DECAY OF A PHOTON INTO TWO PHOTONS IN A HOMOGENEOUS MAGNETIC FIELD

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IT follows from Furry's theorem[1] that the decay of a photon into two photons in vacuum is impossible. For decay into a larger number of photons the statistical weight of the final state goes to zero (from the laws of conservation of energy and momentum it follows that the photons produced in the decay must have the same direction of motion as the original photon). Therefore an interesting question is that of the possibility of spontaneous decay of a photon in a homogeneous external field into two photons. Since such a field cannot be regarded as a perturbation, one must use the causal Green's function that takes account of its presence.

Schwinger has shown[2] that in the presence of a weak constant electromagnetic field the electron Green's function (in the system of units $h = c = 1$) is $G(x, x') = \Phi(x, x')\, S^c(x-x')$, where

$$S^c(x) = \frac{1}{(4\pi)^2}\int_0^\infty \frac{ds}{2s^2}\left\{m - \gamma_\alpha \frac{\partial}{\partial x_\alpha},\ 1 + \frac{ie}{2}\sigma_{ik}F_{ik}s\right\} \times \exp\left(\frac{ix_\beta^2}{4s} - im^2 s - \varepsilon s\right), \tag{1}$$

$$\Phi(x, x') = \exp\left\{ie\int_{x'}^{x} dx''_\mu\left[A_\mu(x'') + \frac{1}{2}F_{\mu\nu}(x''-x')_\nu\right]\right\},$$

where $\sigma_{ik} = (i/2)[\gamma_k, \gamma_i]$, $F_{ik} = \partial A_k/\partial x_i - \partial A_i/\partial x_k$ is the tensor of the external field, m is the mass of the electron, $\epsilon > 0$, γ_α are the Hermitian Dirac matrices, and $\{a, b\} = ab + ba$. The function (1) can be used under the condition $eF/m^2 \ll 1$, which is obviously always fufilled.

In the momentum representation the function (1) has the form

$$S^c(p) = \frac{1}{(2\pi)^4}\frac{m - \hat{p}}{m^2 + p^2 - i\varepsilon} + \Delta S(p), \tag{2}$$

$$\Delta S(p) = \frac{1}{(2\pi)^4}\left\{\frac{e}{4}\sigma_{ik}F_{ik},\ \frac{m - \hat{p}}{(m^2+p^2-i\varepsilon)^2}\right\}, \tag{3}$$

where $\hat{p} = i\gamma_\alpha p_\alpha$.

Furry's theorem will be valid also in the presence of a constant external field if under the transformation of charge conjugation

$$\gamma \rightarrow \gamma' = -\gamma^T, \text{ i.e., as } \gamma_{\alpha\beta} \rightarrow -\gamma_{\beta\alpha} \tag{4}$$

we get the relation $S^c(p) = S^{c'}(p) = S^{cT}(-p)$. It is easy to show that actually $\Delta S(p) \rightarrow \Delta S'(p) = -\Delta S^T(-p)$; that is, Furry's theorem does not apply, and we can consider the decay of a photon into two photons in a homogeneous magnetic field.

The corresponding element of the scattering matrix has the form

$$S_3 = \frac{e^3}{3!}\int d^4x_1 d^4x_2 d^4x_3 \operatorname{Sp} \hat{A}(x_1)\, G(x_2, x_1) \times \hat{A}(x_2)\, G(x_3, x_2)\, \hat{A}(x_3)\, G(x_1, x_3), \tag{5}$$

$$A_\alpha(x_i) = (e_\alpha^{(i)}/\sqrt{2\omega_i})\exp(ik^{(i)}x_i),$$

where $A_\alpha(x_i)$ is the matrix element for the absorption or emission of the i-th photon with the four-momentum $k^{(i)}$ and the polarization vector $e^{(i)}$; ω_i is the frequency of the i-th photon.

The calculation of S_3 in the momentum representation leads to the expression

$$S_3 = \frac{\pi^2}{9}e^3\frac{\delta^4(k^{(1)} - k^{(2)} - k^{(3)})}{\sqrt{8\omega_1\omega_2\omega_3}}\frac{e}{m^2} \times \mathbf{H}([\mathbf{k}_1\times\mathbf{e}_1] - [\mathbf{k}_2\times\mathbf{e}_2] - [\mathbf{k}_3\times\mathbf{e}_3]). \tag{6}$$

From this it is easy to calculate the total probability of decay of the photon in unit time:

$$W = \frac{5}{3(144\pi)^2}\left(\frac{e^2}{4\pi}\right)^3\left(\frac{e}{m^2}[\mathbf{H}\times\mathbf{k}_1]\right)^2\frac{1}{\omega_1}. \tag{7}$$

In relativistically covariant form and in ordinary units Eq. (7) has the form

$$W = \frac{5}{3(144\pi)^2}\alpha^3\left(\frac{\mu_0}{mc}F_{\mu\nu}k_\nu^{(1)}\right)^2\frac{1}{\omega_1}, \tag{8}$$

where μ_0 is the Bohr magneton and $\alpha = e^2/4\pi\hbar c$ is the fine structure constant.

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[1] W. H. Furry, Phys. Rev. **51**, 125 (1937).

[2] J. Schwinger, Phys. Rev. 82, 664 (1951) (Russian translation in collection: Latest Developments in Quantum Electrodynamics, IIL 1954).

Translated by W. H. Furry
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