method. The results are shown in the table.

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RADIATION OF A MULTILEVEL SYSTEM MOVING IN A MEDIUM WITH A VELOCITY GREATER THAN THE VELOCITY OF LIGHT

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THE formula for the complex ("superlight") Doppler effect, in which the radiator moves with a velocity $v > c/n$ in a transparent medium characterized by a refractive index $n(\omega)$, is of the form[1]

$$\omega(\theta) = \omega_{ik}\sqrt{1-\beta^2}/|1-\beta n(\omega)\cos\theta|, \tag{1}$$

where ω_{ik} is the frequency in the system in which the radiator is at rest, $\beta = v/c$, and θ is the angle between $\mathbf{v}$ and the wave normal. In a quantum-mechanical analysis Eq. (1) is obtained (cf. reference 2) from the laws of conservation of energy and momentum where the energy and momentum of the photon in the medium are respectively $\hbar\omega$ and $\hbar\omega n/c$. In the quantum-theory formulation, the Cerenkov condition $\beta n \cos\theta_0 = 1$ is obtained in a similar way (cf. reference 3). In the normal Doppler effect [when $\beta n(\omega)\cos\theta < 1$] the radiating system makes a transition from an upper state with energy ϵ_i to a lower state with energy $\epsilon_k = \epsilon_i - \hbar\omega_{ik}$. In the complex Doppler effect, however, [when $\beta n(\omega)\cos\theta > 1$, i.e., inside the Cerenkov cone] the radiation is characterized by transitions of the system in the upward direction — from a level ϵ_k to a level $\epsilon_i = \epsilon_k + \hbar\omega_{ik}$.[2] As a result, even in the absence of any other interaction, the system does not radiate and the probability of its remaining at the levels ϵ_i and ϵ_k is determined by the total probability of radiation at the normal and complex Doppler frequencies. Two points must be kept in mind in considering the possibility of observing the complex Doppler effect. First, if the radiator does not move in the medium itself but in an empty channel or in a slit of width smaller than the wave length of the radiated waves the characteristic features of the "superlight" radiation still obtain.[4,5] The complex Doppler effect can be important in motion of electrons in a magneto-active plasma when the losses are small; this case is of special interest in practice (cf. below). It is the purpose of the present note to point out the interesting possibilities associated with the faster-than-light motion of a multilevel system. If the system is initially at one level, say the ground state, in the course of time it may be found in all states to which it can make a transition as a result of direct or multiple radiative transitions (transitions upward with the radiation of frequency ω are possible only if the relation in (1) obtains with $\beta n(\omega)\cos\theta > 1$). The level populations are determined by the equations:

$$dN_i/dt = -\sum_k A_{ik}N_i + \sum_k B_{ik}N_k,$$

where A_{ik} is the probability for a radiative transition from level i to level k while B_{ik} is the probability for transition from level k to level i (this scheme, similar to that used to describe radioactive decay, can be obtained from the quantum theory of radiation, using certain justifiable assumptions[6]). In the dipole transitions between any states, i and k, the radiation intensity at the normal and complex Doppler frequencies can be determined from the classical formulas[1,2] by replacing the square of the amplitude of the dipole moment by $4|\mathbf{p}_{ik}|^2$. For example, with $n = \text{const}$ and a moment $\mathbf{p}_{ik}$ oriented along the velocity vector $\mathbf{v}$, the energy radiated per unit time into unit solid angle is

$$W(\theta) = \omega_{ik}^4(1-\beta^2)^3|\mathbf{p}_{ik}|^2\sin^2\theta/2\pi c^3|1-\beta n\cos\theta|^5, \tag{2}$$

where the angle θ is related to ω by Eq. (1) and $\mathbf{p}_{ik}$ is the matrix element for the rest system.

When $\beta n\cos\theta \to 1$, it is obviously impossible to neglect dispersion. In a number of cases, however, a reasonable approximation can be obtained by taking n as a step function: $n(\omega) = n$ for $\omega < \omega_c$ and $n = 1$ for $\omega > \omega_c$. This behavior for the refractive index n is reasonable in the case of weak dispersion for motion in a channel or in a slit as well as for an extended radiating system (in this case when $\cos\theta_0 \sim 1$ the frequency $\omega_c \sim 2\pi v/l$ where l is the radius of the channel, the width of the slit or a typical dimension of the system). If the angle θ_c is close to the Cerenkov angle θ_0, it is clear from Eq. (2) that the intensity (or probability) of the radiation is approximately the same at the normal and complex frequencies [basically, frequencies are radiated which are close to $\omega_c(\theta_c)$]. Under these conditions, the populations of levels i and k will be the same in the stationary state.

In a number of multilevel systems we deal with bunches of atoms or molecules with two appropriate levels. The radiation from a bunch which is smaller than the wavelength of interest is coherent and similar to the radiation of a spin system placed in a magnetic field.[7] However, the faster-than-light radiation of such bunches or of individual atoms and molecules or paramagnetic or ferromagnetic particles is scarcely of practical interest (even with $n \sim 30$ the hydrogen atom can acquire a velocity $v > c/n$ only when its kinetic energy is 5×10^6 ev). On the other hand, it is completely feasible to consider faster-than-light radiation of electrons which move along a magnetic field in which the role of the medium is played by a metal slow-wave structure, a dielectric, or a plasma located near the beam. If the velocity component perpendicular to the field $\mathbf{v}_\perp$ is zero only Cerenkov radiation is possible. If, however, $\mathbf{v}_\perp \neq 0$ and the condition $v_\perp/v_\parallel \ll mc^2/E \ll 1$ is fulfilled the electrons radiate in the same way as two perpendicular oscillators which lie in the plane of $\mathbf{v}_\perp$ and move through the field with a velocity $v_\parallel$. The faster-than-light Doppler radiation, which appears when $v_\parallel > c/n(\omega)$, causes electron transition to higher levels, i.e., the excitation of transverse oscillations. As a result the velocity $v_\parallel$ is reduced and, for example, with $n = \text{const}$ the excitation stops when $v_\parallel = c/n$. In a case of great practical interest, namely, the sporadic radio waves from the sun,[8] the motion of the electrons in the magnetoactive plasma medium is anisotropic. Hence in Eq. (1) we must replace n by $n_j(\theta, \omega)$ where j denotes the type of proper wave (ordinary or extraordinary). Dispersion cannot be neglected and the expression for the radiated energy is strongly divergent.[9] The author proposes to consider faster-than-light Doppler electron radiation in detail in a subsequent paper.

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CHANGE IN WIDTH OF BOUNDARY LAYER IN FERROMAGNETS BY MEANS OF THE MAGNETO-OPTICAL KERR EFFECT

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ATTEMPTS to measure the width of the boundary layer between ferromagnetic domains have been heretofore by the method of powder patterns with the use of an electron microscope.[1] In the final analysis, however, this method determines not the true width of the boundary layer, but the width of the path occupied by the magnetic suspension. In this work we employ in principle a method proposed by Krinchik,[2] based on the use of the polar magneto-optical Kerr effect.[3] This method permits direct measurement of the width of the boundary layer.

It can be shown that the average normal component of magnetization of the boundary layer is represented by the quantity $2j_S/\pi$, where J_S is the saturation magnetization of the ferromagnetic region. A photomultiplier (FEU-18) was used to measure the variation, due to the rotation of the