## HALL EFFECT IN PURE NICKEL AT HELIUM TEMPERATURES

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Experimental data are presented on the Hall effect in pure nickel (99.99%) in a wide temperature range down to the temperature of liquid helium. It is shown that the ferromagnetic constant  $R_1$  drops sharply with the temperature T and has a minimum at 20 to 30°K. A physical interpretation, based on the (s-d)-exchange model, is proposed for the observed phenomenon.

A study of the Hall effect in ferromagnets discloses a sharp temperature dependence of the socalled extraordinary Hall constant  $R_1$ , determined from the experimental relation proposed in reference 1:

$$e = R_0 H + R_1 J, \tag{1}$$

where e is the Hall field per unit current density, J the magnetization, H the magnetic field, and  $R_0$  the ordinary Hall constant. The first data on the dependence of  $R_1$  on the temperature T were obtained by Kikoin<sup>2</sup> for nickel for T ranging from room temperature to the Curie point  $\Theta$ . It was observed there that  $R_1$  increases as T rises to  $\Theta$  and diminishes sharply as it passes through  $\Theta$ , evidencing the ferromagnetic nature of  $R_1$ . Jan and Gijsman<sup>3</sup> measured  $R_0$  and  $R_1$  in nickel and in iron for T ranging from room temperature to that of hydrogen and observed that  $R_1$  has a smeared minimum for nickel at 30 to 50°K and for iron at 50 to 70°K. Above these regions,  $R_1$ drops sharply with diminishing T (thus,  $R_1$  in nickel diminishes to  $\frac{1}{20}$  its value from T = 300°K to  $T = 14^{\circ}K$ ). These data<sup>3</sup> are criticized in reference 4, whose authors state that the minimum of  $R_1$  with temperature is in contradiction with general theoretical considerations. Most workers have believed that the ordinary constant  $R_0$  is connected only with the concentration of the current carriers and should not change noticeably with T. Experimental data,<sup>3,5</sup> however, are not in agreement with these considerations. For the extraordinary constant  $R_1$ , the following theoretical relation was obtained in reference 6

$$R_1 = A \rho^2, \tag{2}$$

where  $\rho$  is the specific resistivity and A is a constant. However, an experimental verification of (2), made by Jan,<sup>2</sup> gave for nickel, in the tem-

perature range from that of nitrogen to  $\Theta$ , not 2, but 1.94 as the exponent of  $\rho$ , and for iron the exponent was even 1.42. In connection with this, a hypothesis was proposed<sup>4</sup> that there exist two ferromagnetic effects, one obeying Eq. (2) and another that varies linearly with  $\rho$ , i.e.,  $R_1 = A_1\rho + A_2\rho^2$ .

In view of the above, there is undoubted interest in measuring simultaneously the Hall effect and  $\rho$ in as large a temperature interval as possible, down to the lowest temperatures (helium), using the purest ferromagnetic materials possible. For this purpose we undertook to measure these quantities in pure nickel in the range from room temperature to that of liquid helium  $(4.2^{\circ}K)$ . The measured specimens were made of pure nickel (99.99%) with residual electric resistivity  $\rho_{20.4^{\circ}}/\rho_{293^{\circ}} = 12.36 \times 10^{-3}$ , and  $\rho_{4.2^{\circ}}/\rho_{293^{\circ}} = 10.28$  $\times 10^{-3}$ . The Hall voltage was measured in  $9 \times 4 \times$ 0.3 mm plates using a procedure previously described,<sup>8</sup> which made it possible to obtain an induction B up to 22,000 gausses in the specimen with an electromagnet of magnetic field intensity of 5,000 oersteds. The potentiometric-setup sensitivity was  $2 \times 10^{-8}$  volts. The measurements were performed at room temperature, at 0°C, and in baths of liquid nitrogen, hydrogen, and helium. The specimen temperature was assumed equal to the normal boiling point of the liquid bath. The constants  $R_0$  and  $R_1$  were determined from the slope of the curve e = f(B) in the initial portion and after saturation, using the method proposed in reference 9. The measurement results are given in Figs. 1 and 2, from which it is seen that  $R_1$  diminishes sharply with diminishing T and has a minimum at 20 to 30° [ $R_1$  (T = 300°K) ~  $100 \times 10^{-12}$  v-cm/amp-gauss, R<sub>1</sub> (T = 14°K) ~  $5\times 10^{-12}$  v-cm/amp-gauss]. The constant  $\rm \,R_0$ diminishes from 300° to 4.2°K to approximately  $\frac{1}{3}$  (from 0.6 to  $0.2 \times 10^{-12}$  v-cm/amp-gauss),



and has no minimum. The observed increase in  $R_1$  near the helium temperature raises doubts concerning the universal nature of relations of type (2), at least for a wide range of T. This is seen particularly clearly from Fig. 3, where (on



a logarithmic scale) is shown the connection between  $R_1$  and  $\rho$  as obtained in our measurements. The linear relation is retained up to the liquid-nitrogen temperature, where the exponent in the right half of (2) is 1.02. It must be noted that in references 3 and 7 this index is close to 2 only in the temperature region from  $\Theta$  to room temperature. Farther down, to  $T \sim 100^{\circ}K$  it is closer to unity, and at still lower temperatures  $\log R_1$  and  $\log \rho$  are no longer linearly related. It is peculiar  $\log R_1$  is linear relative to  $\log \rho$ only as long as the quantity proportional to the carrier mobility (calculated from the  $R_0\sigma$  formula) changes little with temperature (see Fig. 4), and ceases to be linear where the latter increases rapidly.

Attempting to understand the physical nature of the observed laws, we note that Vonsovskii et al.,<sup>10</sup> on the basis of sufficiently general semiphenomenological considerations, obtained an ex-



pression for the electric-conductivity tensor, and the anti-symmetrical portion of its inverse value gives the Hall-constant tensor. For temperatures close to the Curie point we have\*

$$R^{0}_{\alpha\beta\gamma} = T \left[ S^{(1)}_{\alpha\gamma} + \overline{m}^{2} S^{(2)}_{\alpha\gamma} \right]; \quad R^{1}_{\alpha\beta\gamma} = T \left[ S^{(3)}_{\alpha\gamma} + \overline{m}^{2} S^{(4)}_{\alpha\gamma} \right] 4\pi \overline{a}_{\beta\gamma}^{-1}.$$
(3)

Here  $S_{\alpha\gamma}^{(i)}$  (i = 1, 2, 3, 4) are tensors that depend on T and on the limiting quasi-momentum of the conduction electron,  $\overline{a}_{\beta\gamma}$  is the parameter of the internal magnetic (spin-orbit etc.) interaction, and  $\overline{m}$  is the relative value of the spontaneous magnetization. From comparison with the variation of  $\rho$  of a ferromagnet below  $\Theta$ ,<sup>11</sup> it can be assumed that the signs of  $S^{(1)}_{\alpha\gamma}$  and  $S^{(3)}_{\alpha\gamma}$ , on the one hand, and of  $S^{(2)}_{\alpha\gamma}$  and  $S^{(4)}_{\alpha\gamma}$ , on the other, are opposite. Therefore, as T diminishes, the values of  $R^0_{\alpha\beta\gamma}$  and  $R^1_{\alpha\beta\gamma}$  should diminish.<sup>†</sup> This can be shown also from thermodynamic considerations.<sup>12,13</sup> The thermodynamic potential of the metal of the ferromagnet at values of T close to  $\Theta$  can be approximately represented in the form of a sum of two terms <sup>14</sup>:  $\Phi_1(\overline{s})$ , which is the potential of the conduction electrons, and depends on their average  $\overline{s}$ , and  $\Phi_2(\overline{s}, \overline{m})$ , which is the ferromagnetism potential, which depends on  $\overline{s}$  and  $\overline{m}$  in addition to the usual quantities (T, pressure). Above  $\Theta$ , the equilibrium conditions are

<sup>\*</sup>From the most general considerations (within the framework of the s-d exchange model) it is possible to assume that  $(R_1/R_0)$  is of the same order of magnitude as  $(\theta/\theta_d)$ , where  $\theta_d$ is the temperature of degeneracy of the Fermi carrier quasiparticles. We then get  $R_1/R_0 \sim 10$ ,<sup>2</sup> as experimentally observed.

<sup>†</sup>It is significant that, according to reference 3 and our measurements, the quantity  $\Re^{0}_{\alpha\beta\gamma}$  in ferromagnets also depends on  $\overline{m}$  and, consequently should have a different dependence on T than in nonferromagnetic materials.

and below  $\Theta$ 

$$\Phi_{1}^{'}(\bar{s}) + \Phi_{2}^{'}(\bar{s}, \bar{m}) = 0.$$

 $\Phi_1''(\bar{s}_0) > 0,$ 

 $\Phi'_{1}(\bar{s}_{0}) = 0,$ 

If one takes into account that hear  $\Theta$  the number  $\overline{s}$  differs little from its equilibrium value  $\overline{s}_0$ , we have

$$\Delta \overline{s} = \overline{s} - \overline{s}_0 = - \Phi_2'(\overline{s}, \overline{m}) / \Phi_1'(\overline{s}_0)$$

Since in the first approximation<sup>14</sup> we have  $\Phi_2(\overline{s}, \overline{m}) \sim \alpha (n-\overline{s})\overline{m}^2$ , where n is the total concentration of the conduction electrons and of the ferromagnetism electrons, and  $\alpha > 0$ , then  $\Phi'_2(\overline{s}, \overline{m}) = -\alpha \overline{m}^2$ , and we have thus  $\Delta \overline{s} > 0$ . From theory it is known that  $R \sim 1/\overline{s}e$ , where  $\overline{s} \approx \overline{s}_0 + \Delta \overline{s}$ . Consequently,

$$R \sim \frac{1}{(\bar{s_0} + \Delta \bar{s})e} \sim \frac{1}{\bar{s_0}e} \left(1 - \frac{\Delta \bar{s}}{\bar{s_0}}\right) \quad (\gamma > 0)$$

If follows therefore that R diminishes with diminishing T. The same conclusion was reached also by Patrakhin<sup>15</sup> in his calculation of  $R_1$  within the framework of the s-d exchange model of ferromagnetism.<sup>16</sup> Thus, it is possible to assume that the mechanism of the appearance of the constant  $R_1$  and its temperature dependence near the point  $\Theta$  is qualitatively understood. However, the form of the function  $R_1(T)$  in the region near very low temperature, and, in particular, the occurrence of a minimum of  $R_1(T)$ , still remain unexplained. One can merely state that the conclusions of references 6 and 4 concerning a simple connection between  $R_1$  and  $\rho$  do not correspond to reality, at least in the region of low temperatures. The function  $R_1(T)$  and its connection with  $\rho$  can be understood only on the basis of a more rigorous s-d exchange theory, in which account is taken of the presence of two branches of the energy spectrum of the electronic system (ferromagnetic and that of the conduction electrons), and in which account is also taken of the collision processes between the carriers and the elementary excitation of the lattice vibrations (phonons) and the spin field of the ferromagnet (ferromagnons) in the presence of a magnetic interaction. The different temperature dependence of the relaxation time for these collision processes (see, for example, reference 17) can lead to a complicated temperature behavior of  $R_1(T)$  at low temperatures.

For a further refinement of the data on the Hall effect in ferromagnets, we have undertaken more detailed theoretical analysis and experimental investigations of the temperature behavior of this phenomenon.

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