walls, then, $U_i = 0$ and $N = 0$, and also $NR_{ij} = 0$.

Equations (1) to (3) are found to be in satisfactory quantitative agreement with experiment.[1] In this case the following approximate values are obtained for the constants: $\alpha = 5$; $\beta = 0.16$; $\gamma = 0.022$.

The approximate solution for the boundary layer can be obtained in the form of an expansion in powers of R_{yy}/R (x is directed along the current and y along the normal to the wall). The first approximation gives the following dependencies:

$$R_{xy}\frac{dU_x}{dy} \sim \frac{1}{y}, \quad R^3 \sim \log\frac{l}{y}, \quad N \sim y\left(\log\frac{l}{y}\right)^{2/3}, \qquad (4)$$
$$R_{xy} \sim a - y, \quad R_{yy} \sim R_{xy}^2/R, \quad R_{zz} \sim R.$$

For flow in the tube, $2l = a$, where a is the radius of the tube.

These dependencies are confirmed by experiment. In particular, the logarithmic dependence for R^3 is confirmed.

[1] J. Laufer, NACA Report 1174, 1954. A. A. Townsend, The Structure of Turbulent Shear Flow (Cambridge, 1956).

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ON THE DEVIATION OF THE EQUILIBRIUM SHAPE OF ATOMIC NUCLEI FROM AXIAL SYMMETRY

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AN analysis of the experimental data on the levels of atomic nuclei indicates that a number of nuclei do not have an axis of symmetry.[1] Therefore it appears advisable to investigate the behavior of nucleons in a potential field without axial symmetry. As an example of such a field we consider an infinitely deep potential well with vertical walls which has the shape of an ellipsoid with semi-axes $a_x R_0$, $a_y R_0$, and $a_z R_0$, where R_0 is the radius of a sphere of equal volume. The problem of determining the nucleon states in such a potential well is reduced to the solution of the equation

$$(2M)^{-1}(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2)\psi_i(\mathbf{r}) = E_i\psi_i(\mathbf{r}) \qquad (1)$$

inside the ellipsoid with zero boundary conditions. By considering the deviations from spherical shape as being small we set

$$a_x^{-1} = 1 + \lambda, \quad a_y^{-1} = 1 + \nu, \quad a_z^{-1} = 1 + \kappa,$$

where κ is related to λ and ν by the condition that the volume remains the same. By going over in (1) to the variables

$$x' = xa_x^{-1}, \quad y' = ya_y^{-1}, \quad z' = za_z^{-1}$$

and by restricting ourselves to quantities of the second order of smallness, we obtain

$$[(2M)^{-1}\hat{p}^2 + V(\mathbf{r}')]\psi_i(\mathbf{r}') = E_i\psi(\mathbf{r}'),$$
$$\psi(\mathbf{r}') = 0 \quad \text{for } r' = R_0,$$
$$V(\mathbf{r}') = (2M)^{-1}[(\alpha^2 + {}^1/_3\delta^2)\hat{p}^2$$
$$+ (\alpha - {}^3/_4\alpha^2 - {}^1/_{12}\delta^2)(\hat{p}^2 - 3\hat{p}_{z'}^2) + \delta(1 + {}^1/_2\alpha)(\hat{p}_{x'}^2 - \hat{p}_{y'}^2)],$$
$$\alpha = \lambda + \nu, \quad \delta = \lambda - \nu.$$

We regard $V(\mathbf{r}')$ as a perturbation. The wave functions of the unperturbed problem (up to the normalization factor) and the corresponding eigenvalues are equal to

$$\psi_i(\mathbf{r}') \equiv \psi_{nlm}(\mathbf{r}') = r'^{-1/2}j_{l+1/2}(\mu_{nl+1/2}r'/R_0)Y_{lm}(\vartheta'\varphi'),$$
$$E_i^0 \equiv E_{nl}^0 = \hbar^2\mu_{nl+1/2}^2/2MR_0^2,$$

where $\mu_{nl+\frac{1}{2}}$ is the n-th root of the Bessel function $j_{l+\frac{1}{2}}(z)$, and Y_{lm} is a spherical harmonic.

Already in the first order of perturbation theory the degeneracy with respect to m is completely removed. Calculations carried out up to second-order perturbation theory inclusive lead to quite awkward expressions for the energies, which in the case $\delta = 0$ (axial symmetry) reduce to the corresponding expressions given by Moszkowski[2] (when $\delta = 0$, his parameter d is related to the parameter α by the expression $d = \alpha + \alpha^2/4$). Qualitatively the behavior of the nucleon levels can be studied as with the s and p shells as examples. In the case of s nucleons we obtain

$$E_{n0} = E_{n0}^0\left\{1 + \left[1 - \frac{144}{5}s_2(\mu_{n^1/_2})\right]\left(\alpha^2 + \frac{1}{3}\delta^2\right)\right\}. \qquad (2)$$

and the p level splits up into three:

$$(E_{n1})_1 = E_{n1}^0\left\{1 - \frac{4}{5}\alpha + \alpha^2\left[\frac{8}{5} - \frac{432}{7}s_3(\mu_{n^3/_2})\right] + \delta^2\left[\frac{2}{5} - \frac{80}{7}s_3(\mu_{n^3/_2})\right]\right\},$$
$$(E_{n1})_2 = E_{n1}^0\left\{1 + \frac{2}{5}\alpha - \frac{2}{5}\delta + \alpha^2\left[\frac{7}{10} - \frac{288}{7}s_3(\mu_{n^3/_2})\right] + \delta^2\left[\frac{3}{10} - \frac{128}{7}s_3(\mu_{n^3/_2})\right] - \alpha\delta\left[\frac{1}{5} - \frac{96}{7}s_3(\mu_{n^3/_2})\right]\right\},$$
$$(E_{n1})_3 = E_{n1}^0\left\{1 + \frac{2}{5}\alpha + \frac{2}{5}\delta + \alpha^2\left[\frac{7}{10} - \frac{288}{7}s_3(\mu_{n^3/_2})\right] + \delta^2\left[\frac{3}{10} - \frac{128}{7}s_3(\mu_{n^3/_2})\right] + \alpha\delta\left[\frac{1}{5} - \frac{96}{7}s_3(\mu_{n^3/_2})\right]\right\},$$

$$s_{l+2}(\mu_{nl+1/2}) = \sum_{n'} \mu^2_{n'l+3/2} / (\mu^2_{n'l+3/2} - \mu^2_{nl+1/2})^3$$

$$= \frac{2l+7}{16(2l+3)^2} - \frac{\mu^2_{nl+1/2}}{8(2l+3)^3}. \quad (3)$$

The wave-functions corresponding to these states are given in zeroth approximation (up to the normalization factor) by

$$(\psi_{n1})_1 = \psi_{n10}, \; (\psi_{n1})_2 = \psi_{n11} + \psi_{n1-1}, \; (\psi_{n1})_3 = \psi_{n11} - \psi_{n1-1}.$$

From (2) and (3) it may be easily seen that for a filled shell the spherical shape is energetically the most favorable one. As the p shell is gradually filled the equilibrium shape, starting with the third nucleon, will deviate not only from the spherical one but also from the axially-symmetric one. For example, for the $(1s)^2(1p)^4$ configuration, in which the levels E_{10}, $(E_{11})_1$ and $(E_{11})_2$ are filled, the minimum total energy is at $\alpha \approx 0.015$ and $\delta \approx 0.3$.

A detailed analysis of the deviations of the equilibrium shape of actual nuclei from axial symmetry will be published later.

Similar calculations for the oscillator potential have been made by Garskii (cf. Wilets and Jean[3]) and Geilikman[4].

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[1] A. S. Davydov and G. V. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 440 (1958), Soviet Phys. JETP **8**, 303 (this issue).

[2] S. A. Moszkowski, Phys. Rev. **99**, 803 (1955).

[3] L. Wilets and M. Jean, Phys. Rev. **102**, 788 (1956).

[4] B. T. Geilikman, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 989 (1958), Soviet Phys. JETP **8**, (in press).

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SECOND RELAXATION IN A SPIN SYSTEM IN CERTAIN COMPOUNDS OF ELEMENTS OF THE IRON GROUP

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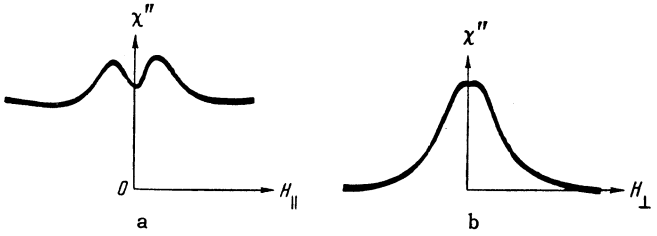
SPIN-SPIN absorption in parallel fields, $\chi''(H_{||})$, decreases monotonically with increase of the constant magnetic field $H_{||}$.[1-5] The experimental curves of $\chi''(H_{||})$ that have been obtained are described by the theory of Shaposhnikov.[6]

In a number of cases,[7-9] the spin-spin absorption curves of hydrated salts of elements of the iron group have a maximum in $\chi''(H_{||})$, dependent both on the frequency of the alternating field H_ν and on the intensity of $H_{||}$. Gorter and de Vrijer[7] suggested that the maximum in the spin-spin absorption curve in chromium-potassium alums at 20°K is due to relaxation between two spin systems (second relaxation in the spin system).

Conditions for production of second relaxation are created also as a result of nonuniform heating (or ventilation) of certain hydrated salts of Mn^{++} Fe^{+++}, and Cu^{++}. In such slightly hydrous salts, the ions in different elementary cells have different environments, and this leads to production of second relaxation in the spin system. Similar conditions should be present likewise in substances with residual antiferromagnetism.



Double absorption curves χ'' (arbitrary units) in $FeNH_4(SO_4)_2 \cdot H_2O$; ν = 296 Mc/sec, T = 300°K. a – absorption in parallel fields; b – absorption in perpendicular fields.

Actually, in $Mn(COCH_3)_2$, $FeNH_4(SO_4)_2 \cdot H_2O$, $Fe_2(SO_4)_3 \cdot 3H_2O$, and $CuCl_2$, in the frequency range 600 to 150 megacycle/sec, by the method described earlier,[10] we have succeeded in obtaining curves of the absorption in parallel fields with a maximum in $\chi''(H_{||})$. In these substances the position of the maximum is independent of temperature* (in the range 300 to 90°K); on decrease of the frequency ν, it moves toward lower magnetic fields $H_{||}$ (cf. table).

In the magnetically concentrated salts listed