mass different from zero, because such a speed is not attainable.

It may turn out that for a given $a_H = a_H^*$ the amplitude of the shock wave will be a maximum. In the case of a further increase in the speed a_H, a part of the energy of the particles will be converted into energy of radiation, pairs will begin to be created in the photon gas, and the amplitude of the shock wave will be reduced.

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[2] K. P. Staniukovich, Неустановившиеся движения сплошной среды (Unsteady Motion of Continuous Media), Gostekhizdat, 1955, Ch. 15, § 87.

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THE ENERGY OF EXCITONS FOR VERY SMALL QUASI-MOMENTA

S. I. PEKAR

Institute of Physics, Academy of Sciences, Ukrainian S.S.R.

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LET us divide the crystal into cubic regions, let the edge of the cube L contain a large number of lattice constants, and let the exciton quasi-momentum k have an absolute magnitude much less than 1/L. We construct the wave function of the crystal in the form of a linear combination of antisymmetrized products of wave functions of the separate regions:

$$\Psi_{\mathbf{k}} = A \sum_{\mathbf{m} l \lambda} e^{i\mathbf{k}\cdot\mathbf{m}} (-1)^{\lambda} \prod_{\substack{\mathbf{n}=1 \\ \mathbf{n}\neq\mathbf{m}}}^{N} \psi_{\mathbf{n}}(\lambda)\, c_l \varphi_{\mathbf{m}}\ (\lambda). \qquad (1)$$

Here $\mathbf{n}$ (or $\mathbf{m}$) is a vector determining the position of the center of the cube and at the same time an index, numbering the cube. $\psi_{\mathbf{n}}$ is the (antisymmetrized) ground state of the separate cube, $\varphi_{\mathbf{m}l}$ an excited state of the cube, l numbers the degenerate excited states of the cube, and λ is the number giving the distribution of the electrons over the cube. The distribution is numbered arbitrarily but in such a way that each subsequent distribution is obtained from the previous one by the interchange of two electrons belonging to different cubes.

The exciton energy is evaluated from the wave function (1) in the same way as in the Heitler-London-Heisenberg method (see reference 1, § 2, case b). The only difference is that the Heitler-London-Heisenberg method involves the wave functions of the elementary cells of the crystal, while here we have instead the wave functions of the above-mentioned cubes. We can neglect the exchange interaction energy for electrons belonging to different cubes. The energy is then equal to

$$\mathscr{E}(\mathbf{k}) = \mathscr{E}_0 + u(\mathbf{k}), \quad u(\mathbf{k}) = \sum_{\mathbf{m}} L^{(\mathbf{m})} e^{i\mathbf{k}\mathbf{m}}, \qquad (2)$$

where $\mathscr{E}_0$ is a constant that does not depend on the quasi-momentum $\mathbf{k}$, and

$$L^{(\mathbf{m})} = \sum_{ll'} c_l^* c_{l'} \int \varphi_{0l}^*(r_0)\, \psi_{\mathbf{m}}^*(r_{\mathbf{m}})\, V_{\mathbf{m}0}(r_{\mathbf{m}} r_0)\, \varphi_{\mathbf{m}l'}(r_{\mathbf{m}})\, \psi_0(r_0)\, dr_0 dr_{\mathbf{m}}. \qquad (3)$$

$V_{\mathbf{m}0}$ is the potential energy of the Coulomb interaction between charged particles belonging to cubes at the points $\mathbf{m}$ and 0; $r_{\mathbf{m}}$ and r_0 indicate the totality of the coordinates of these particles (for an arbitrary distribution λ). Since the cubes are electrically neutral, $V_{\mathbf{m}0}$ can be reduced to their dipole-dipole interaction. Let $\mathbf{P}(\mathbf{m})$ and $\mathbf{P}(0)$ be the dipole moments of the cube. We introduce the notation

$$\mathbf{P}_l = \int \varphi_{0l}^*(r_0)\, \mathbf{P}(0)\, \psi_0(r_0)\, dr_0, \quad \mathbf{P} = \sum_l c_l^* \mathbf{P}_l. \qquad (4)$$

One can then easily show by expressing $V_{\mathbf{m}0}$ explicitly in terms of $\mathbf{P}(\mathbf{m})$ and $\mathbf{P}(0)$, substituting the result into (3) and using the notation (4) that the quantity $u(\mathbf{k})$ can be interpreted in the following way: u is equal to the electrostatic energy of the interaction of the dipole $\mathbf{P}$ at the origin with all dipoles $\mathbf{P}^* e^{i(\mathbf{k}\cdot\mathbf{m})}$ at the centers of all the cubes. Since $kL \ll 1$ one can replace the collection of dipoles placed at the centers of the cubes by a polarized continuum with a specific dipole moment $L^{-3}\mathbf{P}^* e^{i(\mathbf{k}\cdot\mathbf{r})}$. The latter produces, as is well-known, a fictitious dielectric polarization field which is equal to

$$\mathbf{E}'(\mathbf{r}) = -4\pi L^{-3} \mathbf{s} (\mathbf{P}^*, \mathbf{s})\, e^{i\mathbf{k}\mathbf{r}}, \quad \mathbf{s} = \mathbf{k}/k. \qquad (5)$$

In this way u can be evaluated as the energy of the interaction between the dipole $\mathbf{P}$ and the field $\mathbf{E}'(0)$:

$$u = -\mathbf{P}\cdot\mathbf{E}'(0) = 4\pi L^{-3} |\mathbf{P}\cdot\mathbf{s}|^2 = 4\pi L^{-3} |\mathbf{P}|^2 \cos^2\alpha, \qquad (6)$$

where α is the angle between $\mathbf{P}$ and $\mathbf{s}$. We can show that the quantity (6) does not depend on L, since $\mathbf{P}$ is proportional to $L^{3/2}$. We can thus, for the sake of convenience, put $L = 1$ or put L^3 equal to the volume of the elementary crystal cell. The meaning of $\mathbf{P}_l$ is then that it is the matrix element of the dipole moment per unit volume, or, respectively, of the elementary cell. Below we shall consider only excited states for which $\mathbf{P} \neq 0$.

The coefficients of the linear combination of the degenerate functions c_l are determined from the condition that $\mathscr{E}(\mathbf{k})$ or the quantity $|(\mathbf{P}\cdot\mathbf{s})|$ is extremum under the supplementary condition $\Sigma|c_l|^2 = 1$.

If the excited state of the cube (cell) is nondegenerate, the single coefficient c is equal to 1. This occurs for all excited states of a rhombic crystal and also, for instance, for those states of a tetragonal crystal for which $\mathbf{P}_l$ is directed along the tetragonal axis. Then α is the angle between the uniquely determined direction in the crystal $\mathbf{P} = \mathbf{p}_l$ and the direction $\mathbf{s}$. Depending on the direction of $\mathbf{s}$, $\cos^2\alpha$ takes on all values from 0 to 1.

Apart from the just mentioned states there are in a tetragonal crystal still two-fold degenerate excitations of the cube for which $\mathbf{P}$ lies in the plane perpendicular to the tetragonal axis, z. There are thus two sets of extremal values c_1 and c_2 and correspondingly two exciton states for a given direction of $\mathbf{s}$:

(1) $\mathbf{P} \perp \mathbf{s}$, $u = 0$; (2) $\mathbf{P}$ coplanar with $\mathbf{s}$ and z,

$$u = 4\pi L^{-3}|\mathbf{P}|^2\sin^2(s,z). \tag{7}$$

In both cases $|\mathbf{P}|$ is unique and is independent of the direction of $\mathbf{s}$.

In cubic crystals there are two kinds of exciton for any given direction of $\mathbf{s}$:

(1) $\mathbf{P} \perp \mathbf{s}$ and otherwise arbitrary in direction; $u = 0$,

$$\text{(2)}\quad \mathbf{P} \parallel s \quad \text{and}\quad u = 4\pi L^{-3}|\mathbf{P}|^2. \tag{8}$$

In both cases $|\mathbf{P}|$ is unique and is independent of the direction of $\mathbf{s}$.

The dependence of $u(\mathbf{k})$ on the direction of $\mathbf{k}$ as $|\mathbf{k}| \to 0$, just proven, means that the function $\mathscr{E}(\mathbf{k})$ has a discontinuity at $\mathbf{k} = 0$. One must thus introduce the corresponding corrections in many earlier papers in which $\mathscr{E}(\mathbf{k})$ was expanded in a power series in k_x, k_y, k_z, in particular in reference 2. However, for each given direction $\mathscr{E}(\mathbf{k})$ can be considered to be an analytical function of the absolute magnitude k and can be expanded in a power series in it. In this way one obtains all basic results of reference 2, including Eqs. (39) to (44). One must only consider the parameter $\mathscr{E}(0)$ to be a constant for a given direction of $\mathbf{s}$, and generally speaking, a quantity depending on the angle between the direction of $\mathbf{s}$ and $\mathbf{P}_{0\mathbf{k}}(0)$.

The discontinuous dependence of the exciton energy on k of the above discussed type was earlier obtained for particular models for the exciton: for a Frenkel exciton in molecular crystals (naphthalene and anthracene)[3] and for a plasmon in an anisotropic medium.[4]

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ON THE STABILITY OF A FERMI LIQUID

I. Ia. POMERANCHUK

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In the theory of the Fermi liquid proposed by Landau[1,2] the energy of the system is a functional of the distribution of the excitations among the quantum states (in particular, among values of the momentum). When the distribution function of the excitations has only small deviations δn from the equilibrium distribution corresponding to the filling up of all states with $p < p_0$ (all the notations are the same as in reference 1), the energy functional E has the form

$$E = \sum_{\sigma}\int \varepsilon(p)\,\delta n(\mathrm{p})\,\frac{d^3p}{(2\pi\hbar)^3} + \frac{1}{2}\sum_{\sigma}\sum_{\sigma'}\iint f(\mathrm{pp}')\,\delta n(\mathrm{p})\,\delta n(\mathrm{p}')\,\frac{d^3p\,d^3p'}{(2\pi\hbar)^6} \tag{1}$$

(σ is the excitation spin).

Stability will exist for small δn if $E > 0$ for arbitrary δn. The values of p actually involved in the integrals of Eq. (1) are those close to p_0 (the temperature is zero). Therefore $\epsilon(p) = (d\epsilon/dp)_{p_0}(p - p_0) = v_0(p - p_0)$. The variations δn appearing in Eq. (1) are due to deformations of the Fermi surface, and we shall find the criterion for stability with respect to such deformations (see diagram).