

(d, p) STRIPPING REACTIONS IN THE SILICON ISOTOPES

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Thick emulsions were used to measure the angular distribution of protons from the ground and first excited states in the reactions $\text{Si}^{29,30}(\text{d}, \text{p})\text{Si}^{30,31}$ at a deuteron energy of 4.3 Mev. The orbital angular momenta l of the captured neutrons were determined, as well as the absolute differential reaction cross sections and the reduced neutron widths of levels in the final nuclei. The experimental proton angular distributions were compared with the theoretical curves of Butler and Bhatia. The spin and parity of the ground and first excited states in Si^{31} confirm the calculations of Goldhammer.

THE angular distributions of protons or of neutrons in stripping reactions of the type (d, p) or (d, n) have become a generally accepted tool of nuclear spectroscopy for investigating the quantum numbers of states in the product nucleus.¹⁻³ As is well known, the orbital angular momentum l of the neutron captured by the nucleus in the (d, p) reaction is determined by the position of the peak in the angular distribution of the emitted protons, and can be used to find the parity π and a set of possible spins I_f of the final nucleus. The absolute value of the cross section for the stripping reaction is proportional to the probability that a particle with orbital angular momentum l will be captured on the surface of the nucleus, provided that the core remains unexcited. This probability is proportional to the reduced widths γ_l^2 and Θ_l^2 for the captured particle⁴ or to the nuclear matrix element Λ_l .² If the states of the final nucleus can be described by the shell model, then the values of these parameters should be relatively large; they should at least be of the same order of magnitude for the excited states as they are for the ground states.

(d, p) reactions on enriched Silicon isotopes Si^{29} and Si^{30} were studied at deuteron energies of 4.3 Mev.⁵ The energy spectra were calibrated using magnetic field measurements of the proton spectra⁶ from the reactions $\text{Si}^{28,29,30}(\text{d}, \text{p})\text{Si}^{29,30,31}$.

DESCRIPTION OF THE EXPERIMENT

A 4.3-Mev deuteron beam from the 72 cm cyclotron of the Institute of Nuclear Physics, Moscow State University was electromagnetically focused and, passing through a system of collimating slits, fell at 45° on the target, placed at the center of a

cylindrical camera. The energy spread in the beam had been determined earlier in previous experiments⁷ and was less than 40 kev; the beam did not diverge by more than 20'. Since the deuteron energy varied from one bombardment to another, the value of E_d used was obtained from the proton group of longest range and from a reaction with known Q . The protons emitted from the target were detected by type YA-2 NIKFI emulsions 100 and 200 μ thick and placed about 10 cm from the target at angles of 6.5 to 150° with the incident deuteron beam. Aluminum absorbers were placed between the target and the emulsions to suppress elastically-scattered deuterons and low-energy protons. The beam current was measured with a Faraday cylinder and integrator.

The targets for reactions on the enriched isotopes were made by depositing finely divided SiO_2 powder on a piece of gold tinsel 0.15 mg/cm² thick stretched over a frame. The silicon dioxide was enriched with Si^{29} or Si^{30} (see table I). The thickness of the target was found by weighing and turned out to be 0.30 mg/cm² for Si^{29} and 0.34 mg/cm² for Si^{30} . The Si^{28} target was made by evaporating a silicon sample with the natural isotopic abundances in vacuum. Only chemically pure materials were used in making the targets. Bombardments carried out as controls showed that there was a negligible amount of impurities in the targets and underlying tinsel foil.

The tracks were counted using an MBI-2 microscope with magnification $1.5 \times 90 \times 5$. In the histograms below, the scale along the abscissa is in units of the scale in the ocular of the microscope. In the angular distributions shown below, the errors are statistical; the dotted lines show the isotropic

TABLE I

Isotope	Natural Abundance, %	Composition of the Si ²⁹ sample, %	Composition of the Si ³⁰ sample, %
Si ²⁸	92.28	34.6	35.3
Si ²⁹	4.67	63.9	12.5
Si ³⁰	3.05	1.5	52.2

part of the angular distribution, which is presumably due to formation of the compound nucleus.

RESULTS

Si²⁹ (d, p) Si³⁰. Figure 1 shows the proton spectra at angles $\theta_{\text{lab.}} = 6.5^\circ$ and 40° . According to Dzhelepov and Peker,⁸ Si³⁰ has an excited state at 1.250 mev. It does not appear in our experiment. The dotted line in Fig. 1 shows the position of the proton group which should correspond to such a state. If this state is formed its intensity of formation is 0.3% of the corresponding quantity for the ground state of Si³⁰ at $\theta_{\text{lab.}} = 6.5^\circ$ and less than 0.6% for $\theta_{\text{lab.}} = 40^\circ$. The energy resolution of the nuclear emulsions is insufficient to distinguish between the proton groups Si³⁰ (1) corresponding to reactions on the first excited state of Si³⁰, and the proton group Si²⁹ (0) corresponding to reactions

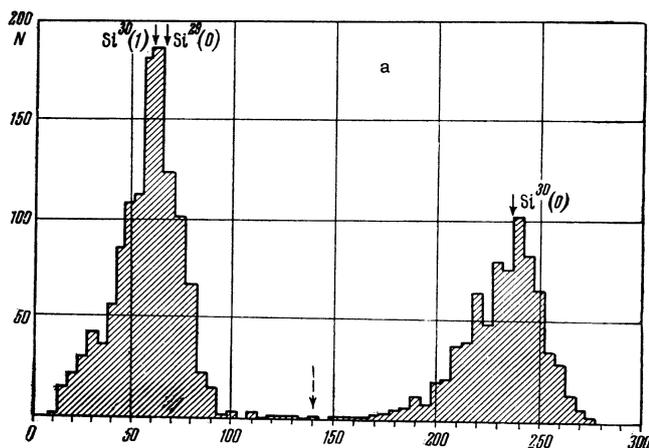


FIG. 1a. Proton spectrum at $\theta_{\text{lab.}} = 6.5^\circ$ from the reaction Si²⁹ (d, p) Si³⁰.

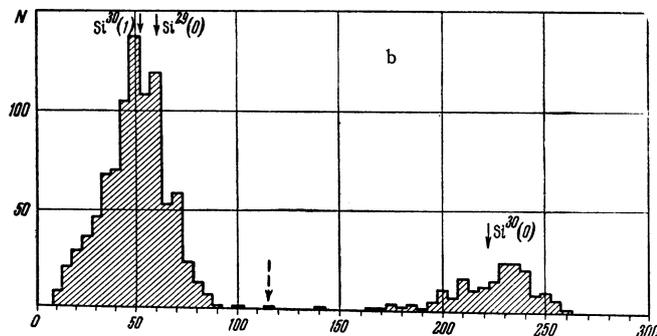


FIG. 1b. Proton spectrum at $\theta_{\text{lab.}} = 40^\circ$ from the reaction Si²⁹ (d, p) Si³⁰.

leading to the ground state of Si²⁹. The Q's for these reactions differ only by $\Delta Q = 0.098$ mev. The sum of the angular distributions has been plotted for these groups. Figure 2 shows the measured angular distributions together with the theoretical curve of Bhatia.² When the neutron was captured with orbital angular momentum $l = 0$, the radius of the target nucleus was taken to be $R = R_0 + 2 \times 10^{-13}$ cm, while $R = R_0 + 1 \times 10^{-13}$ cm was used for $l = 2$. R_0 was calculated as $R_0 = (1.7 + 1.22A^{1/3}) \times 10^{-13}$ cm. In Fig. 2a the angular distribution of the group Si³⁰ (0) agrees satisfactorily with theoretical curve for $l = 0$. The angular

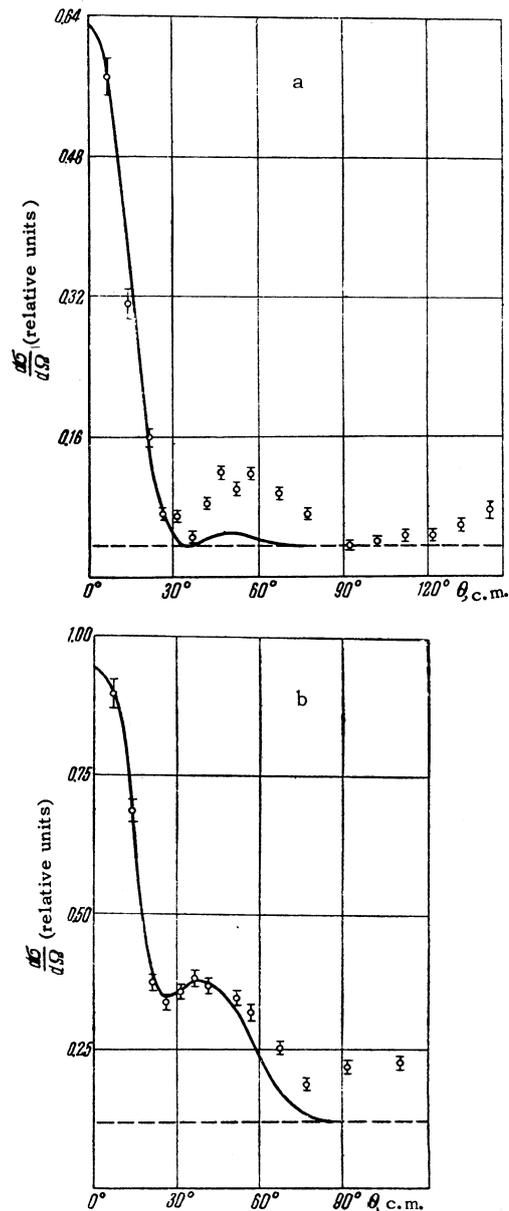


FIG. 2. Angular distribution of protons from the reaction Si²⁹ (d, p) Si³⁰: a - ground state of Si³⁰, $l = 0$; b - first excited state of Si³⁰ $l = 2$ and the ground state of Si²⁹, $l = 0$. $d\sigma/d\Omega$ is in units of 7.30 mbn/sterad.

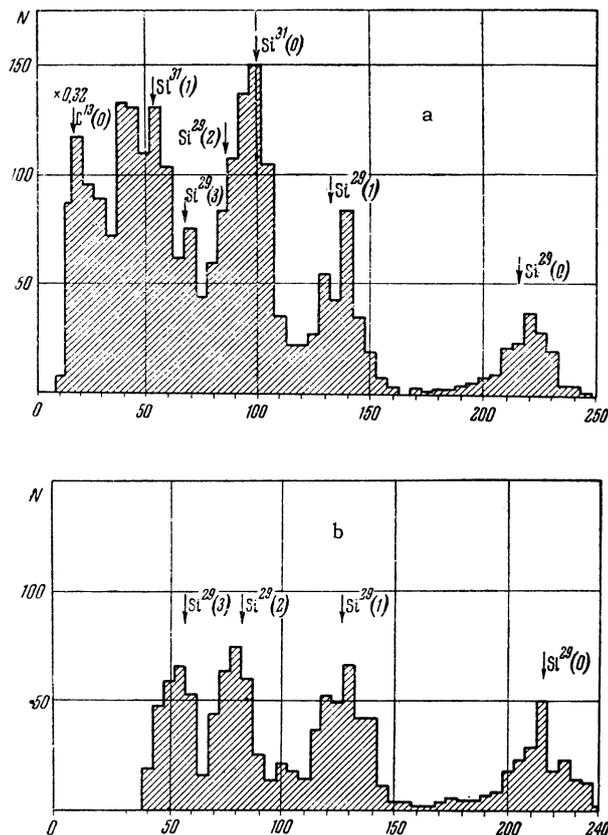


FIG. 3. Proton spectra: a - from the reactions $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$ and $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$ at an angle $\theta_{\text{lab.}} = 40^\circ$; b - from the reaction $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$ at an angle $\theta_{\text{lab.}} = 40^\circ$.

distribution shown in Fig. 2b for the two groups $\text{Si}^{30}(1)$ and $\text{Si}^{29}(0)$ agrees with the sum of the curves for $l = 2 + 0$. Since the neutron is captured by the ground state of Si^{29} into an $l = 0$ state,^{9,10} the angular distribution of the proton group $\text{Si}^{30}(1)$ corresponds to neutron capture with $l = 2$.

$\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$. The proton spectra from the reactions $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$ and $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$ are shown in Fig. 3. The energy resolution is not enough to separate the proton groups $\text{Si}^{31}(0)$ from $\text{Si}^{29}(2)$ or $\text{Si}^{31}(1)$ from $\text{Si}^{29}(3)$ (the Q 's for these reactions differ only by 0.144 and 0.212 mev). Figure 4 shows the sum of the angular distributions for these two groups. The bottom part of the figure shows the angular distributions of the proton groups $\text{Si}^{29}(2)$ and $\text{Si}^{29}(3)$, drawn to a scale which shows the contributions these groups make to the total angular distribution. A correction was made for the variation in deuteron energy ($\Delta E_d = 0.4$ mev), using the data of Tobocman and Kalos.¹¹ The correction never exceeded 0.5%. The angular distribution of the $\text{Si}^{29}(3)$ group is isotropic, while the $\text{Si}^{29}(2)$ protons correspond to capture of a neutron with $l = 2$ (the theoretical curve is drawn using Bhatia's formula with $R = R_0 + 1 \times 10^{-13}$ cm). These results agree with those of Holt and Mar-

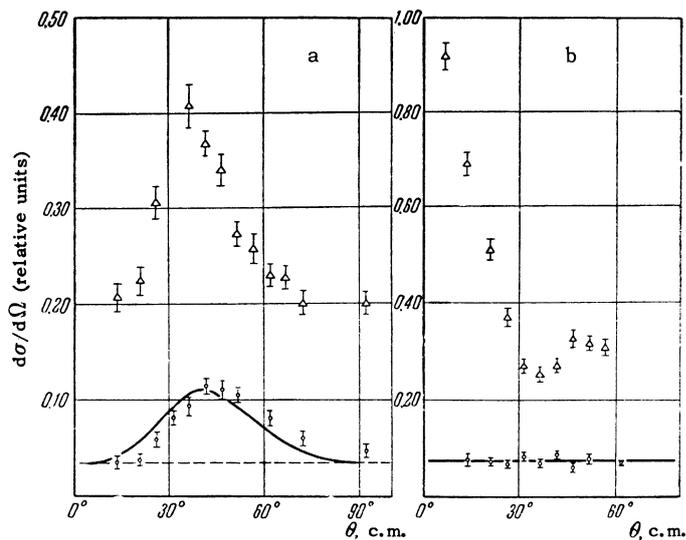


FIG. 4. Angular distributions of protons from the reactions $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$ and $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$: a) Δ - ground state of Si^{31} , $l = 2$, and the second excited state of Si^{29} , $l = 2$; \circ - for the proton group $\text{Si}^{29}(2)$; b) Δ - first excited state of Si^{31} , $l = 0$, and the third excited state of Si^{29} (isotropic). $d\sigma/d\Omega$ is in units of 9.30 mbn/sterad.

sham,⁹ taken at a deuteron energy of 8 Mev. The final angular distributions for the proton groups $\text{Si}^{31}(0)$ and $\text{Si}^{31}(1)$ (obtained from the difference between the angular distributions of Fig. 4) are shown in Fig. 5, together with theoretical curves calculated from the formulas of Bhatia (solid curves, $R = R_0 + 1 \times 10^{-13}$ cm for $l = 2$ and $R = R_0 + 2 \times 10^{-13}$ cm for $l = 0$) and Butler (dotted curves, $R = R_0 + 1 \times 10^{-13}$ cm). The radius R was increased by 1×10^{-13} cm for $l = 2$ in order to obtain better agreement with experiment. The change displaces the peak in the theoretical curve by 3 or 4° toward the smaller angles.

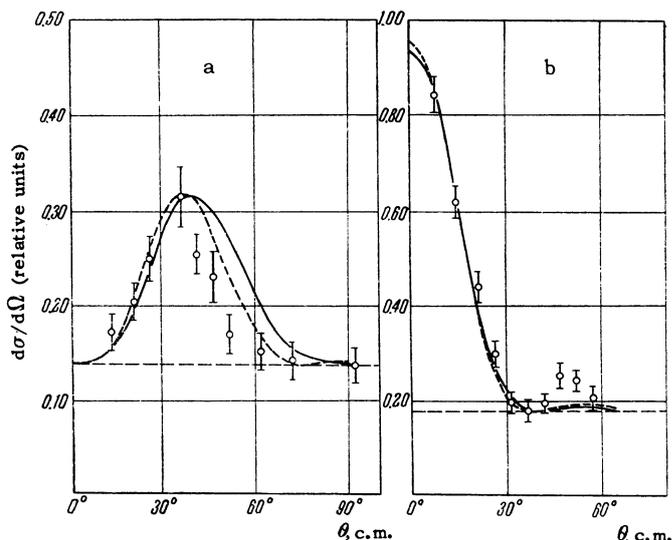


FIG. 5. Angular distribution of protons from the reaction $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$ a - ground state of Si^{31} , $l = 2$; b - first excited state of Si^{31} , $l = 0$. $d\sigma/d\Omega$ is in units of 9.30 mbn/sterad.

TABLE II

Reaction	Level position E Mev from reference 6	Level position E Mev from our data*	l	I_f, π	$d\sigma/d\Omega$, mbn/steradian	$10^2 \gamma_l^2$, mev-cm $\times 10^{-30}$	Θ_l^2	Shell model configuration
$\text{Si}^{29}(d, p)\text{Si}^{30}$	0	0	0	$0^+, (1^+)$	4.57 (0°)	0.88	0.029	$(2s_{1/2})^2$
	2.239 ± 0.020	2.25 ± 0.09	2	$2^+, (1^+, 3^+)$	1.55 (38°)	0.57	0.019	$(2s_{1/2})^1 (1d_{3/2})^1$
$\text{Si}^{30}(d, p)\text{Si}^{31}$	0	0	2	$3/2^+, (3/2^+)$	1.66 (38°)	0.31	0.011	$(2s_{1/2})^2 (1d_{3/2})^1$
	0.757 ± 0.007	0.76 ± 0.08	0	$1/2^+$	7.25 (0°)	0.17	0.006	$(2s_{1/2})^1 (1d_{3/2})^2$

*The indicated uncertainty is the root-mean-square error.

The fact that the experimental angular distribution is shifted toward the smaller angles and is somewhat narrower than the theoretical curve for $R = R_0$ indicates that there is a nuclear interaction not accounted for in the theories of Butler and Bhatia. In our case the deuteron energy was 4.3 Mev, almost equal to the height of the Coulomb barrier for the nuclei investigated, so that Coulomb effects cannot be neglected; their effect is to shift the peak in a direction opposite to that given by the nuclear forces, that is, toward larger angles. The Coulomb barrier of the nucleus Si^{29} is 3.70 Mev, while for the nucleus Si^{30} it is 3.66 Mev. However, when E_d is about the same as the Coulomb barrier, the shape of the angular distribution is not deformed enough to give a wrong value for l , at least for nuclei up to $Z = 19$ and for $l = 2,^{12}$ even though the Coulomb forces, together with other nuclear interactions, can have a big effect on the magnitude of the stripping cross section.

DISCUSSION

The results of the experiment are shown in Table II. The level spins were obtained from the selection rules corresponding to the formula $I_f = I_i + l + \frac{1}{2}$, the values in parentheses being less reliable. The values of the absolute differential cross sections were calculated without taking into account the isotropic part of the angular distributions at the maxima of the theoretical curves (for $\theta = 0^\circ$ for $l = 0$ and $\theta = 36$ to 38° for $l = 2$). The accuracy of $d\sigma/d\Omega$ is about 40%. The reduced neutron widths γ_l^2 and Θ_l^2 for levels of the final nuclei were calculated from the results of Friedman and Tobocman³ and from the tables of reference 13, without taking into account the Coulomb and nuclear interactions. Θ_l^2 is given in units of the Wigner sum rule limit.

In principle, a measurement of the absolute value of σ_{str} gives the value of γ_l^2 and the kind of nuclear excitation. However, it is not possible

to make a direct determination of the reduced width because one cannot take properly into account the nuclear interaction between the particles taking part in the reaction. The effects of the Coulomb forces can be calculated exactly, but such a calculation is very complicated. The reduced widths obtained without taking these interactions into account are too small. Hence the values of γ_l^2 and Θ_l^2 quoted in table II should be regarded as only relative. From the calculations of Tobocman and Kalos, the neglected interactions should increase γ_l^2 and Θ_l^2 by a factor $N > 10$. Calculations carried out by Neudachin and Teplov¹⁴ for the reaction $\text{Ca}^{40}(d, p)\text{Ca}^{41}$ for $E_d \approx 4$ Mev give a value of N about 40 or 50. Since the nuclear interactions have but little effect on the cross section, one can carry out an approximate calculation of the Coulomb corrections. Such estimates were obtained by Sawicki¹⁵ and Butler¹⁶ but the corrections obtained were small compared with the numerical computations of Tobocman and Kalos. According to Butler, a correction factor $\{R_\infty\}^2$ can be calculated in the limit of small l and for $E_d \gg Ze^2/R$, i.e., for relatively large angular momenta of the deuteron, λ_d , and proton L_p . (The correction factor $\{R_\infty\}^2$ is applied to decrease the value of $\sigma_{\text{stripping}}$ as calculated taking Coulomb corrections into account). The approximation improves with increasing λ_d and L_p , because of the short range of nuclear forces. For $E_d \approx Ze^2/R$, the factor $\{R_\infty\}^2$ is too small, but still can be used to obtain a lower limit for $\sigma_{\text{stripping}}$. The factor $\{R_\infty\}^2$ is almost exactly equal to $\exp[-\pi(\eta_p + \eta_d)]$ where η is the Coulomb parameter, $\eta = Ze^2m/\hbar^2k$ (k is the wave number). The second factor in $\{R_\infty\}^2$ is usually little different from one ($E_d \gtrsim Ze^2/R$, $Q >$ binding energy of the deuteron).

For the reaction $\text{Si}^{29}(d, p)\text{Si}^{30}$ at $E_d = 4.30$ Mev, the correction decreases σ_{str} by a factor 10^3 , which is clearly wrong but does provide a lower limit to the reaction cross section. From the values of the reduced width Θ_l^2 for the ground

state of Si^{30} (see table II), σ_{str} cannot be decreased by more than a factor 30, since $\{\Theta_2^2\}_{\text{Coul.}} = \Theta_2^2 / \{R_\infty\}^2$ cannot be more than unity ($\{\Theta_2^2\}_{\text{Coul.}}$ is the reduced width taking Coulomb corrections into account). The reason for this discrepancy with experiment is not hard to find, since the orbital angular momentum of the incident deuteron is

$$\lambda_d = \hbar^{-1} R [2M_d(E_d - Ze^2/R)]^{1/2} \approx 1.$$

For the ground state of Si^{30} , the orbital angular momentum of the captured neutron is $l = 0$, i.e., the values $\lambda_d = L_p = 1$ are too small. In the case of the reaction ${}^9\text{F}^{19}(d, p){}^9\text{F}^{20}$, proceeding from the ground state with $E_d = 14.3$ Mev, the Butler correction is small compared to the result given by the calculations of Tobocman and Kalos. Here $l = 2$, $\lambda_d = 4$, $E_d \gg Ze^2/R = 2.4$ Mev, but $\{R_\infty\}^2$ is less than the result given by Tobocman and Kalos by a factor 7.5. We may conclude that the approximations made by Butler are too crude, at least for deuteron energies up to 14 Mev.

The nucleus Si_{16}^{30} is even-even and has isotopic spin $T = 1$. The spin and parity of the ground state are 0^+ . For even-even nuclei, the first excited state usually has spin and parity 2^+ , which for this nucleus means that one of the two neutrons outside a closed shell has undergone the transition $(2s_{1/2}) \rightarrow (1d_{3/2})$. The close agreement between the values of the reduced widths for the ground and first excited states suggests that these can be pictured as two neutrons interacting outside a closed shell.

The nucleus Si_{17}^{31} is even-odd, with $T = 3/2$. Possible values for the spin and parity of the ground state are $3/2^+$, $5/2^+$; according to Dzheleпов and Peker the ground state is $3/2^+$. The spin of the first excited state is determined uniquely by this experiment; it is $1/2^+$. The values $3/2^+$, $1/2^+$ for the spins and parities of the ground and first excited states agree with the calculations carried out by Goldhammer;¹⁷ for three neutrons outside a spherical core, his calculations involved a coupling between nucleons in orbits of the same parity introduced by surface oscillations of the core.

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