CASCADE CURVES FOR COPPER

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Electron cascade curves in copper are calculated for primary electron or photon energies $10^7 \lesssim E \lesssim 10^{12}$ ev by the method of moments. The calculation is performed with account taken of the dependence of the total photon absorption coefficient on energy and of Rutherford scattering of the charged particles. A brief analysis of the numerical results is carried out.

In experimental work on cosmic rays one uses copper or iron ionization chambers shielded by absorbers made of different materials. Thick layers of copper or iron are sometimes used as absorbers. For the purpose of analyzing the results obtained with such apparatus it is necessary to have quite accurate cascade curves in copper and iron* for primary electron or photon energies E_0.

Cascade curves for copper have been calculated by the method of moments.[1] In calculating the moments of the distribution function $\{N_p(E_0, 0, t)\}^{p,\Gamma}$ for electrons with energy greater than zero at depth† t in a shower generated by a primary electron or photon of energy E_0, by means of recursion formulas,[2] we took into account the energy dependence of the total photon absorption coefficient $\sigma(E)$ as well as Rutherford scattering of charged shower particles. $\sigma(E)$ for different E was taken from Heitler's book.[3] The expression for the "equilibrium" spectrum taking account of scattering, which served as the zero moment, was taken from the paper of Belen'kii and Maksimov.[2] The cascade unit (t-unit) in copper was taken to be 11.6 g/cm^2 with the critical energy $\beta = 16.6$ Mev. Results for the first two moments $\bar{t}(\epsilon_0, 0)$ and $\bar{t^2}(\epsilon_0, 0)$ with primary particle energies‡ ϵ_0 from 2 to 1.4×10^5 are given in the table.

The values obtained for $\bar{t}$ and $\bar{t^2}$ were used to plot cascade curves. In earlier papers[1] a method was presented for computing cascade curves and the energy spectrum of particles generated by primary electrons or photons, for not-very-high energies in both light and heavy substances. In the present paper we shall compute cascade curves for copper over a broad energy range of the primary particles. Therefore in selecting a weighting function we must use more information regarding the behavior of the cascade curve. Cascade curves for a primary photon are approximated by means of polynomials which are orthogonal in the interval $(0, \infty)$ with the weighting function*

$$w(t) = \gamma^{i+1} t^i e^{-\gamma t} / \Gamma(i+1). \qquad (1)$$

Here $\gamma = 0.376$ is the minimum value of the total photon absorption coefficient in copper. We determine i by equating the first moment of the weighting function to the sought distribution function. The ordinary theory of orthogonal polynomials[4] was used to construct an orthogonal system of polynomials with weighting w(t) in the interval $(0, \infty)$.

Figures 1 and 2 show the resulting curves. It may be noted that for $\epsilon_0 \sim 1$ we obtain $i < 1$. In this case $dN/dt|_{t=0} = \infty$, so that it is more convenient to assume $i = 1$ and to determine γ accordingly. For $\epsilon_0 = 2$, 6, 10, and 14 we obtain $\gamma = 0.64$, 0.55, 0.51, and 0.47, respectively. The series obtained in this way converges rapidly and $\{N_p\}^{\Gamma}$ for $\epsilon_0 = 14$ practically coincides with the value obtained with $\gamma = \sigma_{min}$.

The cascade curves for a primary electron were approximated by means of polynomials that were orthogonal in the interval $(0, \infty)$ with the weighting function

$$w(t) = \gamma^{i+1}(a + t^i) e^{-\gamma t} / (\Gamma(i+1) + a\gamma^i). \qquad (2)$$

Here i and a are determined by equating the

*Z of copper is ~ 10% different for that of iron, so that the derived cascade curves can also be used for calculations of cascade processes in iron.

†In cascade theory depth is measured in the so-called "cascade unit" of length.[3]

‡ϵ is the particle energy measured in units of β/q, where β is the critical energy for the given material and q is a constant equal to 2.29

*For convenience in subsequent calculations, the weighting function w(t) will always be assumed to be normalized to unity.

ε_0	$\{\bar{t}_p(\varepsilon_0, 0)\}^p$	$\{\overline{t_p^2}(\varepsilon_0, 0)\}^p$	$\{\bar{t}_p(\varepsilon_0, 0)\}^\Gamma$	$\{\overline{t_p^2}(\varepsilon_0, 0)\}^\Gamma$	ε_0	$\{\bar{t}_p(\varepsilon_0, 0)\}^p$	$\{\overline{t_p^2}(\varepsilon_0, 0)\}^p$	$\{\bar{t}_p(\varepsilon_0, 0)\}^\Gamma$	$\{\overline{t_p^2}(\varepsilon_0, 0)\}^\Gamma$
2	0,96	2.99	3.13	17.8	$3.5\cdot10^2$	6,62	56.4	7.48	69.7
6	2.02	8.22	3.61	21.2	$5.6\cdot10^2$	7.10	64.0	7.92	77.1
10	2.64	12.1	3.94	23.8	$7.7\cdot10^2$	7.42	69.2	8.22	82.7
14	3.06	15.1	4.24	26.2	$9.8\cdot10^2$	7.66	73.3	8.47	87.2
35	4.13	24.9	5.16	36.3	$1.4\cdot10^3$	8.01	79.5	8.83	94.1
56	4.66	30.6	5.65	42.3	$7.4\cdot10^3$	9.68	112	10.5	129
77	5.01	34.6	5.97	46.7	$1.4\cdot10^4$	10.3	126	11.1	144
98	5.27	37.9	6.22	50.1	$7.4\cdot10^4$	12.0	166	12.8	187
$1.4\cdot10^2$	5.66	42.9	6.58	55.3	$1.4\cdot10^5$	12.6	183	13.4	205

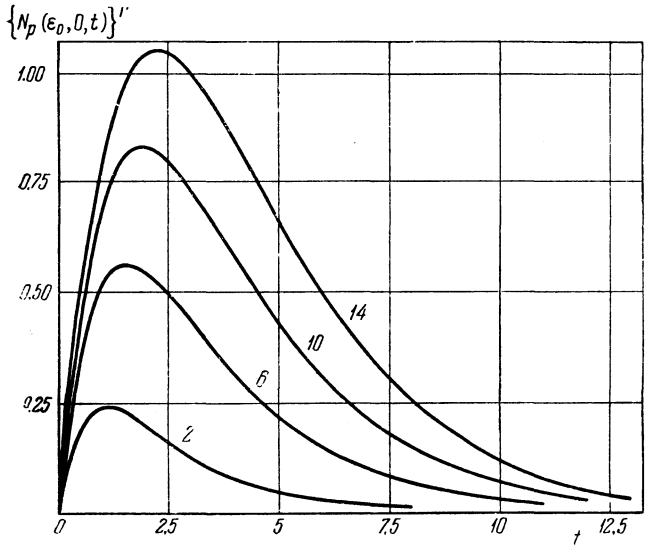

FIG. 1. Cascade curves in copper for a primary photon. The numerals over the curves denote the primary photon energy in units of β/q.

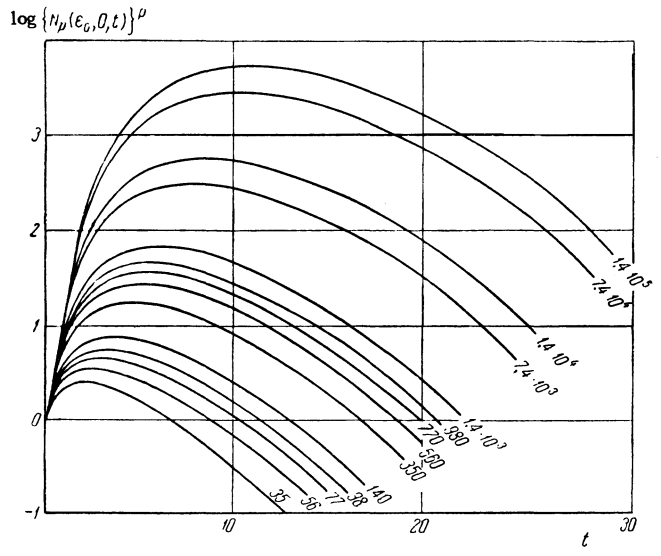

FIG. 2. The same as in Fig. 1.

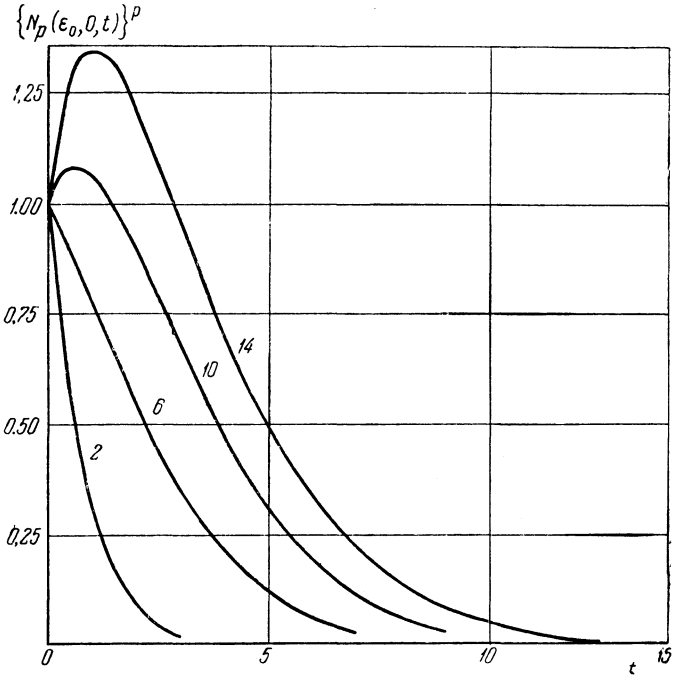

FIG. 3. Cascade curves in copper for a primary electron. The numerals over the curves denote the primary electron energy in units of β/q.

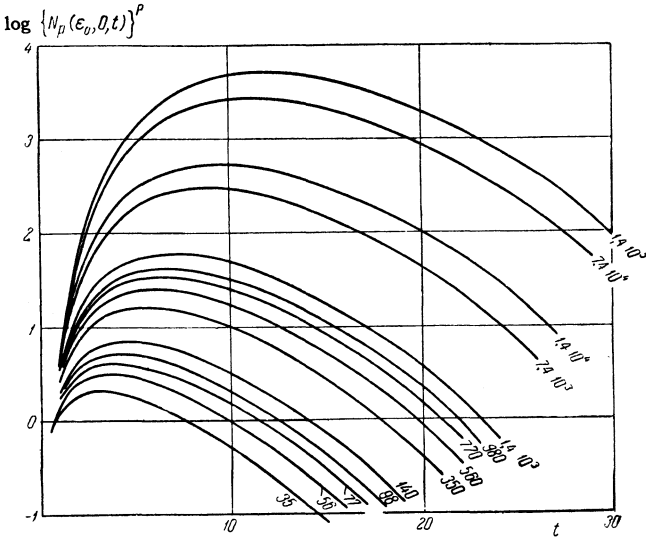

FIG. 4. The same as in Fig. 3.

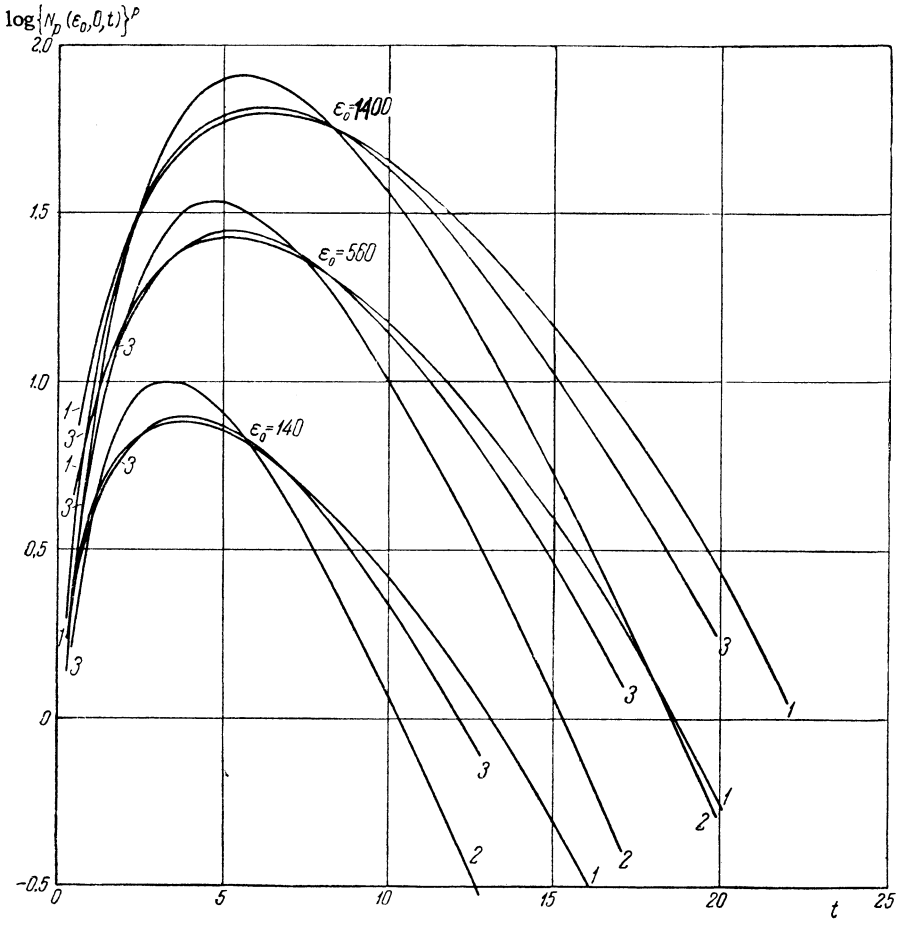

FIG. 5. Cascade curves in copper for a primary electron with several values of ε_0. Curves 1 were calculated by the method of moments, curves 2 from the equations of Belen'kii and Maksimov[2] and curves 3 from Ott's equations.[6]

first moments of the weighting function to the value $\tilde{t}(\epsilon_0, 0)$ of the sought distribution function and from the boundary condition $\{N_p(\epsilon_0, 0, 0)\}^p = 1$.

The resulting curves are shown in Figs. 3 and 4. For $\epsilon_0 \sim 1$ we obtain $0 < i < 1$. To avoid obtaining an infinite derivative at $t = 0$, in the first few curves we assume i to be equal to the nearest integer, after which a is determined. As in the case of a primary photon the series converge quite rapidly, owing to the proper choice of a weighting function that provides a good representation of all essential features of the distribution function. We note that by means of only a slight complication this method can be used to obtain cascade curves that are more exact for small values of t. For this purpose it is sufficient to introduce one additional parameter into the weighting function. The values of this parameter can be determined from the boundary condition at $t = 0$ for the derivative of the cascade curve with respect to t, which can be obtained from the equations of the cascade theory. An investigation of the computed functions (see reference 1, for example, which contains a detailed analysis of similar functions) shows that in the present case over the entire interval of primary energies and depths with the exception of the first cascade unit the error in the cascade curves does not exceed 10%.

It is of interest to compare our results with the cascade curves of ordinary shower theory[5] which do not take into account the energy dependence of the total photon absorption coefficient nor Rutherford scattering, and with the cascade curves obtained for copper from the approximate formula of K. Ott (see reference 6). Figure 5 shows that curves 1 and 3 are in good agreement around the maximum, while curves 1 and 2 differ by a factor of 1.5, with the difference diminishing as ϵ_0 increases. In the first few cascade t-units the curves differ by a factor of about 1.3. The tails of curves 1 and 3 [where $N(t) \sim 1$] differ by a factor of 1.4 to 1.7. Thus Ott's approximate equation[6] satisfactorily describes the development of the cascade process in copper only in the vicinity of the shower maximum. Bernstein[7] solved the cascade equations by successive approximations, the zero approximation being that obtained by using asymptotic expressions for the cross sections of elementary processes.[5] A correction was calculated by means of Bethe and Heitler's more exact

approximation for the cross sections. However the results are valid only when there is little difference between the exact and asymptotic cross sections and become incorrect when $Z \approx 30$. The tails of curves 1 and 2 differ by a factor of 2.5 to 4.5 and the difference increases with ϵ_0 as in the case of curves 1 and 3.

Belen'kii[5] obtained the following equations for the position and number of particles of a shower maximum:

$$t_m = \bar{t}, \quad N_m = (E_0/\beta)[2\pi(\overline{t^2} - \bar{t}^2)]^{-1/2}. \qquad (3)$$

A comparison of the values of t_m and N_m for the curves of Figs. 1 to 4 with the values calculated from (3) by means of the table shows that (3) can be used for t_m when $\epsilon_0 > 10^2$ and for N_m when $\epsilon_0 > 10^3$. The accuracy of (3) is somewhat diminished for cascade curves from a primary photon.

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