THE SCATTERING OF MUONS IN LEAD

A. I. ALIKHANYAN and F. R. ARUTYUNYAN

Physics Institute, Academy of Sciences, Armenian S.S.R.

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Muons with momenta between 1.0 and 1.8×10^8 ev were scattered in 7-mm thick lead plates. The experimental angular distribution is compared with the theoretical multiple scattering curve for a finite nucleus. It is shown that after all necessary corrections are made the experimental results are in good agreement with the calculations.

1. INTRODUCTION

In recent years a number of investigators have discovered that the scattering of muons with momenta above 100 Mev/c does not agree with the calculated angular distribution based purely on a Coulomb interaction between muons and nuclei. Whittemore and Shutt[1] studied the scattering of muons with momenta 0.3 to 3.1 Bev/c in a 5-cm lead plate mounted inside a cloud chamber and observed considerable scattering at larger angles than are predicted by Olbert's theory[2] for finite nuclei. The experiments of George, Redding, and Trent[3] at 60 m water equivalent (m.w.e.) underground also revealed an apparent excess of muon scattering in 2-cm lead plates, especially at angles greater than 15°. A cross section of the order of 4×10^{-27} cm^2/nucleon was obtained at 100 to 300 Mev, where the scattering results exceeded the theory.

McDiarmid,[4] working with a cloud chamber at a depth of 26 m.w.e., found that the scattering in lead of muons with momenta 200 to 600 Mev/c agrees with scattering theory, whereas at 500 to 1700 Mev/c the experimental results for angles greater than 4° considerably exceed the predictions of the theory for a finite nucleus. Leontic and Wolfendale[5] have also found a larger number of muons scattered in lead above 285 Mev/c.

The discrepancy between the observed angular distribution of scattered muons and the theoretical distribution calculated for Coulomb scattering by a finite nucleus has been called the "anomalous" scattering of muons.

All of the experimental work mentioned above was performed with fairly thick plates in which a considerable fraction of the mesons underwent multiple scattering. Kannangara and Shrikantia[6] investigated pure single scattering of muons at 100-600 Mev/c in an emulsion irradiated 60 m.w.e. underground, obtaining the anomalous scattering cross section $\sigma_{an} = (1.5 \pm 1.0) \times 10^{-27}$ cm^2/nucleon. Alikhanov and Eliseev[7] obtained the same order of magnitude for the anomalous muon scattering cross section in graphite at 200 to 800 Mev/c.

Alikhanyan and Kirillov-Ugryumov[8] investigated slow muon scattering in thin copper plates within a narrow momentum range from 80 to 140 Mev/c and detected excess scattering at projected angles greater than 15°. The small number of observed events did not permit any final conclusion regarding the existence of anomalous scattering in this energy region.

Establishment of the existence of anomalous muon scattering would be of fundamental significance for the theory of elementary particles but would disagree strongly with the known properties of muons. Specifically, it is known that the spectra of x-ray emission in μ-mesonic atomic transitions very accurately establishes the absence of appreciable non-Coulomb interaction between slow muons and nucleons.

It has been found impossible to account for the observed anomalous muon scattering by means of incoherent scattering accompanied by nuclear excitation, or by excess scattering resulting from anomalous muon and nucleon magnetic moments.

The present paper describes an experimental investigation of muon scattering in lead plates at $(1.0 \text{ to } 1.8) \times 10^8$ ev/c. An exact nuclear model is required to establish excess muon scattering by heavy nuclei. Experiments on fast electron scattering by Hofstadter[9] and Schiff[10] have permitted a direct experimental determination of a nuclear form factor for various nuclei, including lead.

2. EXPERIMENTAL PROCEDURE

Muon scattering was investigated at the cosmic-ray station on Mt. Aragats (3200 m above sea level) using a magnetic mass spectrometer in conjunction with two multiplate cloud chambers. A detailed description of the apparatus has been given in references 12 and 13. Particle trajectories were selected which satisfied the conditions given in references 8 and 13. The present paper gives data on muon scattering in 7-mm lead plates.

Particle Identification

Momentum and range measurements by means of the mass spectrometer were used to determine the particle masses. Momenta were determined from the radius of curvature of trajectories in the magnetic field and ranges were determined from the amount of matter traversed by particles inside the chamber before stopping. Range measurements took into account the entrance angles of particles into the scattering plate and the systematic path lengthening due to multiple Coulomb scattering.[14] For 812 particles with masses measured at 150 to 360 m_e the experimental mass distribution is well approximated by two normal distribution curves with half-width corresponding to a root mean square error of 10%.

Assuming that all particles with measured mass below 240 m_e are muons and that all particles with mass greater than 250 m_e are pions, the average mass for the muon group is 209 m_e and for the pion group 278 m_e. An analysis of this mass spectrum with corrections for the transmission of the apparatus shows that the muon group should contain no more than 2% pions and that the pion group should not contain more than 12% muons.

Reference 8 gives the mass spectra when the chamber contained 5-mm and 2-mm copper plates, in which case the separation of pions and muons is considerably less clearcut.

Measurement of Angles

Large numbers of angle measurements, obtained by means of a special goniometer, were averaged, the accuracy being limited principally by cloud-chamber distortions (diffuseness of tracks, turbulent eddies, etc.). The root mean square error of angle measurements due to these factors is 0.6°. The error is constant over the entire angular range so that the relative error decreases as the angle increases.

The multiple Coulomb scattering of charged particles is usually determined theoretically for projections of the scattering angles on a plane passing through the original trajectory, which generally differs from projections on the plane of the photographic plate. The two angles are equal only when a trajectory before the scattering event is parallel to the photographing plane. The original trajectory makes an angle γ with the photographing plane because of scattering in the previous plate, so that the angle measured in the photograph is greater than the projection of the scattering angle on a plane passing through the original trajectory. Also, when a particle passes through the cloud chamber at a relatively great distance from the optical axis of the camera objective (compared with the distance from the objective to the scattering point inside the chamber) a nonorthogonal projection of the scattering angle is photographed. The true projection of the scattering angle is thus increased or decreased, depending on the orientation of the trajectory inside the chamber; this was pointed out to us by M. I. Daĭon.

The trajectories of mesons which, because of their large curvature in the magnetic field, enter the side regions of the chamber and are scattered in the plates at relatively large angles are subject to the indicated angular discrepancy. For mesons scattered at angles greater than 20° a stereocomparator was used for each angle separately to determine the trajectory in space, i.e., the coordinates of the intersections of the trajectory and the plate, after which the true projection of the scattering angle was determined from simple geometric relations.

From the distribution of the angle differences $\Delta\theta_\varphi = \theta'_\varphi - \theta_\varphi$, where θ'_φ is the true projection of the scattering angle and θ_φ is the angle measured directly in the photograph, we determined that the systematic excess in these directly measured angles was 0.45°. All angles smaller than 20° were corrected by this amount.

Measurement of Momentum at the Point of Scattering

The particle momentum at the point of scattering must be known for comparison of the observed and the expected scattering angle. When the particle mass is known the momentum can be determined in two ways: (1) From the residual range after scattering, which is taken to be the amount of matter traversed by the particle from the middle of the scattering plate to the middle of the plate in which the particle is stopped. (2) From the momentum measured in the magnetic field and the amount of matter traversed by the particle up to the scattering point.

When the plate in which a particle is stopped is denoted by $i = 0$ and the other plates above it are numbered serially $i = 1, 2, 3$ etc., the two methods yield, for a plate of the same number i values of the average momentum which agree to within 2 to 3%.

The distribution function of multiple Coulomb scattering is derived on the assumption that the momentum loss in the scattering plate is negligibly small, but a calculation shows that when this momentum loss is taken into account the scattering curve is broadened. The broadening depends on the plate number i and for $i > 1$ does not exceed 5% (depending on the stopping point of the particle in plate $i = 0$); for $i = 1$ it varies from 5% to ∞. We therefore do not investigate scattering in plates with the number $i = 1$.

The residual range after scattering is thus known with maximum inaccuracy $\Delta R = t/2$, where t is the thickness of plate $i = 0$. This value of ΔR corresponds to a certain momentum difference Δp, which must be taken into account in plotting the scattering curves. Corrections were obtained for the multiple Coulomb scattering curves plotted for the mean momentum p_m by taking this uncertainty of the momentum measurement into account. At large scattering angles these corrections reach 30 or 40%.

Material and Thickness of the Scattering Plates

The lead plates in which scattering was investigated contained impurities which we took into account in calculating the rms scattering angle and distribution function. For example, 7% of antimony reduces by 1.5% the rms scattering angle calculated for pure lead. Particle tracks in a cloud chamber form different angles with the normal to a scattering plate first, because they enter the chamber at different angles and secondly, because of scattering in the plates, so that there was a certain distribution of scattering plate thickness which had to be taken into account.

For a given single scattering angle θ and particle momentum p, different scattering plate thicknesses $t_1, t_2, t_3, \ldots, t_k$ occurring with frequencies $n_{t_1}, n_{t_2}, \ldots, n_{t_k}$, respectively, correspond to different probabilities of observing the angle θ_φ: $f_{t_1}(\theta_\varphi), f_{t_2}(\theta_\varphi), \ldots, f_{t_k}(\theta_\varphi)$. We select a certain effective scattering plate thickness t_{eff} such that the relation

$$f_{t_{\mathrm{eff}}}(\theta_\varphi) = \sum_{i=1}^{k} n_{t_i} f_{t_i}(\theta_\varphi) \Big/ \sum_{i=1}^{k} n_{t_i} \tag{1}$$

is valid for the entire investigated angular range. For 7-mm lead plates this t_{eff} equalled the rms thickness calculated from the experimental thickness distribution and was 9% greater than the geometrical thickness of the plate.

Experimental Geometry

Our apparatus defined a certain region of space in which particle scattering was investigated. The projection of the total scattering angle θ on the plane in which scattering is investigated will be denoted by θ_φ, and the projection on a plane perpendicular to the plate of investigation will be denoted by θ_ψ. Because of the geometrical limitations in the investigated plane the probability of registration varies for different scattering angles. This was also pointed out by McDiarmid[4] and more recently by Cousins et al.[15] The same result is produced by geometrical limitations in the plane perpendicular to the plane of investigation, but this depends also on the distribution function of scattering angles. The registration of particles scattered at projected angle θ_φ will depend on the corresponding angle θ_ψ; hence the observed distribution of θ_φ will depend on the allowed distribution of θ_ψ.

The experimental geometry thus leads to a correction function $C(\theta_\varphi)$ consisting of two parts, $C_1(\theta_\varphi)$ and $C_2(\theta_\varphi)$. $C_1(\theta_\varphi)$ results from the experimental geometry in the investigated scattering plane and gives the probability that the scattering angle θ_φ will be observed if there are no geometrical limitations in the perpendicular plane.

When a particle impinges on scattering plate i at angle γ to its normal and at the distance ϵl from one end of the chamber (l is the length of the illuminated region of the chamber in the photographing plane, which is generally simply the length of the chamber) and is then scattered at angle θ_φ, then we can always find in plate $i + 1$ a length b such that if the particle passes through this length it must be stopped in plate $i - 1$ within the illuminated region of the chamber.

Then, if above the scattering plate there is a certain angular distribution $\zeta(\gamma)\,d\gamma$ of incident particles, simple geometry leads to an expression for the total relative probability of observing the angle θ_φ, as follows:

$$P(\theta_\varphi) = \int_{\varepsilon=0}^{1} \left\{ 1 - \varepsilon - \frac{\varepsilon - (1/n)\tan\theta_\varphi}{1 + n\varepsilon\tan\theta_\varphi} \right\} \frac{n\zeta[\tan^{-1}(n\varepsilon) - \theta_\varphi]}{[1 + (n\varepsilon)^2]}\, d\varepsilon, \tag{2}$$

where $n = l/a$, a is the plate separation and the correction function $C_1(\theta_\varphi)$ is given by

$$C_1(\theta_\varphi) = P(\theta_\varphi) / P(0). \tag{3}$$

It was very difficult to determine the exact angular distribution of incident particles in our apparatus; in calculating $C_1(\theta_\varphi)$ we used the experimentally observed angular distributions of incident particles. $C_1(\theta_\varphi)$ was then calculated

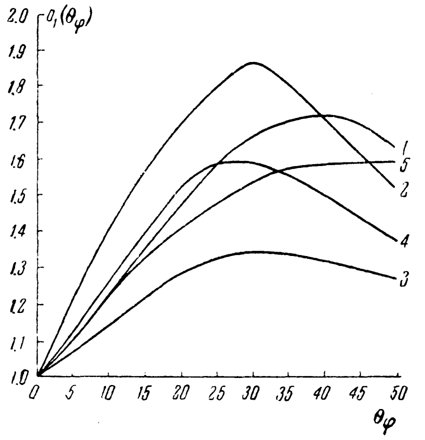

FIG. 1. Correction function C_1 (θ_φ). 1) Muons in momemtum range (1.0 to 1.4) $\times$ 10^8 ev/c in 7-mm lead plates; 2) muons in momentum range (1.4 to 1.8) $\times$ 10^8 ev/c; 3) pions in momentum range (1.2 to 1.6) $\times$ 10^8 ev/c in 7-mm lead plates; 4) pions in momentum range (1.6 to 2.0) $\times$ 10^8 ev/c; 5) C_1 (C_φ) assuming $\zeta(\gamma)\,d\gamma = \cos^2\gamma d\gamma$.

by numerical integration of (2). The angular distributions differ according to the plate number i and also for muons and pions; $C_1(\theta_\varphi)$ is therefore not identical for muons and pions in different momentum ranges. Figure 1 shows values of $C_1(\theta_\varphi)$ for muon and pion scattering in 7-mm lead plates. $C_1(\theta_\varphi)$ is also shown when we assume $\zeta(\gamma)\,d\gamma = \cos^2\gamma\, d\gamma$.

$C_2(\theta_\varphi)$ is the correction function for the experimental geometry in a plane perpendicular to the plane of investigation. The multiple Coulomb scattering curve for a finite nucleus can be approximated very accurately by a normal distribution; it is easily shown that there is identical probability of registering different angles so that $C_2(\theta_\varphi)$ is independent of θ_φ.

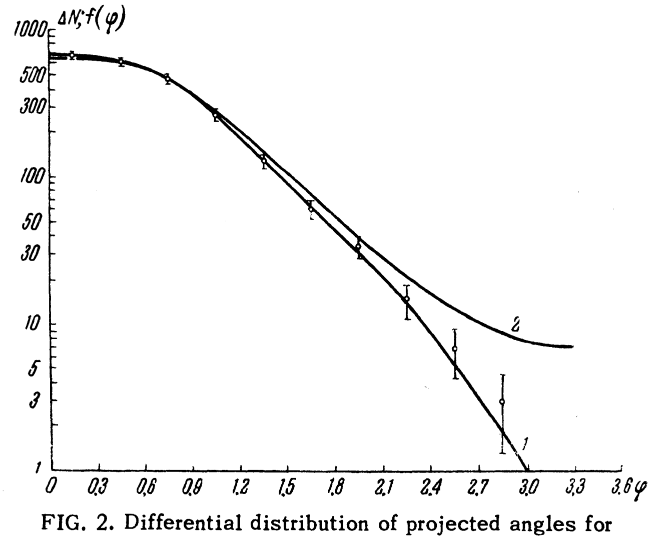

FIG. 2. Differential distribution of projected angles for muons scattered in 7-mm lead plates. Curves 1 and 2 represent multiple Coulomb scattering for an extended and a point nucleus, respectively.

Thus the multiple Coulomb scattering curves with which experimental results are to be compared need be corrected only by the function $C_1(\theta_\varphi)$.

3. EXPERIMENTAL RESULTS AND DISCUSSION

We investigated a total of 2337 muon scatterings and 818 pion scatterings in 7-mm lead plates. The total range in lead of muons with $p = (1.0 \text{ to } 1.8) \times 10^8$ ev/c is 19 m; for pions with $p = (1.2 \text{ to } 2.0) \times 10^8$ ev/c it is 6.7 m.

The experimental results for muon and pion scattering in 7-mm lead plates were compared with multiple Coulomb scattering for a point[16] and an extended[11] nucleus. Multiple Coulomb scattering curves for a finite nucleus were obtained by Ter-Mikaelian; in the calculation experimental results on the nuclear scattering of fast electrons were used.

In comparing the experimental and theoretical curves we used all corrections that were analyzed in Sec. 2. These corrections depend principally on the number of the plate in which scattering is investigated, i.e., on the momentum at the point of scattering; therefore the entire momentum range was divided into two groups, each of width 0.2×10^8 ev/c. After suitable corrections the muon and pion scattering curves were plotted for each group separately.

Figures 2 and 3 show the total scattering curves for muons and pions in lead for all groups combined. The horizontal axis represents the dimensionless quantity φ defined by

$$\varphi = \theta_\varphi / \chi_c B^{1/2},$$

where θ_φ is the projected scattering angle in degrees and $\chi_c B^{1/2}$ is the characteristic angle of multiple Coulomb scattering.[11] The advantage de-

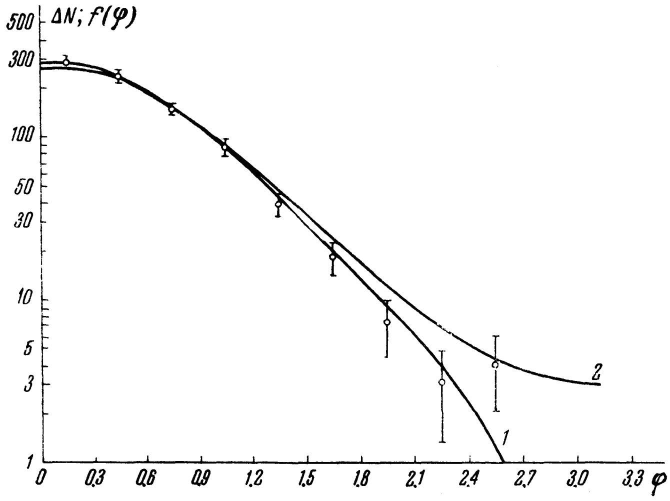

FIG. 3. Differential distribution of projected scattering angles for pions in 7-mm lead plates; 1) extended nucleus, 2) point nucleus.

rived from the use of φ is that scattering data in different momentum ranges and for different plate thicknesses can be combined.

Figures 2 and 3 show the statistical errors combined with errors in determining the correction function $C_1(\theta_\varphi)$; values of the latter were based on the experimental angular distribution of incident particles $\zeta(\gamma)\,d\gamma$. The error in measuring the scattering angle is much less than $\Delta\varphi = 0.3$ and is not shown in the figures.

It is evident from Fig. 2 that the experimental points are well fitted by the multiple Coulomb scattering curve for a finite nucleus.

A χ^2-type test showed that the experimental angular distribution is in very good agreement with theory for a finite nucleus ($P_{\chi^2} = 95\%$) and differs sharply from the curve for a point nucleus ($P_{\chi^2} = 0.33\%$).

Experimental data on pion scattering in lead also agree with the corresponding curves for multiple Coulomb scattering. It would seem at first that such agreement could not occur because of diffraction scattering of pions, but an investigation of multiple proton scattering in reference 18 showed that when the proton energy and scattering substance obey the relation $\lambda\!\!\!^{-}/R > \chi_c B^{1/2}$, which also holds true for the pions, diffraction scattering resulting from the absorption of particles by nuclei can occur only at very large angles which are outside of the investigated region.

As a control of our results we plotted the scattering curves of the muons with measured mass $< 220\ m_e$ and of pions with mass $> 270\ m_e$. These curves do not differ at all from those given above and are not reproduced here.

$\varphi >$	Number of scattered muons	
	Experimental	Theoretical
1.5	120	119.4
1.8	59	54.0
2.1	25	22.0
2.4	10	8.2
2.7	3	3.0

The magnitude of the possible anomaly can be determined from the table, which contains data (with all corrections) for the total number of scattered muons with φ larger than a given value. The small excess of the observed number of scattered muons over the values given by Ter-Mikaelyan lies within the limit of statistical fluctuations. The total amount of material traversed by the muons is 2.16×10^4 g/cm^2 of lead. It follows that the cross section for anomalous scattering, if it exists, does not exceed 10^{-28} cm^2/nucleon in the investigated momentum range.

It should be noted that if we neglect all of the corrections and refinements given in Sec. 2 (especially with reference to the geometry) we obtain strong disagreement between the experimental data and the theoretical multiple Coulomb scattering curves. In this case the experimental points also lie above the theoretical curve for a point nucleus and the cross section for "anomalous" muon scattering is $\sigma_{an} = (2 \text{ to } 5) \times 10^{-27}$ cm^2/nucleon, which has been obtained by many investigators.

In experiments on muon scattering it is very important to determine the momentum accurately at the point of scattering. We therefore cannot trust data on muon scattering at a few hundred million electron volts in which approximate momentum values were used.

Just as serious is the insufficient consideration (or, in the majority of experiments, the complete disregard) of the experimental geometry. The dimensions of the apparatus are often arbitrary and in some instances strongly distort the multiple Coulomb scattering curves. Incomplete consideration of these two factors has apparently led to "anomalous" muon scattering, at least for slow muons. In any event our data indicate that if anomalous muon scattering exists in the investigated momentum range it is smaller by a factor of tens than the observed value given by a number of investigators. This question can be answered by experiments in which single acts of scattering are observed.

Our data provide a useful indication of muon spin, which is usually assumed to be ½, although there is still no direct experimental verification. Ter-Mikaelian's[11] calculations show that for spin 3/2 the angular distribution curve lies considerably above our experimental curve.

In conclusion we wish to thank M. L. Ter-Mikaelian for very useful discussions and assistance. We are also indebted to B. A. Dolgoshein and B. I. Luchkov, who assisted with the preparations and measurements, and to M. I. Daion and V. G. Kirillov-Ugryumov for discussions.

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