Here $w = \beta^2/a^2$ is the energy of the magnetic interaction of two atoms, and a is the lattice constant.

The relaxation times of the spin waves and phonons relative to interaction among the spin waves and among phonons do not change in the presence of an external magnetic field and have the form

$$\tau_k^{(2)} \approx \frac{\rho}{\hbar}\left(\frac{a\Theta_c}{\Theta}\right)^5\left(\frac{\Theta^2}{\Theta_c T}\right)^{3/2}\exp\left\{\frac{\Theta^2}{4\Theta_c T}\right\}; \quad T \ll \frac{\Theta^2}{\Theta_c},$$

$$\tau_f^{(2)} \approx \frac{\rho}{\hbar}\left(\frac{a\Theta_c}{\Theta}\right)^5\frac{\Theta^2}{\Theta_c T}\exp\left\{\frac{\Theta^2}{4\Theta_c T}\right\}; \quad T \ll \frac{\Theta^2}{\Theta_c},$$

$$\tau_f^{(1)} \approx \frac{\rho}{\hbar}\left(\frac{a\Theta_c}{\Theta}\right)^5\left(\frac{\Theta^2}{\Theta_c T}\right)^5; \quad T \ll \Theta;$$

where ρ is the density of the material.

As a result, it follows that for $H/T \ll 10^4$ and $T \ll \Theta^2/\Theta_c$, the dissipation function and the sound absorption coefficient Γ are determined by the relaxation time $\tau^{(1)}$:

$$\Gamma = C\omega^2(\lambda_{ij}\dot{u}_{ij})\ln^2\frac{w + 2\beta H}{T},$$

where $\dot{u}_{ij}$ is the change in the deformation tensor of the crystal, λ_{ij} is some tensor, C is a constant, and ω is the sound frequency. In this case, the magnetic field increases the sound absorption.

For $H/T \gg 10^4$ and $T \ll \Theta^2/\Theta_c$, the spin-spin interaction has a small probability, and the principal contribution to the distribution function is made by the interaction of the spin waves with the phonons. The absorption coefficient in this case is determined by the time $\tau_k^{(2)}$ and has the form

$$\Gamma = C'\omega^2(\lambda_{ij}\dot{u}_{ij})\exp\{\Theta^2/4\Theta_c T\}.$$

In a sufficiently large magnetic field, the sound absorption depends exponentially on the temperature and not on the field.

In both cases, we can show that $\tau_f \ll \tau_k$ for $T \ll \Theta^2/\Theta_c$, and, consequently, the phonons play a small role in the sound absorption.

The author thanks Prof. A. I. Akhiezer for discussion of the problem.

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Translated by R. T. Beyer
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INFLUENCE OF ACOUSTIC VIBRATIONS ON THE PARAMETERS OF THE BANDS OF IMPURITY ABSORPTION IN CRYSTALS

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Submitted to JETP editor April 1, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 287-289 (July, 1958)

In a previous paper[1] the authors considered a system of an ionic crystal and an F-center in a macroscopic approximation and obtained expressions for the energies of the ground and excited states of the system and for the parameters of the corresponding absorption bands. These expressions depend, according to reference 1, on the sums $q_1 = \Sigma\cos^2\theta_\alpha/f_\alpha^2$ and $q_2 = \Sigma\cos^2\theta_\alpha/f_\alpha$, where θ is the angle between the displacement vector of the medium $\mathbf{u}$ and the wave vector κ for the α-th branch of the elastic vibrations of the crystal $\alpha = 1, 2, 3$); f_α determines the angular dependence in the dispersion law for the frequencies of the elastic vibrations $\omega_{\kappa\alpha}: f_\alpha = \omega_{\kappa\alpha}/|\kappa|$.

In reference 1 we used for the evaluation of the sums q_1 and q_2 the results of a paper by Tolpygo[2] where the consideration was based on a microscopic approximation. It turned out, however, that it was possible to perform, in general form, a consistent macroscopic consideration for a large group of crystals for which the anisotropy was not too large (for crystals which did not satisfy this condition it was necessary to resort to numerical calculations).

The quantities f_α are, as is well known, the solutions of the characteristic determinant

$$n_1^2/(n_1^2 - \gamma_1) + n_2^2/(n_2^2 - \gamma_2) + n_3^2/(n_3^2 - \gamma_3) = 1, \qquad (1)$$

where

$$\gamma_i = (A - C)n_i^2 + C - x; \quad A = \lambda_{1111}/(\lambda_{1122} + \lambda_{1212});$$

$$C = \lambda_{1212}/(\lambda_{1122} + \lambda_{1212});$$

$$x = \rho f_\alpha^2/(\lambda_{1122} + \lambda_{1212});$$

λ_{iklm} are the elasticity moduli of a cubic crystal; n_i a unit vector in the direction κ, and ρ the crystal density. It is convenient to rewrite Eq. (1) in the form

$$x^3 + bx^2 + cx + d = 0, \tag{2}$$

where

$$\begin{aligned} b &= -(1 + 3C + \eta); \quad c = C(2 + 3C) \\ &\quad + 2\eta(C + L_0) + L_0\eta^2; \\ d &= -[C^2(C + 1) + \eta(C^2 + 2CL_0) \\ &\quad + \eta^2(CL_0 + 3L_1) + \eta^3 L_1]; \\ L_0 &= n_1^2 n_2^2 + n_1^2 n_3^2 + n_2^2 n_3^2; \quad L_1 = n_1^2 n_2^2 n_3^2. \end{aligned} \tag{2a}$$

The quantity $\eta = A - C - 1$ can be considered to be the "anisotropy parameter" (in the isotropic case $\eta \to 0$). Approximate solutions of Eq. (2) can be written as series in powers of η. For many crystals retaining terms of the first order in η is already a good approximation:

	AgBr	NaBr	NaCl	Ge	Si	MgO	LiF
$\eta =$	0.22	—0.27	0.35	—0.46	—0.46	—0.46	—0.48.

In zeroth approximation the roots of Eq. (2) are

$$x_{1,2}^0 = C; \quad x_3^0 = C + 1. \tag{3}$$

Looking for corrections in the first and the second approximation, we get

$$\begin{aligned} x_{1,2} &= C + L_0\left(1 \pm \sqrt{1 - \frac{3L_1}{L_0^2}}\right)\eta \\ &+ \frac{(4L_0^3 - L_0^2 - 3L_0L_1)\sqrt{1 - 3L_1L_0^{-2}} \pm (4L_0^3 - L_0^2 - 9L_0L_1 + 2L_1)}{2L_0\sqrt{1 - 3L_1L_0^{-2}}}\eta^2 + \dots \\ x_3 &= C + 1 + (1 - 2L_0)\eta + (L_0 - 4L_0^2 + 3L_1)\eta^2 + \dots . \end{aligned} \tag{4}$$

Using the equations of motion for the elastic vibrations of the lattice we get easily the expressions

$$\cos^{-2}\theta_\alpha = n_1^2/(n_1^2 - \gamma_1)^2 + n_2^2/(n_2^2 - \gamma_2)^2 + n_3^2/(n_3^2 - \gamma_3)^2. \tag{5}$$

Substitution of (4) into (5) enables us to get final expressions for $\cos^2 \theta_\chi$ and after that to evaluate the sums q_1 and q_2. Restricting ourselves to linear terms in η we get

$$\begin{aligned} q_1 &= \frac{\rho}{(\lambda_{1122} + \lambda_{1212})(C + 1)}\left(1 - \frac{1 - 2L_0}{C + 1}\eta\right), \\ q_2 &= \left[\frac{\rho}{(\lambda_{1122} + \lambda_{1212})(C + 1)}\right]^{1/2}\left[1 - \frac{1 - 2L_0}{2(C + 1)}\eta\right]. \end{aligned} \tag{6}$$

Use of (6) leads to final expressions for the quantities J_0, J_1, σ_1, σ_2, with the aid of which the required energy levels are determined according to reference 1 and also the frequency of the corresponding absorption band and its half-width (in the limiting cases of high and low temperatures).

$$\begin{aligned} J_0[\alpha_0] &= \frac{1}{14}\left\{3\frac{\hbar^2}{\mu_0}\alpha_0^2 - 3\frac{e^2}{\varepsilon}(3z + \varepsilon c)\alpha_0 - \frac{\alpha_0^3}{\mu_0 g_0 e^2}\right\}; \\ g_0 &= \frac{2^8 \cdot 7\pi(C + 1)^2(\lambda_{1122} + \lambda_{1212})}{437a^2\mu_0 e^2(C + 1 - 0.6\eta)}; \\ J_1[\beta] &= \frac{1}{2}\left\{\frac{\hbar^2}{\mu_0}\beta^2 - \frac{e^2}{\varepsilon}(z + 0.3914\varepsilon c)\beta - \frac{\beta^3}{3e^2 g_1\mu_0}\right\}; \\ g_1 &= \frac{2^8\pi(C + 1)^2(\lambda_{1122} + \lambda_{1212})}{3a^2\mu_0 e^2[9(C + 1) - 41\eta/7]}; \\ \sigma_1 &= \frac{\alpha_0^3}{\hbar}\left\{\frac{1}{2\mu_0 e^2}\left(\frac{1}{7g_0} + \frac{1}{3v^3 g_1}\right) - \frac{a^2(27v^2 + 14v + 2)}{14\pi(1 + v)^7}\frac{1 - 0.6\eta/(C + 1)}{(C + 1)(\lambda_{1122} + \lambda_{1212})}\right\}; \\ \sigma_2 &= \frac{a^2\alpha_0^4}{\hbar\sqrt{\rho(C + 1)(\lambda_{1122} + \lambda_{1212})}}\left\{\frac{183}{7^3 \cdot 10\pi^2}\left(1 - \frac{0.3\eta}{C + 1}\right) + \frac{1}{70\pi^2 v^4}\left(\frac{73}{15} - \frac{23}{14}\frac{\eta}{C + 1}\right) - \frac{4}{7\pi^2}\left(1 - 0.3\frac{\eta}{C + 1}\right)\right. \\ &\quad \left.\times\left[\frac{-7v^8 - 480v^6 - 930v^4 - 32v^2 + 9}{6(v^2 - 1)^6} + 2\frac{v^2\ln v}{(v^2 - 1)^7}(27v^6 + 141v^4 + 77v^2 - 5)\right]\right\}. \end{aligned} \tag{7}$$

We have used in these formulae the notation of reference 1.

The approximation used above can also be used to consider the behavior of an electron of an impurity center in homopolar crystals.

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Translated by D. ter Haar
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