ELASTIC SCATTERING OF PHOTONS ON EXCITED NUCLEONS

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The cross section for elastic scattering of γ quanta is computed with allowance for the excited nucleon states as described by the Markov model. The calculated cross sections are found to agree satisfactorily with the available experimental data.

1. It is well known that the cross sections of many processes (scattering and photoproduction of π-mesons etc.) have been found to show only one or a few maxima. To explain the first of these maxima, Tamm has developed a semiphenomenolgical theory involving a nucleon isobar state. In terms of this theory he analyzed π-meson[1] and γ-ray[2] scattering and the photoproduction of π mesons[3] on nucleons. He obtained a qualitative agreement with the experimental data.

Markov has given another explanation of the observed maxima of the cross sections.[4] It, too, is founded on the picture of an excited nucleon state which is identified to be a hyperon instead of an isobaric state. We have shown earlier[5] that there exist two distinct classes of excited nucleon states in Markov's model. If these excited states are identified with the hyperons, then accounting for the latter in the intermediate states leads indeed to resonance effects. In reference 5 this was illustrated with the example of π^- scattering on protons. However, in that paper a number of essential assumptions were made. They were: applicability of perturbation theory, pseudoscalar meson-nucleon interaction, and identical form of the interaction of the meson both with the nucleon and the excited state of the nucleon. This example can therefore be considered only as illustration. In this connection it is of interest to investigate the elastic scattering of photons on excited nucleons in terms of the Markov model, since the above questions can be unequivocally answered in this case. Such an investigation is furthermore of interest because some experimental data have recently been obtained on that process.

A more consequent treatment of the process should be made in terms of damping theory, since the cross section at the resonance energy is generally infinite. However, in a preliminary calculation of the differential cross section at energies differing from the resonance energy, we can limit ourselves to a perturbation-calculation treatment. We can also disregard for the time being effects due to the meson cloud around the nucleon. These simplifications are taken into account in the treatment of the problem in the present paper. Furthermore, we assume that the first excited state of the nucleon is the Λ^0-particle, and we take the excitation energy to be 140 Mev (the excitation energy in this case can lie anywhere between 140 and 180 Mev). Then the obtained cross sections are close to the experimentally-observed values.

2. In the lowest nonvanishing order of perturbation theory the process

$$\gamma + P \rightarrow \gamma' + P' \qquad \text{(I)}$$

is described by the diagrams of Fig. 1. The lower diagrams correspond to the case where the intermediate state consists of the excited nucleon state. We shall perform the calculation in the laboratory system.

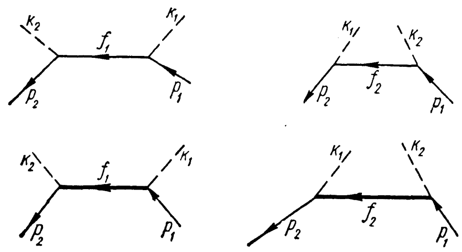

FIG. 1

The matrix elements corresponding to the lower diagrams are given (the notation is that employed by Akhiezer and Berestetskii[6]) by

$$S_{nn'} = -\frac{ie^2}{2\sqrt{\omega_0\omega}} J_{nn'} \bar{u}_2 \left\{\hat{e}_2 \frac{i\hat{f}_1 - M}{f_1^2 + M^2}\hat{e}_1\right\} u_1,$$
$$S'_{nn'} = -\frac{ie^2}{2\sqrt{\omega_0\omega}} J'_{nn'} \bar{u}_2 \left\{\hat{e}_1 \frac{i\hat{f}_2 - M}{f_2^2 + M^2}\hat{e}_2\right\} u_1, \qquad (1)$$

where $f_1 = p_1 + k_1 = p_2 + k_2$; $f_2 = p_1 - k_2 = p_2 - k_1$;

ω_0, e_1 and ω, e_2 are the energy and polarization vectors of the incoming and outgoing photon respectively; $M = m + a(n_1 + n_2 + n_3 + n_0)$ is the mass of the excited nucleon state; a is the excitation energy; n_1, n_2, n_3, n_0 are numbers that characterize the excitation of the nucleon; $J_{nn'} = (I_{nn''}I_{n''n'})_1 + (I_{nn''}I_{n''n'})_2 + (I_{nn''}I_{n''n'})_3 + \ldots$,

$$I_{nn'} = N \int H_{n_1}(\xi_1) \ldots H_{n_0'}(\xi_0)$$
$$\times \exp\left\{-\xi_\nu^2 - \frac{1}{m^2}(p_\nu \xi_\nu)^2 - \frac{1}{M^2}(f_\nu \xi_\nu)^2 - ik_\nu \xi_\nu r_0\right\} d^4\xi,$$

k is the energy-momentum vector of the photon; $I_{nn''}$ corresponds to the transition from the initial to the intermediate state, and $I_{n''n'}$ to the transition from the intermediate to the final state. The subscripts 1, 2 . . . refer to the different diagrams of a given excited state* (which, in general, is degenerate).

We have assumed that the photons interact with the excited state in the same way as with the usual nucleons.[7]

The two upper diagrams correspond to the following transition-matrix elements:

$$S_{00} = \frac{-ie^2 J_{00}}{2\sqrt{\omega_0\omega}} \bar{u}_2 \left\{\hat{e}_2 \frac{i\hat{f}_1 - m}{f_1^2 + m^2} \hat{e}_1\right\} u_1,$$
$$S'_{00} = \frac{-ie^2 J'_{00}}{2\sqrt{\omega_0\omega}} \bar{u}_2 \left\{\hat{e}_1 \frac{i\hat{f}_2 - m}{f_2^2 + m^2} \hat{e}_2\right\} u_1, \qquad (1')$$

where m is the mass of the nonexcited nucleon, while J_{00} and J'_{00} are the respective integrals over the variables of the intermediate state.

3. To calculate $J_{nn'}$ which enters the expression for the cross section, we must know $I_{nn'}$. In reference 5 we have obtained an expression for $I_{nn'}$ for the case where either the initial or final nucleon momentum vanishes. For our present purposes we must know $I_{nn'}$ also for the case when both these momenta differ from zero (this need arises in the diagrams where a photon is emitted first and absorbed afterwards).

By the same method as used in reference 5 we obtain for an arbitrary coordinate system

$$N \exp\{-t_1^2 + \ldots + t_0^2 - t_1'^2 + \ldots + t_0'^2\}$$
$$\times \int_{-\infty}^{+\infty} \exp\{-\xi_\nu\xi_\nu - (p'_\nu\xi_\nu)^2 - (f'_\nu\xi_\nu)^2 + K_\nu\xi_\nu\} d^4\xi \qquad (2)$$
$$= N \sum_{n_1=0}^{\infty} \cdots \sum_{n_0'=0}^{\infty} I_{nn'} \frac{1}{n_1!} \cdots \frac{1}{n_0'!} t_1^{n_1} \ldots t_0^{n_0'},$$

*In diagrams where the nucleon is not excited, M = m.

where $p'_\nu = p_\nu/m$, $f'_\nu = f_\nu/M$, $K_\nu = 2(t_\nu + t'_\nu) - ik_\nu r_0$.

Expressing the quadratic form in the exponent as a sum of squares, we obtain the left hand side of (2) in the form

$$N \frac{\pi^2}{\sqrt{-\Delta_4}} \exp\left[\frac{1}{2}(K_\nu a_\nu) - t_\nu^2 - t_\nu'^2\right], \qquad (2')$$

where

$$K_\nu a_\nu = \frac{1}{2}K^2 - (p_\nu K_\nu)(f_\nu K_\nu)/(p_\nu f_\nu)^2,$$
$$\Delta = -(p_\nu f_\nu)^2/m^2M^2.$$

Thus we have according to (2)

$$I_{nn'} = N \frac{\pi^2}{\sqrt{-\Delta_4}} \left\{\frac{\partial^{n_1+\ldots+n_0'}}{\partial t_1^{n_1} \ldots \partial t_0'^{n_0'}} \right.$$
$$\left. \times \exp\left[\frac{1}{2}(K_\nu a_\nu) - t_\nu^2 - t_\nu'^2\right]\right\}_{t_1=\ldots t_0'=0} \qquad (3)$$

In particular, if $n_1 = n_2 = \ldots = n_0' = 0$ one sees from (3) that

$$I_{00} = J_{00} = \frac{1}{\sqrt{-\Delta}} \exp\left\{-\frac{r_0^2}{4}\left[\frac{(p_\nu k_\nu)(f_\nu k_\nu) - k^2(p_\nu f_\nu)}{-(p_\nu f_\nu)}\right]\right\}$$
$$\approx \frac{1}{\sqrt{-\Delta_4}}. \qquad (4)$$

In the following we shall assume that the excited state of the nucleon has a lifetime of the order 10^{-10} sec (e.g., a Λ^0 particle). Then[8], $r_0 \sim 10^{-19}$ cm and we therefore shall neglect quantities $\sim r_0 p$.

If $N = n_1 + n_2 + n_3 = 0$, $n_0 = 0$ and $N' = 1$, $n_0' = 0$ then, for $n_1' = 1$, we have

$$I_{01} = -i\frac{r_0}{\sqrt{2}} I_{00}\left[k_1 - \frac{(f_\nu k_\nu)p_1 - (p_\nu k_\nu)f_1}{(p_\nu f_\nu)}\right] \qquad (5)$$

and analogously for $n_2' = 1$, $n_3' = 1$.

If $N = n_0 = 0$ and $N' = n_0' = 1$ then, for $n_1' = 1$,

$$I_{02} = \frac{1}{2} I_{00}\left[\frac{2(p_0 f_1 + p_1 f_0)}{(p_\nu f_\nu)} + O(r_0 p)\right] \qquad (6)$$

and analogously for $n_2' = 1$, $n_3' = 1$.

As can be seen, the selection rules for the transition-matrix elements with respect to the internal coordinates are in the present case the same as for f = 0, which was treated in reference 5. This means that the most probable transitions will be those for which $N' = n_0'$.*

*It is easy to verify that Eqs. (4) to (6) go over into Eqs. (10), (11) of reference 5 when f = 0. We use the opportunity to point out that in Eqs. (10) and (11) of reference 5 a term $2(t_0 + t_0')$ should be added in the exponent and the sign of β and of $2q_1/E$ should be inverted. The same holds for Eqs. (13) and (14). In the last expressions q signifies that q_1 stands for n_1', q_2 for n_2' etc.

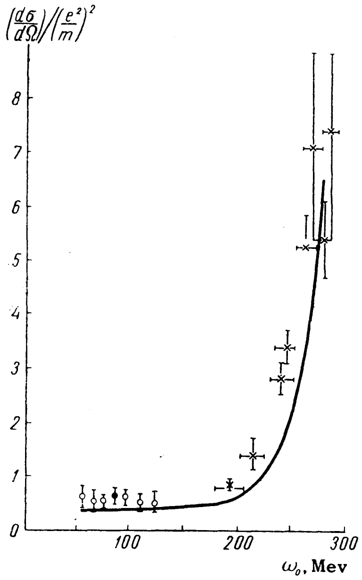

FIG. 2. Dependence of the differential scattering cross section on the energy of the incoming photon for the angle $\vartheta = 90°$ in the center-of-mass system. (The energy, ω_0, is given in the lab. system).

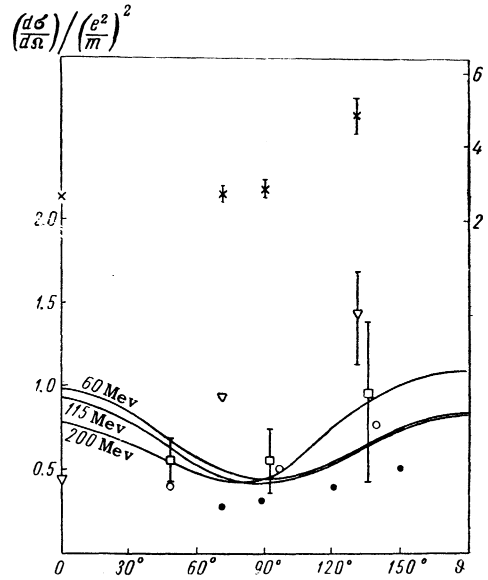

FIG. 3. Angular distributions of the scattered photons for different primary energies: calculated (solid curves) and measured: × – 239 Mev (right scale), ∇ – 193 Mev, ○ – 100 Mev, □ – 90 Mev, ● – 65 Mev.

4. We now calculate the differential cross section of the process (I).

We limit ourselves to such excitations in the intermediate state for which $N = n_0 = 1$. Then J_{02} will be made up of three contributions according to the threefold degeneracy of the diagram containing excited nucleons in the intermediate state. Thus, according to (6) we have

$$J_{02} = \frac{\dfrac{\omega_0^2 - \omega\omega_0\cos\vartheta}{Mm}\sqrt{1 + \dfrac{\omega_0^2}{M^2} + \dfrac{\omega_0^2}{M^2}\left(1 + \dfrac{\omega_0}{m} - \dfrac{\omega}{m}\right)}}{\left(1 + \dfrac{\omega_0^2}{M^2}\right)\left[\dfrac{\omega_0^2 - \omega_0\omega\cos\vartheta}{Mm} - \sqrt{1 + \dfrac{\omega_0^2}{M^2}\left(1 + \dfrac{\omega_0}{m} - \dfrac{\omega}{m}\right)}\right]^2}, \tag{7}$$

$$J'_{02} = \frac{\dfrac{\omega^2}{M^2}\left(1 + \dfrac{\omega_0}{m} - \dfrac{\omega}{m}\right) + \dfrac{\omega^2 - \omega_0\omega\cos\vartheta}{Mm}\sqrt{1 + \dfrac{\omega^2}{M^2}}}{\left(1 + \dfrac{\omega^2}{M^2}\right)\left[\dfrac{\omega\omega_0\cos\vartheta - \omega^2}{Mm} + \sqrt{1 + \dfrac{\omega^2}{M^2}\left(1 + \dfrac{\omega_0}{m} - \dfrac{\omega}{m}\right)}\right]^2}. \tag{7'}$$

Furthermore, according to (4) we have

$$J_{00} = -\left\{\sqrt{1 + \frac{\omega_0^2}{m^2}}\left[\frac{\omega_0^2}{m^2} - \frac{\omega\omega_0\cos\vartheta}{m^2} - \sqrt{1 + \frac{\omega_0^2}{m^2}\left(1 + \frac{\omega_0}{m} - \frac{\omega}{m}\right)}\right]\right\}^{-1}, \tag{8}$$

$$J'_{00} = -\left\{\sqrt{1 + \frac{\omega^2}{m^2}}\left[\frac{\omega^2}{m^2} - \frac{\omega\omega_0\cos\vartheta}{m^2} - \sqrt{1 + \frac{\omega^2}{m^2}\left(1 + \frac{\omega_0}{m} - \frac{\omega}{m}\right)}\right]\right\}^{-1}. \tag{8'}$$

We finally obtain for the differential cross section of process (I) the expression

$$\begin{aligned}\frac{1}{r_0^2}\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\omega}{\omega_0}\right)^2 \Big\{&\left(1 + \frac{\omega_0}{m} + \frac{\omega_0\omega}{m^2}\right)\Phi_1^2 + \left(1 - \frac{\omega}{m} + \frac{\omega_0\omega}{m^2}\right)\Phi_2^2 \\ &+ \frac{2a^2}{m^2}\left(\frac{\omega_0}{m} + 5 - \frac{\omega}{m}\right)(\Phi_3^2 + \Phi_4^2) - \left(2 - \frac{\omega}{m} + \frac{\omega_0}{m}\right)\Phi_1\Phi_2 \\ &+ \frac{2a}{m}\left(1 - \frac{\omega}{m} - \frac{3\omega_0}{m}\right)\Phi_1\Phi_3 - \frac{2a}{m}\left(1 - \frac{\omega}{m} + 3\frac{\omega_0}{m}\right)\Phi_1\Phi_4 \\ &- \frac{2a}{m}\left(1 - 3\frac{\omega}{m} + \frac{\omega_0}{m}\right)\Phi_2\Phi_3 + \frac{2a}{m}\left(1 + 3\frac{\omega}{m} + \frac{\omega_0}{m}\right)\Phi_2\Phi_4 \\ &+ \frac{4a^2}{m^2}\left(1 + \frac{\omega}{m} - \frac{\omega_0}{m}\right)\Phi_3\Phi_4\Big\},\end{aligned} \tag{9}$$

where

$$r_0^2 = \left(\frac{e^2}{m}\right)^2, \quad \Phi_1 = \frac{m}{\omega_0}J_{00} + \Phi_3, \quad \Phi_2 = \frac{m}{\omega}J'_{00} + \Phi_4,$$

$$\Phi_3 = J_{02}\left(\frac{\omega_0}{m} - \frac{2a^2}{m^2} - \frac{2a}{m}\right)^{-1}, \quad \Phi_4 = J'_{02}\left(\frac{\omega}{m} + \frac{2a^2}{m^2} + \frac{2a}{m}\right)^{-1}.$$

The elastic photon-proton scattering cross section was evaluated from (9) (in units of e^4/m^2) for an angle of 90° in the center-of-mass system and for $a = 140$ Mev. It is plotted in Fig. 2 using experimental points taken from reference 9. As can be seen, there is rather good agreement between the calculations (in the range of their applicability) and the experimental data.

The dependence of the differential cross section (in units of e^4/m^2) on the center-of-mass angle, according to Eq. (9) with $a = 140$ Mev, has been plotted in Fig. 3 for several photon energies. The experimental points are from references 9 and 10.

We see that these experimental data are poor and contain large experimental errors. The experimental points evidently indicate a characteristic tendency. As the photon energy increases, almost up to the resonance energy, the backward scattering becomes more prominent.* If this is so, then the tendency of our angular distributions are in opposition with the experimental distributions up to an energy ~ 100 Mev but have a similar tendency at higher energies. However, a definite comparison will be possible only after the calculations have been performed in the damping theory, with account of meson effects.

It should be further noted that the cross section for the scattering of photons on nucleons has been calculated in several papers. Some have used the picture of an isobaric state (see, e.g., reference 3, where further references are given) while others have used actual meson-theoretical approaches (see, e.g., reference 12 which also contains further references). The results of the different calculations are rather contradictory.

In conclusion, I would like to thank M. A. Markov for valuable suggestions and I. A. Egorova for the numerical computations.

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*The opposite result has been obtained in reference 11.

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