THE INELASTIC SCATTERING OF DEUTERONS BY Mg^{24}

T. I. KOPALEISHVILI and V. I. MAMASAKHLISOV

Physics Institute, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor May 20, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1017-1019 (October, 1958)

The angular distribution of deuterons inelastically scattered from Mg^{24} with excitation of the 2^+ level (4.23 Mev), which results from excitation of a single quantum of deformation, is investigated. Both nuclear and electric interaction of the deuteron and nucleus are taken into account. The theory is compared with the experimental data.

A paper of Hinds, Middleton, and Parry[1] gives results of an experimental study of the inelastic scattering of 8.9 Mev deuterons by Mg^{24}, with excitation of the 2^+, 4^+, 2^+ levels which have energies of 1.37, 4.12, and 4.23 Mev respectively. The angular distributions of the scattered deuterons when the first level (2^+; 1.37 Mev) and the third level (2^+; 4.23 Mev) are excited differ both in shape and in magnitude. The probability of scattering with excitation of the 4.23-Mev level is smaller in order of magnitude than the probability for scattering with excitation of the 1.37 Mev level, even though the spin and parity of the levels are the same. The scattering of deuterons by Mg^{24} with excitation of the 1.37 Mev level was treated in our previous paper.[2] In the present paper we investigate the scattering of deuterons by Mg^{24} with excitation of the 4.23-Mev level.

The Mg^{24} nucleus is known to be highly deformed, so that it has a spectrum of rotational levels corresponding to the different values K of the projection of the total angular momentum I of the nucleus on its symmetry axis.

The lowest terms in the spectrum have $K = 0$ and $I = 0$, 2^+, 4^+, etc. In treating the inelastic scattering of deuterons by Mg^{24} with excitation of the 1.37-Mev level, we made the assumption that the target nuclei are excited to the $K = 0$, $I = 2^+$ rotational level, and that the deformation of the nucleus is not changed, i.e., no quanta associated with β and γ deformations are emitted ($n_\beta = 0$, $n_\gamma = 0$).

Since the cross section for inelastic scattering of deuterons by Mg^{24} with excitation of the 4.23-Mev state is smaller in order of magnitude than that for excitation of the 1.37-Mev state, it is natural to assume that the 4.23-Mev level arises from the creation of one quantum of γ deformation and therefore corresponds to $K = 2$. Thus this level is characterized by the quantum numbers $K = 2$, $I = 2$, $n_\beta = 0$ and $n_\gamma = 1$. (Values of the parameter γ different from 0 and π show that the nuclear shape deviates from axial symmetry.) An indication that the 4.23-Mev level in Mg^{24} corresponds to $K = 2$ is also contained in a paper of Rakavy.[3]

We shall calculate the inelastic scattering of deuterons by Mg^{24} on the basis of the above assumptions.

The interaction between the deuteron and the Mg^{24} nucleus is taken in the form

$$V = \left[\pm V_0 R_0 \delta(r_p - R_0) + \frac{3Ze^2 R_0}{5r_p^3}\right] \times \sum_{\mu\nu} \alpha_{2\nu} D_{\mu\nu}^{*2}(\vartheta, \varphi, \psi) Y_{2\mu}(\vartheta_p, \varphi_p) \pm V_0 R_0 \delta(r_n - R_0) \sum_{\mu\nu} \alpha_{2\nu} D_{\mu\nu}^{*2}(\vartheta, \varphi, \psi) Y_{2\mu}(\vartheta_n, \varphi_n), \tag{1}$$

where ϑ, φ, and ψ are the Euler angles which determine the position of the nuclear axis relative to a fixed system of coordinates; the subscripts p and n refer to the proton and neutron in the deuteron; $D^2_{\mu\nu}$ is the unitary matrix transforming the spherical functions; V_0 and R_0 are the familiar parameters of the uniform model; $\alpha_{2\nu}$ are the deformation parameters and are operators for the creation of phonons.

In Born approximation, the matrix element of the transition is

$$\begin{aligned} <000\,\mathbf{k}\,|V|\,2MK\mathbf{k}'> &= \Pi^{1/2}(q) \times \sum_{\mu\nu} <0\,|\,\alpha_{2\nu}\,|\,1> \int \psi_{000}\psi^*_{2MK} D^{*2}_{\mu\nu}\, d\tau \\ &\times \Big\{ \int \Big[\pm V_0 R_0 \delta(r_p - R_0) + \frac{3Ze^2R_0^2}{5r_p^3}\Big] e^{i\mathbf{q}\mathbf{r}_p} Y_{2\mu}(\vartheta_p, \varphi_p)\, d\mathbf{r}_p \\ &\pm V_0 R_0 \int \delta(r_n - R_0)\, e^{i\mathbf{q}\cdot\mathbf{r}_n} Y_{2\mu}(\vartheta_n \varphi_n)\, d\mathbf{r}_n \Big\}, \end{aligned} \tag{2}$$

where ψ_{000} and ψ_{2MK} (M is the projection of I on the z axis of the fixed system), which are the wave functions of the nucleus in the initial and final states, have the form[4]

$$\psi_{000} = 1/\sqrt{8}\,\pi,$$

$$\psi_{2MK} = \sqrt{\frac{5}{16\pi^2}}\,[D^2_{MK}(\vartheta, \varphi, \psi) + D^2_{M-K}(\vartheta, \varphi, \psi)], \tag{3}$$

$$\Pi(q) = \left[\frac{4\alpha}{q}\tan^{-1}\frac{q}{4\alpha}\right]^2,$$

where $q = |\mathbf{k} - \mathbf{k}'|$ ($\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the center of mass of the deuteron at the start and end of the process); α is the known size parameter of the deuteron; $\langle 0\,|\,\alpha_{2\nu}\,|\,1\rangle$ is the matrix element for creation of a phonon and is equal to[4]

$$\langle 0\,|\,\alpha_{2\nu}\,|\,1\rangle = (\hbar/2\sqrt{B_\nu C_\nu})^{1/2}, \tag{4}$$

where B_ν and C_ν are, respectively, the mass coefficient and the coefficient of deformability of the nucleus.

Using formulas (3) and (4) and the fact that the selection rules require $\nu = K$, we get for the matrix element (2)

$$\langle 000\ \mathbf{k}\,|\,V\,|\,2MK\mathbf{k}'\rangle = (20\pi/\sqrt{2})\ \langle 2200\,|\,2200\rangle$$
$$\times \langle 22K-K\,|\,2200\rangle \sqrt{\frac{\hbar}{2\sqrt{B_K C_K}}}\,V_0 R_0^3 \Pi^{1/2}(q) \tag{5}$$
$$\times \left[\pm\frac{J_{5/2}(qR_0)}{\sqrt{qR_0}} + \frac{0.3Ze^2}{R_0 V_0}\,\frac{J_{3/2}(qr_0)}{(qr_0)^{3/2}}\right]\delta_{M0},$$

where $\langle \ldots | \ldots \rangle$ are Clebsch-Gordan coefficients, the J_n are Bessel functions, and r_0 is the electric radius of the nucleus.

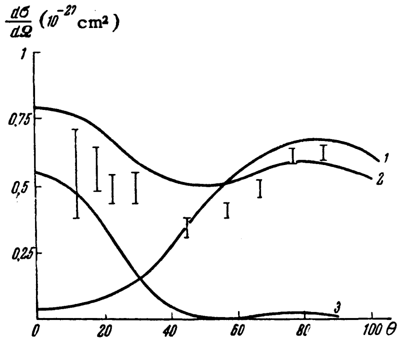
We finally get for the differential cross section for inelastic scattering of deuterons by Mg^{24} (with K = 2)

$$\frac{d\sigma}{d\Omega} = \frac{2\mu^2 (V_0 R_0^3)^2 R_0^2 \Pi(q)}{\hbar^3 \sqrt{B_2 C_2}} \tag{6}$$
$$\times \left[\pm\frac{J_{5/2}(qR_0)}{\sqrt{qR_0}} + \frac{0{,}3Ze^2}{V_0 R_0}\,\frac{J_{3/2}(qr_0)}{(qr_0)^{3/2}}\right]^2 \sqrt{1 + \frac{Q}{E_0'}},$$

where μ is the reduced mass of the (d, Mg^{24}) system and E_0' is the energy of the incident deuteron in the center of mass system.

In order to compare our formulas with experimental data, we must choose values for the parameters. We evaluated the quantities B_2 and C_2 using the formulas[4]

$$B_2 = \frac{3}{8\pi} AMR_0^2, \quad C_2 = 4R_0^2 S - \frac{3}{10\pi}\frac{Z^2 e^2}{R_0},$$



where $4\pi R_0^2 S = 15.4\ A^{2/3}$ Mev, A is the mass number of the nucleus, and M is the mass of the nucleon.

If we use the values of reference 2 for V_0 and r_0 ($V_0 = 1.84$ Mev, $r_0 = 6 \times 10^{-13}$ cm), take $R_0 = 3 \times 10^{-13}$ cm, and assume that the interaction of the deuteron with the nuclear vibrations is repulsive [i.e., choose the + sign before the nuclear term in formula (1)], the angular distribution of the deuterons, as shown in the figure, is in good agreement with the experimental data (curve 2).

Curve 1 corresponds to the inclusion only of the nuclear interaction of the deuteron and the nucleus, and curve 3 takes account only of their Coulomb interaction.

Thus, under our assumptions, simultaneous inclusion of both Coulomb and nuclear interactions gives satisfactory agreement with experiment. It is interesting that the occurrence of a second maximum in the deuteron distribution is explained.

It should be stated that there is some difference between the theoretical results and the experimental data with respect to the absolute value of the scattering cross section. However, since the coefficients B_2 and C_2 are evaluated approximately, no special significance should be attributed to this discrepancy.

[1] Hinds, Middleton, and Parry, Proc. Phys. Soc. **A70**, 900 (1957).

[2] V. I. Mamasakhlisov and T. I. Kopaleishvili, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1169 (1958), Soviet Phys. JETP **7**, 809 (1958).

[3] G. Rakavy, Nucl. Phys. **4**, 375 (1957).

[4] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. **27**, No. 16 (1953).

Translated by M. Hamermesh
212