THE DE HAAS-VAN ALPHEN EFFECT IN PULSED MAGNETIC FIELDS

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Submitted to JETP editor April 7, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 738-741 (September, 1958)

Equations are obtained for the semi-classical quantization of the motion of an electron in a metal in an inhomogeneous magnetic field, the gradient of which is perpendicular to the direction of the field. The de Haas-van Alphen effect in a pulsed magnetic field is analyzed from the point of view of possibly using it to study the Fermi surface of the electron gas in a metal.

In reference 1 the author investigaged the de Haas-van Alphen effect in a sufficiently slowly changing magnetic field when one can start from the formulae for the quantization of the motion of an electron and the expression for the magnetic moment of an electron gas in a uniform magnetic field.[2] In the present note we obtain the equations for the quantization in an inhomogeneous magnetic field, the gradient of which is perpendicular to the direction of the field, and the role of the inhomogeneity of the field in a pulse method of investigating de Haas-van Alphen effect is elucidated.

1. We shall consider the motion of a particle with an arbitrary dispersion law $E = E(p_x, p_y, p_z)$ and charge e in an inhomogeneous magnetic field **H**, directed along the z axis. If the y axis coincides with the direction* of grad **H**, the classical motion of the particle proceeds according to

$$\mathcal{E}(P_x, P_y, p_z) = \varepsilon = \text{const}, \quad p_x = \text{const}, \quad p_z = \text{const}, \tag{1}$$

where the components of the kinetic momentum **P** are most conveniently expressed in the form

$$P_x = \frac{e}{c}\int\limits_{y_0(p_x)}^{y} H(y)\,dy, \quad P_y = p_y, \quad P_z = p_z, \tag{2}$$

where the function $y_0(p_x)$ is determined by the relation

$$p_x = -\frac{e}{c}\int\limits_{0}^{y_0(p_x)} H(y)\,dy. \tag{3}$$

We shall assume for the sake of simplicity that $H(y) > 0$. It is then clear from (1) and (2) that the classical motion of a particle corresponding to a closed surface $\mathcal{E}(p_x, p_y, p_z) = \epsilon$, is bounded along the y axis, and the coordinate $y_0 = y_0(p_x)$ plays the role of the center of the oscillations along this axis.

In the quantum case the components (2) of **P** correspond to operators with the following commutator relations

$$[\hat{P}_y, \hat{P}_x] = \frac{e}{c}H(\hat{y}), \quad [\hat{P}_x, \hat{P}_z] = [\hat{P}_y, \hat{P}_z] = 0, \tag{4}$$

where $y = y(p_x, P_x)$ is determined from (2).

From (4) the semi-classical quantization condition

$$\oint [P_y / H(y)]\,dP_x = (n+\gamma)\,eh/c, \quad (0<\gamma<1) \tag{5}$$

follows. The integration in (5) is performed along the closed curve (1).

Equation (5) determines implicitly the dependence of the energy levels ϵ on the quantum numbers n, p_z, p_x, or on the quantum numbers n, p_z, y_0, which is the same.

If the distance δ, over which the magnetic field $H(y)$ changes appreciably, is considerably larger than the radius r of the classical particle orbit in the magnetic field $H(y_0)$, we can for fixed p_z and $y_0(p_x)$ on the left hand side of the quantization equation (5) retain only the main terms in an expansion in powers of r/δ:

$$S(\varepsilon, p_z)\left\{1 - \frac{r_1}{H_0}\frac{dH}{dy_0} - \frac{1}{2}\frac{r_2^2}{H_0}\left[\frac{d^2H}{dy_0^2} - \frac{3}{H_0}\left(\frac{dH}{dy_0}\right)^2\right]\right\} = (n+\gamma)\frac{ehH}{c}; \tag{6}$$

$$r_1 S(\varepsilon, p_z) = (c/eH_0)\,J_{1y}, \quad r_2^2 S(\varepsilon, p_z) = (c/eH_0)^2 J_{2y},$$

$$H_0 = H(y_0);$$

where $S(\epsilon, p_z)$ is the area of the cross section of the energy surface $\mathcal{E}(\mathbf{p}) = \epsilon$ with the plane $p_z = \text{const}$, $J_{1y}(\epsilon, p_z)$ the static moment, and $J_{2y}(\epsilon, p_z)$ the moment of inertia of the area $S(\epsilon, p_z)$ with respect to the p_y axis.

To conserve the quantization equations (5) and (6) in an electric field, produced by a non-static

*Such an inhomogeneous magnetic field is necessarily solenoidal (curl $\mathbf{H} \neq 0$) and can, therefore, not be purely magnetostatic.

magnetic field (see reference 1) it is necessary that the energy, gained by a particle in the electrical field during a period of revolution in the classical orbit, be small compared to the distance between the quantized energy levels. We make a corresponding estimate in the case where Eq. (6) is justified, when the simple expression for the distance between the energy levels is well known to be

$$\Delta\varepsilon = \hbar\omega^* \equiv \hbar\,(eH/m^*c) \equiv \mu^* H;$$

where m^* is the effective mass and μ^* the effective magnetic moment of the particle.

From an analysis of the Maxwell equations it follows that if T is a characteristic time of change of the magnetic field, the electrical field strength $E \sim H\delta/cT$. The above mentioned requirement leads thus to the inequality.

$$\omega^* T \gg \delta/\lambda, \qquad (7)$$

where λ is the de Broglie wavelength of a particle with energy ϵ.

If the inhomogeneity of the magnetic field is vanishingly small, Eq. (6) goes over into the well-known quantization equations in a homogeneous field.[2,3] Corrections to the particle energy levels in a uniform magnetic field,[2,3] arising from taking the field inhomogeneity into account in Eq. (6), are meaningful, of course, only if they are considerably larger than the inaccuracy of the semi-classical approximation itself, i.e., when

$$r_1/\delta \gg (\lambda/r)^2 \quad \text{and} \quad r_2^2/\delta^2 \gg (\lambda/r)^2.$$

2. We shall consider the de Haas-van Alphen effect in a pulsed magnetic field. Let the external field which is directed parallel to the surface of the metal specimen increase from zero to a maximum value H_m in a time interval $(0, t_m)$ and the general duration of the pulse $T \sim t_m$. The magnetic field inside the metal is then a function of the y coordinate measuring the depth into the sample from its surface, and a characteristic measure along the y axis $\delta \sim (c^2T/\sigma_m)^{1/2}$, where $\sigma_m = \sigma(H_m)$ is the electrical conductivity of the metal in a magnetic field H_m.

The energy levels of an electron in such a sample are determined by Eqs. (5) or (6)* where y_0 changes within the limits 0 to L, where L is the length of the sample along the y axis.

As is well known,[2] the oscillations of the magnetic moment of the electron gas with a change of the magnetic field is determined by a change of the extreme value $n_m(H)$ — the integral quantum number n, corresponding to the limiting Fermi energy in a factor of the form $2\pi n_m(H)$. Since in a homogeneous field n is very simply connected with $S(\epsilon, p_z)$, an investigation of the periods of the oscillations in a uniform field enables one in principle to reconstruct the Fermi surface for the electrons in a metal.[4] In order that a study of the periods of the oscillations in a pulsed field can also give interesting information about the Fermi surface, it is necessary that n_m be directly connected with the cross sections of that surface. From the foregoing it is clear that this only takes place if $r \ll \delta$, when Eq. (6) is valid. In that case the relative change in period $P = \Delta(1/H_0)$ for the oscillations of the magnetic moment of a thin metal layer at a depth $y = y_0$ as compared to the period in a corresponding uniform field P_0 is, as to order of magnitude, equal to

$$\delta P/P_0 = |P - P_0|/P_0 \sim r_{1m}/\delta + r_{2m}^2/\delta^2, \quad P_0 = eh/cS_m, \qquad (8)$$

where S_m, r_{1m}, and r_{2m} are determined for the extremal cross section of the Fermi surface, corresponding to the given oscillations. If the center of gravity of the extremal cross section lies on the p_y axis (this will, for instance, be the case for a central cross section of the Fermi surface, possessing a center of symmetry), $r_{1m} = 0$ and the order of magnitude of the quantity $\delta P/P_0$ is determined by the second term on the right hand side of (8). Otherwise $\delta P/P_0 \sim r_{1m}/\delta$. We note that in magnetic fields $H_m \sim 10^5$ Oe for the basic electron groups $r_m \sim 10^{-4}$ to 10^{-5} cm.

From (8) it follows that if $r_m \ll \delta$ the above-mentioned periods are practically not changed, while the change in the phase of the oscillations compared with those in a uniform field can be appreciable, since it is of the order of magnitude $(\delta P/P_0)(\epsilon_0/\mu^* H)$ where ϵ_0 is the limiting Fermi energy.

Since the determination of the function $H = H(y)$ is a non-trivial mathematical problem for complicated pulses of the external field and when σ depends essentially on H in strong fields, quantita-

*Strictly speaking, if the external magnetic field vector and the normal to the surface of the sample do not coincide with the principal axes of the electrical conductivity tensor, the direction of the magnetic field inside the metal changes with depth. However, in the practically most interesting cases, considered below, this rotation of the H vector can be neglected when condition (7) is fulfilled. As to inequality (7) in metals, there is a sufficiently wide range of T for which it is satisfied. Indeed, if we take for λ an extreme estimate, $\lambda \sim 10^{-8}$ cm, and take into account that for the basic electron groups in metals at $H \approx 10^5$ Oe the Larmor frequency $\omega^* \sim 10^{12}$, we can write (7) in the form $T/\delta \sim (\delta_m T/c^2)^{1/2} \gg 10^{-4}$ sec/m.

tive conclusions about the Fermi surface based on results of an investigation of the oscillations by means of pulses is possible only in those cases when the measured periods are directly connected with the change in the external field. In reference 1 it was shown that those cases are (1) $\delta \gg L$, (2) $\delta \ll L$. If we take into account that an experimental measurement of the periods of the oscillations enables us to obtain directly the cross sectional area of the Fermi surface only if $r_m \ll \delta$ [see (8)] and $r_m < L$[5], it becomes logical to use in the same connection the following limiting cases (1) $\omega^* T\lambda \gg \delta \gg L > r_m$, (2) $L \gg \delta \gg r_m$; $\omega^* T\lambda \gg \delta$. The experimental conditions of Shoenberg[6] correspond to the first case.

In the first of the cases noted by us it is possible to observe the oscillations with the change in the external field practically during the whole duration of the pulse, but in the second case only for $t < t_0$, where $t = t_0$ corresponds to the moment when on the surface of the sample $dH/dy = 0$. One can show that always $t_0 > t_m$.[1]

The author expresses his thanks to I. M. Lifshitz and M. Ia. Azbel' for discussions.

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Translated by D. ter Haar
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