ON THE THEORY OF PARAMAGNETIC RESONANCE AND PARAMAGNETIC RELAXATION IN METALS

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A kinetic equation is obtained for conduction electrons in metals, which is valid for the investigation of paramagnetic resonance. The case of uniform distribution of an alternating field in a metal is considered in detail. It is shown that for temperatures $kT \gg \mu H_0$, "longitudinal" and "transverse" spin relaxation times can be introduced, these times being practically identical.

1. The recently developed theory of paramagnetic resonance in metals is based either on the use of a representation of the diffusion of electrons from the skin depth[1] or on the use of the kinetic equation for the electron density operator,[2,3] where both the thermal and the spin collision intervals are described phenomenologically: $(f-f_0)/\tau$. In all these researches, the spin and thermal relaxation times entered as parameters. In references 2 and 3, moreover, no distinction is made between "transverse" and "longitudinal" relaxation times, although this is not evident beforehand. The spin relaxation time obtained from experiments on paramagnetic resonance (on resonance width) is "transverse," which was not computed theoretically. Existing data in the literature refer only to "longitudinal" times.[4] It would therefore be appropriate to obtain the equation for paramagnetic resonance in which the collision integral is described in microscopic fashion, and to show the possibility of introducing "longitudinal" and "transverse" spin relaxation times.

In obtaining such an equation, the problem of the choice of the mechanism according to which the perturbed spin system returns to equilibrium is of importance. In the researches of Overhauser[4] and Elliott,[5] different types of spin interactions were investigated; however, only the spin orbit coupling of the electrons with the lattice which was considered by Elliott leads to agreement with the spin relaxation times computed from the experiments. Below, therefore, we shall consider as responsible for the spin relaxation a mechanism associated with consideration of the effect of spin orbit coupling on the interaction of the electrons with the lattice vibrations.

2. To obtain the kinetic equation, we make use of the method of statistical operators for quantum systems.[6] Application of this method to obtain the kinetic equation for conduction electrons without account of their spin was made by Gurzhi.[7] We shall carry out a similar derivation with account of the spins of the electrons and spin orbit interaction of electrons with the periodic field of the lattice.

Let us consider a system of N non-interacting electrons and S phonons. The electrons are located in the periodic field of the lattice and in a constant magnetic and variable electromagnetic external fields. The complete Hamiltonian of such a system is

$$\hat{\mathcal{H}} = \sum_{k=1}^{N} \hat{\mathcal{H}}(\bar{x}_k) + \sum_{k=1}^{S} \hat{\mathcal{H}}(\bar{y}_k) + \sum_{i=1}^{N}\sum_{k=1}^{S} \hat{U}(\bar{x}_i, \bar{y}_k);$$

$$\hat{\mathcal{H}}(\bar{x}) = \frac{1}{2m}\left(\hat{\mathbf{P}} + \frac{e}{c}\mathbf{A}\right)^2 + V(\mathbf{r}) + \mu\hat{\sigma}\cdot\mathbf{H} + \frac{\mu}{2mc}[\nabla V \times \hat{\mathbf{P}}]\cdot\hat{\sigma};$$

$$\hat{\mathcal{H}}(\bar{y}) = \frac{1}{2} h\nu_{q\lambda}(\hat{b}^{+}_{q\lambda}\hat{b}_{q\lambda} + \hat{b}_{q\lambda}\hat{b}^{+}_{q\lambda});$$

$$\hat{U}(\bar{x}, \bar{y}) = \left(\frac{\Delta h}{2V_0 M\nu_{q\lambda}}\right)^{1/2}(\mathbf{e}_{q\lambda}\cdot\nabla V)(\hat{b}^{+}_{q\lambda}e^{-i\mathbf{q}\cdot\mathbf{r}/\hbar} + \hat{b}_{q\lambda}e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}),$$

where $\bar{x}$ and $\bar{y}$ represent, respectively, the set of coordinates and spin of the electron $(\mathbf{r}\sigma)$ and the momentum and polarization of the phonon $(\mathbf{q}\lambda)$; $\mathbf{e}_{q\lambda}$ is the polarization vector of the phonon; M is the mass of the atom; $V(\mathbf{r})$ is the periodic field of the lattice; $h\nu_{q\lambda}$ is the energy of the phonon; $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1(\mathbf{r}, t)$ is the vector potential of the external field; $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1(\mathbf{r}, t)$, where $\mathbf{H}_0$ and $\mathbf{H}_1$ are the constant and variable magnetic field; Δ is the volume of an elementary cell; V_0 is the volume of the crystal; $\mu = |e|\hbar/2mc$;

$\hat{b}^+_{q\lambda}$ and $\hat{b}_{q\lambda}$ are the creation and annihilation operators of the phonon with momentum q and polarization λ. The state of the system is described by the statistical operator F, which satisfies the equation

$$\partial\hat{F}/\partial t = [\hat{\mathcal{H}}, \hat{F}] \equiv (1/i\hbar)(\hat{\mathcal{H}}\hat{F} - \hat{F}\hat{\mathcal{H}}).$$

For the single particle operator $\hat{F}(\bar{x},t)$ similar to what was done in reference 7 and under the same assumptions, we obtain the equation

$$\frac{\partial\hat{F}(\bar{x},t)}{\partial t} = [\hat{\mathcal{H}}(\bar{x}), \hat{F}(\bar{x},t)] + \sum_{k=1}^{S} \mathrm{Sp}_{(\bar{y}_k)}[\hat{U}(\bar{x},\bar{y}_k), \hat{G}(\bar{x},\bar{y}_k,t)],$$
$$\hat{G}(t) = -\int_0^\infty d\tau \cdot \exp\left\{\frac{i}{\hbar}\int_t^{t+\tau} dt'[\hat{\mathcal{H}}(\bar{x}) + \hat{\mathcal{H}}(\bar{y})]\right\}\hat{B}(t+\tau) \quad (1)$$
$$\times \exp\left\{-\frac{i}{\hbar}\int_t^{t+\tau} dt'[\hat{\mathcal{H}}(\bar{x}) + \hat{\mathcal{H}}(\bar{y})]\right\}.$$

Here

$$\hat{B}(\bar{x}_1,\bar{y},t) = [\hat{U}(\bar{x}_1,\bar{y}), \hat{F}(\bar{x}_1,t)\hat{F}(\bar{y})]$$
$$- N\mathrm{Sp}_{(\bar{x}_2)}[\hat{U}(\bar{x}_2,\bar{y}), \hat{P}_{12}\hat{F}(\bar{x}_1,t)\hat{F}(\bar{x}_2,t)\hat{F}(\bar{y})]$$

and it is assumed that the lattice is in thermal equilibrium:

$$(N_{q\lambda}|\hat{F}(\bar{y})|N'_{q\lambda}) = \delta_{N_{q\lambda}N'_{q\lambda}}\delta_{N_{q\lambda}\bar{N}_{q\lambda}};\ \bar{N}_{q\lambda} = [\exp(h\nu_{q\lambda}/kT) - 1]^{-1}.$$

$\hat{P}_{12}$ is the permutation operator of the particles:

$$(n_1n_2|\hat{P}_{12}\hat{F}(\bar{x}_1)\hat{F}(\bar{x}_2)|n'_1n'_2) = (n_1|\hat{F}|n'_2)(n_2|\hat{F}|n'_1).$$

In the collision integral, we limit ourselves to terms of zero order in the vector potential $\mathbf{A}_1(\mathbf{r},t)$. Moreover, we shall not take into consideration the effect of the constant magnetic field on the orbital motion of the electrons which, in any case, is valid if the turning radius of the electron in the magnetic field is much longer than the mean free path. Then the matrix elements of the operator $\hat{G}$ in the proper representation of the operator

$$\frac{\hat{p}^2}{2m} + V(\mathbf{r}) + \frac{\mu}{2mc}[\nabla V\times\hat{\mathbf{p}}]\cdot\hat{\sigma} + \mu\hat{\sigma}\cdot\mathbf{H}_0 + \hat{\mathcal{H}}(\bar{y}) \equiv \hat{\mathcal{H}}_0(\bar{x}) + \hat{\mathcal{H}}(\bar{y})$$

will be:

$$(\mathbf{p}\sigma N_{q\lambda}|\hat{G}|\mathbf{p}'\sigma'N'_{q\lambda})$$
$$= -\int_0^\infty d\tau\cdot\exp\left\{\frac{i\tau}{\hbar}[E_{p\sigma} - E_{p'\sigma'} + (N_{q\lambda} - N'_{q\lambda})h\nu_{q\lambda}]\right\} \quad (2)$$
$$\times(\mathbf{p}\sigma N_{q\lambda}|\hat{B}(t+\tau)|\mathbf{p}'\sigma'N'_{q\lambda}),$$

where E_{ps} are the eigenvalues of the operator $\hat{\mathcal{H}}_0(\mathbf{x})$. The eigenfunctions

$$\psi_{p\sigma} = \sqrt{V_0/\Delta}\,u_{p\sigma}e^{i\mathbf{p}\cdot\mathbf{r}/\hbar},$$

where u_{ps} are periodic with the period of the lattice. By virtue of this fact the matrix elements of the operator $\hat{U}(\bar{x},\bar{y})$ in the representation of $\psi_{p\sigma}$ will be (neglecting processes of "transfer" and transition from zone to zone):

$$(\mathbf{p}\sigma N_{q\lambda}|\hat{U}|\mathbf{p}'\sigma'N'_{q\lambda})$$
$$= (N_{q\lambda}|\hat{b}^+_{q\lambda}|N'_{p\lambda})A^{p\sigma}_{p'\sigma'}\delta_{p'-p-q} + (N_{q\lambda}|b_{q\lambda}|N'_{q\lambda})A^{p\sigma}_{p'\sigma'}\delta_{p'-p+q};$$
$$A^{p\sigma}_{p'\sigma'} = \left(\frac{\Delta h}{2V_0M\nu_{q\lambda}}\right)^{1/2}\int_{(\Delta)} u^*_{p\sigma}(\mathbf{r})(\mathbf{e}_{q\lambda}\nabla V)u_{p'\sigma'}(\mathbf{r})\,d\mathbf{r}. \quad (3)$$

We shall now obtain the kinetic equation. We shall consider the case in which the variable field inside the metal can be considered homogeneous. Such a consideration is suitable if the dimensions of the specimen d are much smaller than the skin depth. Then the matrix of the operator $\hat{F}(\bar{x})$ in the representation of the function $\psi_{p\sigma}$ will be diagonal in the momentum:

$$(\mathbf{P}\sigma|\hat{F}|\mathbf{P}'\sigma') = N^{-1}f_{\sigma\sigma'}(\mathbf{P})\,\delta_{\mathbf{P}-\mathbf{P}'}. \quad (4)$$

In the presence of an electromagnetic field, the distribution function depends upon the generalized momentum $\mathbf{P} = \mathbf{p} - e\mathbf{A}/c$. However, it is appropriate to transform to the distribution function which depends on the kinetic momentum p. In such a transition,

$$\dot{f}(\mathbf{P}) = \dot{f}(\mathbf{p}) + \frac{e}{c}\dot{A}_1(t)\frac{\partial f(\mathbf{p})}{\partial\mathbf{p}}.$$

Making use of (2) to (4) for the functions $f_{\sigma\sigma'}$ we obtain from (1) a set of four equations:

$$\frac{\partial f_{\sigma\sigma'}(\mathbf{p})}{\partial t} + \frac{e}{c}\dot{A}_1(t)\frac{\partial f_{\sigma\sigma'}}{\partial\mathbf{p}} = (\mathbf{p}\sigma|\hat{\mathcal{H}}_0(\bar{x})|\mathbf{p}\sigma'')f_{\sigma''\sigma'}(\mathbf{p})$$
$$- f_{\sigma\sigma''}(\mathbf{p})(\mathbf{p}\sigma''|\hat{\mathcal{H}}_0(\bar{x})|\mathbf{p}\sigma')$$
$$+ \frac{1}{\hbar^2}\int_0^\infty d\tau\sum_{\mathbf{q}}\sum_{\lambda=1}^{3}\left\{\exp\left[\frac{i\tau}{\hbar}(E_{p\sigma} - E_{p+q\sigma''} - h\nu_{q\lambda})\right]\right. \quad (5)$$
$$\times F^{(1)}_{\sigma\sigma'} + \exp\left[\frac{i\tau}{\hbar}(E_{p+q\sigma''} - E_{p\sigma'} + h\nu_{q\lambda})\right]F^{(2)}_{\sigma\sigma'} + \exp\left[\frac{i\tau}{\hbar}(E_{p\sigma}\right.$$
$$\left.\left. - E_{p+q\sigma''} + h\nu_{q\lambda})\right]F^{(3)}_{\sigma\sigma'} + \exp\left[\frac{i\tau}{\hbar}(E_{p+q\sigma''} - E_{p\sigma'} - h\nu_{q\lambda})\right]F^{(4)}_{\sigma\sigma'}\right\},$$

where

$$F^{(1)}_{\sigma\sigma'} = N_{q\lambda}A^{p\sigma}_{p+q\sigma'''}A^{p+q\sigma''}_{p\sigma'}f_{\sigma'''\sigma''}(\mathbf{p}+\mathbf{q}) - (N_{q\lambda}+1)$$
$$\times A^{p\sigma'''}_{p+q\sigma''}A^{p+q\sigma''}_{p\sigma'}f_{\sigma\sigma'''}(\mathbf{p}) + A^{p\sigma_2}_{p+q\sigma'_2}A^{p+q\sigma''}_{p\sigma'}f_{\sigma\sigma_2}(\mathbf{p})f_{\sigma'_2\sigma''}(\mathbf{p}+\mathbf{q});$$

$$F^{(2)}_{\sigma\sigma'} = N_{q\lambda}A^{p\sigma}_{p+q\sigma''}A^{p+q\sigma'''}_{p\sigma'}f_{\sigma''\sigma'''}(\mathbf{p}+\mathbf{q}) - (N_{q\lambda}+1)$$
$$\times A^{p\sigma}_{p+q\sigma''}A^{p+q\sigma''}_{p\sigma'''}f_{\sigma'''\sigma'}(\mathbf{p}) + A^{p\sigma}_{p+q\sigma''}A^{p+q\sigma_2}_{p\sigma'_2}f_{\sigma'_2\sigma'}(\mathbf{p})f_{\sigma''\sigma_2}(\mathbf{p}+\mathbf{q}).$$

Repeated index in σ indicates summation. The expressions for $F^{(3)}_{\sigma\sigma'}$ and $F^{(4)}_{\sigma\sigma'}$ are obtained from $F^{(1)}_{\sigma\sigma'}$ and $F^{(2)}_{\sigma\sigma'}$ by the substitution $N_{q\lambda} \rightleftarrows N_{q\lambda} + 1$ and by change of sign for the term which is nonlinear in f.

We note that in the quasi-classical approximation on the coordinates and momenta of the electrons, the resulting equations are applicable also to the case of an inhomogeneous distribution of the variable field in the metal, i.e., in analysis of the skin effect. In this case it suffices to substitute the commutator $[\hat{\mathcal{H}}_0(\bar{x}), \hat{F}(\bar{x})]$ which enters into (5) for the classical Poisson bracket:

$$\mathbf{v}\frac{\partial f_{\sigma\sigma'}}{\partial \mathbf{r}} + \frac{e}{c}[\mathbf{v}\times\mathbf{H}_0]\frac{\partial f_{\sigma\sigma'}}{\partial \mathbf{p}}$$

$$+ (\sigma|\mu\hat{\sigma}\cdot\mathbf{H}|\sigma'')f_{\sigma''\sigma'} - f_{\sigma\sigma''}(\sigma''|\mu\hat{\sigma}\cdot\mathbf{H}|\sigma'),$$

while the collision integral remains as before (taking into consideration the fact that the desired functions depend on the coordinates). The collision integral remains as before because the matrix elements $(\mathbf{p}\sigma|\hat{F}|\mathbf{p}'\sigma')$, which are nondiagonal in the momentum, are small since the dimensions of the inhomogeneity of the field (skin depth) are always large in comparison with the de Broglie wavelength of the electron $\hbar/p$ (p extends to the limiting Fermi surface since only electrons with energies close to the limiting energy ϵ_0 make a contribution to the resonance).

3. To simplify Eq. (5), it is necessary to know the form of the matrix elements $A^{\mathbf{p}\sigma}_{\mathbf{p}'\sigma'}$. Here we limit ourselves to the alkali metals, for which the wave functions $\psi_{\mathbf{p}\sigma}$ with consideration of spin orbit coupling were computed by Yafet:[8]

$$\hbar u_{p1,2} = \{\hbar u_0 + i(\mathbf{p}\cdot\mathbf{r})u_1$$

$$\pm C_1[\mathbf{p}\cdot\mathbf{r}]_z u_2\}\chi_{1/2,-1/2} + C_2[\mathbf{p}\cdot\mathbf{r}]_{x\pm iy}u_3\chi_{-1/2,1/2},$$

where $\chi_{1/2}$ and $\chi_{-1/2}$ are the eigenfunctions of the operator $\hat{\sigma}_z$; the z axis is directed along the field H_0. It was shown in reference 5 that $u_1(r) \sim u_2(r) \sim u_3(r)$, and $C_1 \sim C_2 \sim g - 2$, where g is the splitting factor of the conduction electron. The matrix elements consequently will be:

$$-(A^{p2}_{p'2})^* = A^{p1}_{p'1} = \left(\frac{\Delta}{2V_0 M\hbar\nu_{q\lambda}}\right)^{1/2} C\{i\,\mathbf{e}_{q\lambda}(\mathbf{p}'-\mathbf{p})$$

$$+ C_1[\mathbf{e}_{q\lambda}\times(\mathbf{p}'+\mathbf{p})]_z\};$$

$$A^{p1}_{p'2} = \left(\frac{\Delta}{2V_0 M\hbar\nu_{q\lambda}}\right)^{1/2} CC_1\{[\mathbf{e}_{q\lambda}\times(\mathbf{p}'+\mathbf{p})]_x - i[\mathbf{e}_{q\lambda}\times(\mathbf{p}'+\mathbf{p})]_y\};$$

$$C = \int_{(\Delta)} u_0(r)u_1(r)(\nabla V\cdot\mathbf{r})\,d\mathbf{r}.$$

the quantity C is of the order of the mean energy of the electron in the crystal.

In the following we shall assume a simple Debye model for the lattice vibrations, in which the vector $\mathbf{e}_{q1}$ is parallel to the momentum of the phonon $\mathbf{q}$ while $\mathbf{e}_{q2}$ and $\mathbf{q}$ are mutually orthogonal. We transform from a summation over $\mathbf{q}$ to integration:

$$\sum_{\mathbf{q}} \to \frac{V_0}{(2\pi\hbar)^3}\int_{q_{min}}^{q_{max}} q^2 dq \int d\Omega_q,\quad q_{max} = \frac{2\pi\hbar}{a}\left(\frac{3}{4\pi}\right)^{1/3};$$

a is the lattice constant; q_{min} is determined from the law of conservation of energy. Moreover, for simplicity, we do not distinguish between transverse and longitudinal sound velocities: $v_\perp = v_\parallel = v_s$ (i.e., the phonon energies $h\nu_{q\lambda} = v_s q$, $\lambda = 1, 2, 3$). We also assume that $E_{p1} = \epsilon - \mu H_0$, $E_{p2} = \epsilon + \mu H_0$, $\epsilon = p^2/2m^*$, m^* is the effective mass of the electron.

It is not difficult to prove that in the equations for $2f_z = f_{11} - f_{22}$ and f_{12}, we can discard terms with $\dot{A}$. In this case the desired functions will depend only on $|\mathbf{p}|$, inasmuch as it is assumed that $\epsilon = \epsilon(|\mathbf{p}|)$. We can then show that the terms in the collision integral containing matrix elements of the type $A^{p1}_{p'2}A^{p'1}_{p2}$ and $A^{p1}_{p'2}A^{p'1}_{p2}$ disappear in the integration over $\mathbf{p}$ and $\mathbf{q}$. The equations in this case are significantly simplified.

4. Multiplying the equations for f_{11} and f_{22} by $\mu(2\pi\hbar)^{-3}d\mathbf{p}$ and integrating over the entire momentum space, we obtain for the "longitudinal" component of the magnetization

$$M_z = \frac{\mu}{(2\pi\hbar)^3}\int(f_{11}-f_{22})\,d\mathbf{p} \equiv \frac{2\mu}{(2\pi\hbar)^3}\int f_z d\mathbf{p}$$

the equation

$$\frac{\partial M_z}{\partial t} = \frac{2\mu}{\hbar}\mathrm{Im}[(H_{1x}-iH_{1y})M^*_{12}]$$

$$+ \frac{4\mu}{\hbar^2(2\pi\hbar)^6}\int_0^\infty d\tau\int d\mathbf{p}\int d\mathbf{q}\sum_{\lambda=1}^{3}|A^{p1}_{p+q2}|^2$$

$$\times\Big\{\cos\frac{\tau}{\hbar}(E_{p1}-E_{p+q2}-v_s q)[N_{q\lambda}f_{22}(\mathbf{p}+\mathbf{q})$$

$$-(N_{q\lambda}+1)f_{11}(\mathbf{p})+f_{11}(\mathbf{p})f_{22}(\mathbf{p}+\mathbf{q})] \qquad (6)$$

$$+\cos\frac{\tau}{\hbar}(E_{p1}-E_{p+q2}+v_s q)[(N_{q\lambda}+1)f_{22}(\mathbf{p}+\mathbf{q})$$

$$-N_{q\lambda}f_{11}(\mathbf{p})-f_{11}(\mathbf{p})f_{22}(\mathbf{p}+\mathbf{q})]\Big\}.$$

Here,

$$M_{12} = \frac{2\mu}{(2\pi\hbar)^3}\int f_{12}d\mathbf{p} \equiv \frac{\mu}{(2\pi\hbar)^3}\int(f_x - if_y)\,d\mathbf{p} \equiv M_x - iM_y.$$

Inasmuch as thermal equilibrium is obtained much more rapidly than spin equilibrium, we can make use of the representation of partial equilibrium, i.e., we can write the functions f_{11} and f_{22} which enter into (6) as

$$f_{11}(\mathbf{p}) = f_0(\varepsilon - \mu H_0 + \Delta\varepsilon);\ f_{22}(\mathbf{p}) = f_0(\varepsilon + \mu H_0 - \Delta\varepsilon);$$

$$f_0(x) = (e^{(x-\varepsilon_0)/kT}+1)^{-1}, \qquad (7)$$

where $\Delta\epsilon \to 0$ as the system tends toward complete equilibrium; $\Delta\epsilon \leq \mu H_0$. We then obtain

$$M_{0z} - M_z = \Delta\varepsilon(\mu m^* p_0/\pi^2\hbar^3) \equiv (\chi/\mu)\Delta\varepsilon;\quad M_{0z} = \chi H_0. \qquad (8)$$

the solutions for M_z must be time independent if we take a rotating magnetic field: $H_{1x} - iH_{1y} = H_{10}e^{-i\omega t}$ and the solution M_{12} must be sought in the form $M_{12}e^{-i\omega t}$. Then, carrying out the integration in (6) over τ and over all angles except the angle between $\mathbf{p}$ and $\mathbf{q}$ we get after summation over λ:

$$\begin{aligned}
&-\operatorname{Im}(H_{10}M_{12}^*)\\
&=\frac{4(2\pi)^3C^2C_1^2}{3Mv_s(2\pi\hbar)^6}\int_0^\infty p^2dp\int_{q_{\min}}^{q_{\max}}qdq\int_0^\pi(2\mathbf{p}+\mathbf{q})^2\sin\theta d\theta\\
&\times\left\{\delta\left(\frac{pq}{m^*}\cos\theta+\frac{q^2}{2m^*}+2\mu H_0+v_sq\right)[N_qf_{22}(\mathbf{p}+\mathbf{q})\right.\\
&-(N_q+1)f_{11}(\mathbf{p})+f_{11}(\mathbf{p})\times f_{22}(\mathbf{p}+\mathbf{q})]\\
&+\delta\left(\frac{pq}{m^*}\cos\theta+\frac{q^2}{2m^*}+2\mu H_0-v_sq\right)[(N_q+1)f_{22}(\mathbf{p}+\mathbf{q})\\
&\left.-N_qf_{11}(\mathbf{p})-f_{11}(\mathbf{p})f_{22}(\mathbf{p}+\mathbf{q})]\right\}.
\end{aligned}\tag{9}$$

After integration over θ, the right hand side of (9) reduces to a sum of two integrals in which the regions of integration of p and q are bounded by the inequalities

$$|q/2p+2\mu H_0m^*/pq\pm m^*v_s/p|\leqslant 1$$

by virtue of the fact that $|\cos\theta|\leq 1$. The values of p close to p_0, the limiting momentum of the electron, make the principal contribution to the integrals. Therefore the inequalities are automatically satisfied while the condition

$$q_{\min}\approx 2\mu H_0m^*/(p_0\pm m^*v_s)$$

applies to q. Inasmuch as $p_0 \gg m^*v_S$, we can consider the regions of integration in the two integrals to be identical. On integration over θ we obtain the expressions $4p^2 - q^2 - 8\mu H_0m^* \pm 4m^*v_Sq$ under the integral sign. These expressions we can set equal to $4p^2$, since the contribution of the other terms in the integral is significantly less than the contribution from $4p^2$. Thus Eq. (9) reduces to

$$\begin{aligned}
&-\operatorname{Im}(H_{10}M_{12}^*)\\
&=\frac{16(2\pi)^3m^*C^2C_1^2}{3Mv(2\pi\hbar)^6}\int_0^\infty p^3dp\int_{q_{\min}}^{\max}dq\,(e^{\Delta\varepsilon/kT}-e^{-\Delta\varepsilon/kT})\\
&\times\exp\left\{\frac{\varepsilon-\mu H_0-\varepsilon_0}{kT}\right\}f_0(\varepsilon-\mu H_0+\Delta\varepsilon)\\
&\times\{N_qf_0(\varepsilon-\mu H_0-\Delta\varepsilon-v_sq)\\
&+(N_q+1)f_0(\varepsilon-\mu H_0-\Delta\varepsilon+v_sq)\}.
\end{aligned}\tag{10}$$

for $\Delta\epsilon \ll kT$ we can introduce the relaxation time. Actually, expanding the function under the integral sign in $\Delta\epsilon/kT$ and keeping only terms of the expansion that are linear in $\Delta\epsilon$ we get, after regrouping,

$$\begin{aligned}
&\frac{2\Delta\varepsilon}{kT}\exp\left\{\frac{\varepsilon-\mu H_0-\varepsilon_0}{kT}\right\}f_0^{-2}(\varepsilon-\mu H_0)\{2N_q+1\\
&+f_0(\varepsilon-\mu H_0+v_sq)-f_0(\varepsilon-\mu H_0-v_sq)\}.
\end{aligned}\tag{11}$$

If we make use of the fact that

$$\frac{1}{kT}\exp\left(\frac{\varepsilon-\mu H_0-\varepsilon_0}{kT}\right)\cdot f_0^{-2}(\varepsilon-\mu H_0)\approx\delta(\varepsilon-\mu H_0-\varepsilon_0),$$

then (11) reduces to the form

$$4\Delta\varepsilon\left(\sinh\frac{v_sq}{kT}\right)^{-1}\delta(\varepsilon-\mu H_0-\varepsilon_0).\tag{12}$$

Taking into account the coupling (8), substituting (12) in (10), and carrying out integration over p and q, we obtain the equation for M_Z:

$$\operatorname{Im}(\Omega_{10}M_{12}^*)=(M_z-M_{0z})/\tau_l;\ \Omega_{10}=2\mu H_{10}/\hbar.$$

Here τ_l is the "longitudinal" spin relaxation time:

$$\tau_l=\frac{\alpha}{T}\left(\ln\frac{\tanh(T_D/T)}{\tanh(T_m/T)}\right)^{-1};\quad\alpha=\frac{3\mu^2\hbar\rho v_s^2}{16\pi\chi kC^2C_1^2},\tag{13}$$

χ is the paramagnetic susceptibility of the metal; $\rho = M/V$ is the density of the metal: T_D is the Debye temperature; $T_m = (v_S/kv)2\mu H_0$; v_S is the sound velocity; v is the velocity of the electron on the Fermi surface. If the dimensions of the specimen being considered are such that $\hbar/d > 2\mu H_0/v$, we must take $T_m = \hbar v_S/kd$. Equation (13) is valid in the temperature region $kT \gg \mu H_0$ if the resonance is close to saturation, i.e., $M_Z \approx 0$. Far from saturation of the resonance, the region of applicability of (13) extends in the direction of lower temperatures. For $T \gg T_D$

$$\tau_l=\alpha/T\ln(T_D/T_m),$$

which for $T_m = \hbar v_S/kd$ is identical with the expression (58) of the work of Elliott[5] with accuracy to within a coefficient 3/16. However, in reference 5, $\tau_l \sim T^{-3}$ for $T \ll T_D$ while it follows from (13) that $\tau_l = \alpha/T \ln(T/T_m)$ which agrees with experiment.* The quantitative agreement of the relaxation time of (13) with that observed is also satisfactory. Thus for sodium, if we substitute in (13) $\rho = 1$ g/cm^3, $\chi = 6\times10^{-7}$, $C_1 = 3\times10^{-4}$, $C = 10^{-12}$ erg, $v_S = 10^5$ cm/sec, we have: $\alpha = 10^{-5}$. The quantity $\ln[\tanh(T_D/T)/\tanh(T_m/T)] \approx 10$ for all $kT \gg \mu H_0$ (for $H_0 = 10^4$ Oe). Thus, $\tau_l \approx 10^{-6}T^{-1}$ sec, which agrees with data of Thayer and Kip[9] for sodium.

5. We now obtain the equation for the (transverse) component of the magnetization $M_{12} =$

*It is pointed out in reference 9 that Yafet obtained the same result for τ_l: $\tau_l \sim 1/T$ if $T \ll T_D$.

$M_x - iM_y$. We seek a solution of the equation for f_{12} in the form $f_{12}e^{-i\omega t}$. In the equation for f_{12}, we complete the integration over τ. In this case the exponential functions in the integrand lead to appearance of functions of the type $\delta_+(E_{p1} - E_{p+q2} - v_s q - \hbar\omega) \times \delta_+(x) = \delta(x) + i/\pi x)$. We can show that the part of the collision integral due to the second terms in δ_+ is negligibly small. We now note that under the condition $kT \gg \mu H_0$, we can seek a solution of f_{12} in the form $f_{12}\delta(\epsilon - \epsilon_0)$. Actually, the function f_z entering into the equation can be written as $f_z\delta(\epsilon - \epsilon_0)$ for $kT \gg \mu H_0$. Furthermore, if we substitute in the equation for f_{12} the solution for the form $f_{12}(p_0)\delta(\epsilon - \epsilon_0)$ then we can show that $\delta(\epsilon - \epsilon_0)$ will be a general factor of the equation. Multiplying this equation for f_{12} by $\mu(2\pi\hbar)^{-3}dp$ (similarly to what we have done previously), we obtain:

$$i(\omega - \Omega_0)M_{12} + i\Omega_{10}M_z = \frac{2}{3}M_{12}\frac{C^2C_1^2m^*p_0}{\pi\hbar^4Mv_s}\int_{q_{\min}}^{q_{\max}} dq\,\{8N_q + 4$$

$$+ f_{11}(E_{p_0 1} + v_s q) + f_{22}(E_{p_0 2} + v_s q)$$

$$+ f_{22}(E_{p_0 1} + v_s q) + f_{11}(E_{p_0 2} + v_s q) - f_{11}(E_{p_0 1} - v_s q)$$

$$- f_{22}(E_{p_0 2} - v_s q) - f_{22}(E_{p_0 1} - v_s q) - f_{11}(E_{p_0 2} - v_s q)\}.$$

In setting up this equation use is made of the relation $M_{12} = 8\pi m^* p_0 \mu(2\pi\hbar)^{-3} f_{12}(p_0)$. Substituting in the expression under the integral the Eq. (7) we have in first approximation for $\Delta\epsilon/kT$ and $\mu H_0/kT$:

$$i(\omega - \Omega_0)M_{12} + i\Omega_{10}M_z = M_{12}/\tau_t;\ \Omega_0 = 2\mu H_0/\hbar,$$

where τ_t is the "transverse" spin relaxation time which agrees exactly with Eq. (13) for the "longitudinal" relaxation time.

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