,\frac{\sqrt{6}}{3}\,\delta_{R1}$
$\langle r/r\rangle_1\langle (r/r)\sigma\rangle_0$	$[2\mathrm{Re}\,(g'_{va}-g'_{st})\,X_{23}+2\mathrm{Im}\,(g'_{sa}-g'_{vt})\,X_{24}]\,\frac{p}{E}\,\delta_{R1}$
$\langle r/r\rangle_1\langle (r/r)\sigma\rangle_1$	$-\left[2\mathrm{Re}\,(g_{vt}+g_{sa})\left(X_{21}\frac{1}{E}+X_{22}\right)-2\mathrm{Re}(g_{st}+g_{va})\left(X_{21}+\frac{1}{E}X_{22}\right)\right]\sqrt{6}\,\delta_{R0}$
	$+\,[2\mathrm{Re}\,(g'_{va}-g'_{st})\,X_{23}+2\mathrm{Im}\,(g'_{sa}-g'_{vt})\,X_{24}]\,\frac{p}{E}\,\frac{2\sqrt{3}}{3}\,\delta_{R1}$
$\langle (r/r)\sigma\rangle_0\langle (r/r)\sigma\rangle_0$	$\left[2\mathrm{Re}\,g_{ta}\left(X_{21}\frac{1}{E}+X_{22}\right)-(g_{tt}+g_{aa})\left(X_{21}+\frac{1}{E}X_{22}\right)\right]3\delta_{R0}$
$\langle (r/r)\sigma\rangle_1\langle (r/r)\sigma\rangle_1$	$\left[2\mathrm{Re}\,g_{ta}\left(X_{21}\frac{1}{E}+X_{22}\right)-(g_{tt}+g_{aa})\left(X_{21}+\frac{1}{E}X_{22}\right)\right]2\sqrt{3}\,\delta_{R0}+[(g'_{tt}-g'_{aa})\,X_{23}+2\mathrm{Im}\,g'_{at}X_{24}]\,\frac{p}{E}\,\frac{2\sqrt{6}}{3}\,\delta_{R1}$

Comparison of these exact results with experiment makes it possible to check the validity of time reversal invariance for β decay. For unique β transitions the experiments could be performed with the Y^{91} nucleus, for which $j_1 = 1/2$ and $j_2 = 5/2$.[9] Formulas (14) then give

$$\begin{aligned}
\beta_0 &= q^2\left(X_1+\frac{1}{E}X_2\right)+\frac{p^2}{3}\left(X_{13}+\frac{1}{E}X_{14}\right);\\
\beta_1 &= \frac{\sqrt{42}}{18}\,\frac{v}{c}\left(q^2X_3+\frac{p^2}{5}X_{15}\right);\\
\beta_2 &= -\frac{\sqrt{21}}{9\cdot 15}\,p^2\left(X_{13}+\frac{1}{E}X_{14}\right);\quad \beta_3=-\frac{\sqrt{7}}{35}\,p^2\,\frac{v}{c}\,X_{15}.
\end{aligned} \tag{15}$$

To summarize: the study of β-γ angular correlation of circularly polarized γ-rays in first forbidden β-transitions is of interest in two cases: Coulomb transitions in heavy nuclei and unique transitions. Apparently only in these cases is a simple interpretation of the results possible.

The author expresses his gratitude to Prof. I. S. Shapiro, who suggested this research, for valuable advice and constant attention.

APPENDIX

For a general first forbidden β transition the quantity f(J, J′, R) [cf. formula (6)] has the form

$$f(J, J', R) = \sum_{A,B} \langle A\rangle_J \langle B\rangle_{J'} K(A, B, R), \tag{16}$$

where $\langle A\rangle_J$ and $\langle B\rangle_{J'}$ denote arbitrary nuclear reduced matrix elements. Thus f(J, J′, R) is the sum of all possible products of pairs of such elements weighted by the factor K(A, B, R) which depends on their specific form. The table lists all nonzero values of K(A, B, R) for different pairs of matrix elements $\langle A\rangle_J$ and $\langle B\rangle_{J'}$. In the calculations $\langle A\rangle_J$ and $\langle B\rangle_{J'}$ were assumed to be real.

The table contains a number of abbreviations. The quantities g_{yz} and g'_{yz} stand for the following combinations of coupling constants:

$$g_{yz} = C_y C_z^* + C'_y C'^*_z; \quad g'_{yz} = C_y C'^*_z + C'_y C^*_z.$$

Here C_y, C'_y are the coupling constants for the covariant y (y corresponds to s, t, p, etc.), where the primed ones take into account parity nonconservation in β decay.

The quantities X_i have the following values:

$$X_1 = \frac{(\gamma_1+1)^2}{4} + \frac{\alpha^2Z^2}{4}\frac{E^2+1}{p^2}; \quad X_2 = -\frac{\alpha^2Z^2}{2}\frac{E}{p^2}; \quad X_3 = \frac{(\gamma_1+1)^2+\alpha^2Z^2}{4}; \quad X_4 = \frac{\alpha Z}{2p}(\gamma_1+1);$$

$$X_5 = \zeta^*\frac{1}{8}\left[(\gamma_1+1)(\gamma_2+2) + \frac{\alpha^2Z^2(E^2+1)}{p^2} + i\frac{\alpha ZE}{p}(\gamma_2-\gamma_1+1)\right]; \quad X_6 = -\frac{1}{8}\zeta^*\left[\frac{\alpha^2Z^2}{p^2}E + i\frac{\alpha Z}{p}(\gamma_2-\gamma_1+1)\right];$$

$$X_7 = \frac{1}{8p}\zeta^*[p(\gamma_1+1)(\gamma_2+2) + \alpha^2Z^2p + i\alpha ZE(\gamma_2-\gamma_1+1)]; \quad X_8 = \frac{\alpha Z}{8p}(\gamma_2+\gamma_1+3)\zeta^*; \quad X_9 = i\frac{\alpha^2Z^2}{2p}; \quad X_{10} = \frac{\alpha Z}{2}\gamma_1;$$

$$X_{11} = \frac{\alpha ZE}{2p^2}(1-\gamma_1); \quad X_{12} = \frac{\alpha Z}{4p^2}(E^2+1)(\gamma_1-1) - \frac{\alpha Z}{4}(\gamma_1+1); \quad X_{13} = \frac{|\zeta|^2}{16}\left[(\gamma_2+2)^2 + \alpha^2Z^2\frac{E^2+1}{p^2}\right]; \quad X_{14} = -|\zeta|^2\frac{\alpha^2Z^2}{8}\frac{E}{p^2};$$

$$X_{15} = |\zeta|^2\frac{(\gamma_2+2)^2+\alpha^2Z^2}{16}; \quad X_{16} = \frac{\alpha Z}{8p}|\zeta|^2(\gamma_2+2); \quad X_{17} = i\zeta\frac{(\gamma_2+2)(\gamma_1-1)-\alpha^2Z^2}{8p};$$

$$X_{18} = X_{20} = \frac{\zeta}{8p}[\alpha Zp(\gamma_2-\gamma_1+3) + i(\gamma_2+2)(\gamma_1-E) + i\alpha^2Z^2E]; \quad X_{19} = \zeta\left[\frac{\alpha ZE}{p^2}(\gamma_1-1)^2 - i\frac{(\gamma_2+2)(\gamma_1-1)+\alpha^2Z^2}{8p}\right];$$

$$X_{21} = \frac{(\gamma_1-1)^2}{4}\frac{E^2+1}{p^2} + \frac{\alpha^2Z^2}{4}; \quad X_{22} = \frac{(\gamma_1-1)^2E}{2p^2}; \quad X_{23} = \frac{(\gamma_1-1)^2+\alpha^2Z^2}{4}; \quad X_{24} = \frac{\alpha Z}{2p}(1-\gamma_1). \tag{17}$$

The general expression for f(J, J′, R) contains a number of reduced matrix elements satisfying definite selection rules in the total angular momentum J(J′) of the electron-neutrino pair and in parity. Our notation for the reduced matrix elements differs from the generally accepted one, but it simplifies the final results. We give these matrix elements below in the notation of Rose and Osborn[10] together with the appropriate selection rules. Nuclear matrix elements differing by the operator $\beta = \gamma_4$ were assumed to be the same.

$$\langle r\rangle_J = (j_2\|Y_1(\mathbf{r})\|j_1); \quad J=1, \quad |j_2-j_1| \leqslant J \leqslant j_2+j_1, \ \text{yes};$$

$$\langle r\sigma\rangle_J = (j_2\|T_{J1}(\mathbf{r},\sigma)\|j_1); \quad 0 \leqslant J \leqslant 2, \quad |j_2-j_1| \leqslant J \leqslant j_2+j_1, \ \text{yes}; \quad \langle\gamma_5\rangle_J = \frac{1}{M}(j_2\|Y_0(\mathbf{r})\,\sigma\cdot\mathbf{p}\|j_1); \quad J=0, \quad j_2=j_1, \ \text{yes};$$

$$\langle\alpha\rangle_J = -\frac{1}{M}(j_2\|T_{10}(\mathbf{r},\mathbf{p}_n)\|j_1); \quad J=1, \quad |j_2-j_1| \leqslant J \leqslant j_2+j_1, \ \text{yes};$$

$$\langle r/r\rangle_J = \left(j_2\left\|\frac{1}{r}Y_1(\mathbf{r})\right\|j_1\right); \quad J=1, \quad |j_2-j_1| \leqslant J \leqslant j_2+j_1, \ \text{yes};$$

$$\langle(r/r)\sigma\rangle_J = \left(j_2\left\|\frac{1}{r}T_{J1}(\mathbf{r},\sigma)\right\|j_1\right); \quad J=0,1, \quad |j_2-j_1| \leqslant J \leqslant j_2+j_1, \ \text{yes}. \tag{18}$$

Note added in proof (August 28, 1958). Recently M. Morita and S. Morita published an article (Phys. Rev. **109**, 2048 (1958)) containing formulas for β-γ correlation of circularly polarized γ-rays for arbitrary mixtures of covariants but without interference terms between the s, t, p and v, a covariants. The case $\alpha Z \gg 1$ is considered, but the concept of Coulomb β-transitions is not introduced.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957).

[3] G. Racah, Phys. Rev. **62**, 438 (1942).

[4] Biedenharn, Blatt, and Rose, Revs. Modern Phys. **24**, 249 (1952).

[5] Berestezky, Joffe, Rudik, and Ter-Martirosyan, Nucl. Phys. **5**, 464 (1958).

[6] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[7] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 407 (1957), Soviet Phys. JETP **5**, 337 (1957).

[8] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[9] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных изотопов (Decay Schemes of Radioactive Isotopes), Acad. Sci. Press, 1957.

[10] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954).

Translated by A. M. Bincer
26