Equations (4) and (5) together give the most general selection rules in the form of a relation between the statistical tensors.

Let us now consider some simple examples. If the initial state of the $a + b \rightarrow c + d$ reaction is unpolarized, then $\rho = \rho_I$. Then according to (4) and (5) $\rho' = \rho'_I$, or

$$\rho'(q_c, \tau_c, q_d, \tau_d; \vartheta_c) = (-1)^{q_c+\tau_c+q_d+\tau_d}\rho'(q_c, -\tau_c, q_d, -\tau_d; \vartheta_c). \quad (6)$$

In our case the ρ' tensors do not depend on φ_c. Equations (6) are the same selection rules as Simon and Welton obtained for $q = 1$ and the same as those obtained by Shirokov.*

Let us now consider a cascade of the form $a + b \rightarrow c + d$ followed by $c + e \rightarrow f + g$ (the incident beam a, the target b, and e are unpolarized). According to (6), $\rho = \rho_I$ in the initial state of the second reaction, and we obtain

$$\rho'(q_f, \tau_f, q_g, \tau_g; \vartheta_f, -\varphi_f) = (-1)^{q_f+\tau_f+q_g+\tau_g}\rho'(q_f, -\tau_f, q_g, -\tau_g; \vartheta_f, \varphi_f). \quad (7)$$

For the special case in which $q_f = q_g = 0$, Eq. (7) becomes

$$\sigma(\vartheta_f, -\varphi_f) = \sigma(\vartheta_f, \varphi_f). \quad (8)$$

Since φ_f is the azimuth angle of $\mathbf{n}_f$ in the coordinate system in which the y_c axis is directed along $\mathbf{n}_a \times \mathbf{n}_c$, Eq. (8) states the well known fact that the angular distribution is symmetric about the production plane of the incident particle in the second reaction of the cascade. Equations (7) may be regarded as a generalization of this assertion.

In conclusion, we remark that our selection rules can also be obtained by Shirokov's method, but the present approach is simpler.

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*We remark that the first and second selection rules given by Shirokov are actually two different ways of stating the same rule.

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ON THE QUESTION OF THE UNIQUENESS OF PHASE ANALYSIS

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MINAMI[1] has given a transformation of the scattering matrix which leaves the differential cross section invariant for the case in which the colliding particles have spins 0 and $\frac{1}{2}$. The present note gives an analog of this transformation for all spins s_1 and s_2 of the colliding particles.

We express the scattering matrix in terms of the functions ${}_{s_1s_2}Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n})$ describing the state of a system of two particles whose total angular momentum is j. The component of j and the components of the spins of the two particles along the direction given by $\mathbf{n}$ are M, α_1, and α_2, respectively. In terms of these functions, the scattering matrix is[2,3]

$$M(\mathbf{n}_f, \mathbf{n}) = \sum_{\alpha jM} Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}_f)\,[Y^{jM}_{\alpha'_1\alpha'_2}(\mathbf{n}_i)]^*\,A^j_{\alpha_1\alpha_2\alpha'_1\alpha'_2},$$

$${}_{s_1s_2}Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}) = \sum_{s,l}\langle s_1\alpha_1 s_2\alpha_2 \mid s_1s_2sa\rangle \times \langle sal0 \mid slja\rangle \sqrt{\frac{2l+1}{2j+1}}\,{}_{s_1s_2}Y^{jM}_{sl}(\mathbf{n})$$

Let $S(\mathbf{n})$ be a rotation that carries the vector $\mathbf{n}$ into the third axis, and consider the functions $\varphi_{\sigma_1\sigma_2}(\mathbf{n})$ whose components are

$$[\varphi_{\sigma_1\sigma_2}(\mathbf{n})]_{\alpha_1\alpha_2} = D^{s_1}_{\alpha_1\sigma_1}(S^{-1}(\mathbf{n}))\,D^{s_2}_{\alpha_2\sigma_2}(S^{-1}(\mathbf{n})),$$

where $D^j_{m_1m_2}(S)$ are the matrix elements of an irreducible representation of the three-dimensional rotation group.[4] These functions describe a state in which the first and second particles have spins whose components are σ_1 and σ_2, respectively, along $\mathbf{n}$.

The functions $Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n})$ satisfy the relation

$$Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}) = \varphi_{\alpha_1\alpha_2}(\mathbf{n})\sqrt{\frac{2j+1}{4\pi}}\,D^j_{\alpha_1+\alpha_2,M}(S(\mathbf{n})),$$

so that the matrix element for the transition from the state $\varphi_{\alpha'_1\alpha'_2}(\mathbf{n}_i)$ to the state $\varphi_{\alpha_1\alpha_2}(\mathbf{n}_f)$ is*

$$(\alpha_1\alpha_2 \mid M \mid \alpha'_1\alpha'_2) = \sum_j A^j_{\alpha_1\alpha_2\alpha'_1\alpha'_2} \times \sqrt{\frac{2j+1}{4\pi}}\,D^j_{\alpha_1+\alpha_2,\alpha'_1+\alpha'_2}(S(\mathbf{n}_f)\,S^{-1}(\mathbf{n}_i)). \quad (1)$$

Since $Y^{jM}_{\alpha_1\alpha_2}(-\mathbf{n}) = (-)^{s_1+s_2-j} Y^{jM}_{-\alpha_1-\alpha_2}(\mathbf{n})$, the condition that the matrix element be invariant under reflection requires that

$$A'^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2} = A^{j}_{-\alpha_1-\alpha_2,\,-\alpha'_1-\alpha'_2}.$$

It is easily seen that the coefficients

$$A'^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2} = \lambda_{\alpha_1\alpha_2}\lambda_{\alpha'_1\alpha'_2} A^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2}, \qquad (2)$$

where each of the λ's are either $+1$ or -1, and satisfy the same unitary and symmetry conditions as do the $A^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2}$.[5]

If, in addition, $\lambda_{-\alpha_1-\alpha_2} = k\lambda_{\alpha_1\alpha_2}$, where $k = \pm 1$ (if s_1 and s_2 are integers, $k = -1$ must be excluded, since the combinations of α's include one in which $\alpha_1 = \alpha_2 = 0$), the A' coefficients can be used to construct the scattering matrix. It is seen from (1) that this matrix, although different from M, gives the same scattering cross section. It is easily shown that $M'(\mathbf{n}_f\mathbf{n}_i) = A(\mathbf{n}_f)M(\mathbf{n}_f\mathbf{n}_i)A^*(\mathbf{n}_i)$, where $A(\mathbf{n})$ is diagonal and has matrix elements $\lambda_{\alpha_1\alpha_2}$ in the coordinate system whose third axis lies along $\mathbf{n}$. Let us consider double scattering given by

$$\rho_{f2}(\mathbf{n}_{f2}\mathbf{n}_{i2}) = M(\mathbf{n}_{f2}\mathbf{n}_{i2})\,\rho_{i2}(\mathbf{n}_{i2})\,M^*(\mathbf{n}_{f2}\mathbf{n}_{i2}).$$

The M matrix is given on the center-of-mass coordinate system, so that the density matrix $\rho_{i2}(\mathbf{n}_{i2})$ of the initial state before the second scattering must also be given in this system, since it is obtained from the density matrix of the particle after the first scattering by the transformation $\rho_{i2}(\mathbf{n}_{i2}) = S\rho_{f1}(\mathbf{n}_{f1})S^*$. Here S is some rotation in the plane of the first scattering,[6] and $\mathbf{n}_{i2}$ and and $\mathbf{n}_{f1}$ are vectors related in the usual way. Now consider the density matrices ρ' obtained from the scattering matrix by

$$M' = A(\mathbf{n}_f)\,M(\mathbf{n}_f\mathbf{n}_i)\,A^*(\mathbf{n}_i),$$
$$\rho'_{2f} = M'\rho'_{2i}M'^*, \quad \rho'_{2i} = S\rho'_{1f}S^*,$$
$$\rho'_{1f}(\mathbf{n}_{f1}) = A(\mathbf{n}_{f1})\,\rho_{f1}(\mathbf{n}_{f1})\,A^*(\mathbf{n}_{f1}).$$

The cross section for double scattering is then given by

$$\sigma_2 \sim \mathrm{Sp}\,(MS\rho_{1f}S^*M^*);\quad \sigma'_2 \sim \mathrm{Sp}\,(MS'\rho_{f1}S'^*M^*),$$

where $S' = A^*(\mathbf{n}_{i2})SA(\mathbf{n}_{f1})$.

If the spins of the particles involved are no greater than $\frac{1}{2}$, the transformations of Eq. (2) contain one for which $S'\rho_{f1}S'^* = \rho_{f1}$.[7] If at least one of the particles has spin greater than $\frac{1}{2}$, however, there is no such transformation among those given by (2). Thus for processes involving particles of spin greater than $\frac{1}{2}$, there is in general no arbitrariness such as that given by (2). An obvious exception is scattering by an infinitely heavy target, when $\mathbf{n}_{i2} = \mathbf{n}_{f1}$ and $S = 1$; in this case there is no multiplicity of scattering which can be used to differentiate between the set of phases given by (2).

In conclusion, I express my gratitude to Professor M. A. Markov, Professor Ia. A. Smorodinskii, R. M. Ryndin, M. I. Shirokov, and Chou Kuang-Chao for discussion of the work and valuable comments.

*We note that the cross section and tensor moments are easily expressed in terms of the parameters $A^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2}$ and generalized spherical functions[4] without the use of Racah coefficients.

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COMPUTING THE SPECTRA OF FISSION NEUTRONS

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USACHEV,[1] Watt,[2] Fraser,[3] and Gurevich and Mukhin[4] have studied the theoretical interpretation of fission neutron spectra. These authors used a model in which the neutrons are assumed to be evaporated from the moving fission fragments. In the center-of-mass system the spectrum of neutrons evaporated from a fragment whose excitation energy is E_0 is of the form

$$n(\varepsilon) \sim \sigma(\varepsilon, E_0)\,\varepsilon\omega(E_0 - \varepsilon).$$