SCATTERING BY A SCHWARZSCHILD FIELD IN QUANTUM MECHANICS

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WHEN dealing with a metric which is almost Galilean, we can write

$$\mathfrak{G}^{\mu\nu} = g^{\mu\nu}\sqrt{-g} = \delta^{\mu\nu} - k\gamma^{\mu\nu}; \quad k = \sqrt{16\pi\varkappa}/c;$$
$$\delta^{00} = -\delta^{11} = -\delta^{22} = -\delta^{33} = 1. \tag{1}$$

It is then possible to expand the Lagrangians of all the fields in powers of the constant k. For the scalar (or pseudoscalar), spinor (as developed by Rumer[1]), electromagnetic, and gravitational fields these expansions are, respectively

$$\mathscr{L}_{\mathrm{sc}} = \frac{1}{2}(\delta^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - m^2\varphi^2)$$
$$+ \frac{k}{2}\left(\frac{1}{2}\gamma^{\nu}_{\nu}m^2\varphi^2 - \gamma^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\right) + O(k^2); \tag{2}$$

$$\mathscr{L}_{\mathrm{sp}} = \frac{i}{2}(\bar\psi\gamma(\alpha)\psi_{,\mu} - \bar\psi_{,\mu}\gamma(\alpha)\psi)\delta^{\mu}(\alpha)$$
$$+ k\left[-\frac{i}{4}(\bar\psi\gamma(\alpha)\psi_{,\mu} - \bar\psi_{,\mu}\gamma(\alpha)\psi)\right. \tag{3}$$
$$\left.\times\left(-\frac{1}{2}\delta_{\omega\varepsilon}\delta^{\mu}(\alpha) + \delta^{\mu}_{\varepsilon}\delta_{\omega}(\alpha)\right) + \frac{1}{2}m\bar\psi\psi\delta_{\omega\varepsilon}\right]\gamma^{\omega\varepsilon} + O(k^2);$$

$$\mathscr{L}_{\mathrm{em}} = -\frac{1}{4}F_{\mu\nu}F_{\alpha\beta}\left[\delta^{\mu\alpha}\delta^{\nu\beta}\right.$$
$$\left. + k\left(\frac{1}{2}\gamma^{\lambda}_{\lambda}\delta^{\mu\alpha}\delta^{\nu\beta} - 2\gamma^{\mu\alpha}\delta^{\nu\beta}\right)\right] + O(k^2); \tag{4}$$

$$\mathscr{L}_{g} \equiv \frac{\sqrt{-g}}{k^2}R = -\frac{1}{k}\left(\gamma^{\mu\nu}_{,\mu\nu} + \frac{1}{2}\gamma_{,\mu\nu}\delta^{\mu\nu}\right) + \left(\frac{1}{2}\gamma^{\mu\nu}\gamma_{,\mu\nu}\right.$$
$$-\frac{1}{2}\gamma^{\alpha\beta}\gamma_{\alpha\beta,\mu\nu}\delta^{\mu\nu} - \frac{1}{8}\gamma_{,\mu}\gamma_{,\nu}\delta^{\mu\nu} - \frac{1}{4}\gamma^{\alpha\beta}_{,\mu}\gamma_{\alpha\beta,\nu}\delta^{\mu\nu}$$
$$\left.+\frac{1}{2}\gamma_{,\lambda}\gamma^{\lambda\nu}_{,\nu} - \frac{1}{2}\gamma^{\rho\nu}_{,\lambda}\gamma^{\lambda}_{\nu,\rho}\right) + k\left(\frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\gamma_{\alpha\beta,\mu\nu}\right.$$
$$-\frac{1}{2}\gamma^{\alpha\beta}\gamma_{\beta\varepsilon}\gamma^{\varepsilon}_{\alpha,\mu\nu}\delta^{\mu\nu} + \frac{1}{8}\gamma^{\mu\nu}\gamma_{,\mu}\gamma_{,\nu} + \frac{1}{4}\gamma^{\mu\nu}\gamma^{\rho\tau}_{,\mu}\gamma_{\rho\tau,\nu} \tag{5}$$
$$-\frac{1}{2}\gamma^{\mu\nu}\gamma^{\rho}_{\mu,\lambda}\gamma^{\lambda}_{\nu,\rho} - \frac{1}{2}\gamma^{\alpha\beta}\gamma^{\lambda}_{\alpha,\mu}\gamma_{\lambda\beta,\nu}\delta^{\mu\nu}$$
$$\left.-\frac{1}{4}\gamma^{\alpha\beta}\gamma_{\alpha\beta,\mu}\gamma^{\sigma}_{\sigma\nu}\delta^{\mu\nu} + \frac{1}{2}\gamma^{\alpha\beta}\gamma_{\alpha\beta,\lambda}\gamma^{\lambda\nu}_{,\nu}\right) + O(k^2);$$
$$\delta^0(0) = 1,\ \delta^1(1) = \delta^2(2) = \delta^3(3) = i$$

From these expressions, it is an easy matter to obtain the cross sections for scattering of the quanta associated with these fields by a static spherically-symmetric gravitational field given by

$$\gamma^{\mu\nu}_{\mathrm{st}} = \gamma^{\lambda}_{\mathrm{st}\,\lambda}\delta^{\mu}_{0}\delta^{\nu}_{0};\ \gamma^{\lambda}_{\mathrm{st}\,\lambda} = -Mk/4\pi r. \tag{6}$$

It is clear that one of these processes, namely the scattering of a graviton by a Schwarzschild field, is entirely nonlinear. We remark that this effect is similar to the well-known Delbrück scattering effect (scattering of a photon by the Coulomb field of the nucleus), which lies at the frontier of modern experimental techniques. Graviton scattering by a Schwarzschild field, however, takes place at a lower order of perturbation theory. We shall here make use of the usual expressions for the commutation relations for all the fields involved (see, for instance, Bogoliubov and Shirkov[2]) including the gravitational,[3] since we shall be using the interaction representation (which was first used for the gravitational field by Gupta[4]). The matrix elements for our processes are

$$F_{\mathrm{sc}}(k, k')\,\delta(k_0 - k_0') = -\frac{k^2M}{32\pi^2}\,\frac{m^2 - 2k_0^2}{2k_0\mathbf{k}^2}\,\frac{\delta(k_0 - k_0')}{\sin^2(\theta/2)}; \tag{7}$$

$$F_{\mathrm{sp}}(k, k') = -\frac{k^2M}{32\pi^2}\,\frac{k_0 v^{+}_{\tau}(\mathbf{k}')\,\gamma(0)\,\bar v^{-}_{\sigma}(\mathbf{k})}{\mathbf{k}^2\sin^2(\theta/2)}; \tag{8}$$

$$F_{\mathrm{em}}(k, k') = \frac{ik^2M}{32\pi^2}\,\frac{1}{k_0\sin^2(\theta/2)}\left(e^{\tau'}_{k}e^{\tau}_{k}\cos^2\frac{\theta}{2} - \frac{k_i' k_k}{2k_0^2}e^{\tau'}_{k}e^{\tau}_{i}\right); \tag{9}$$

$$F_{g}(k, k') = \frac{ik^2M}{128\pi^2}\,\frac{1}{k_0\sin^2(\theta/2)}$$
$$\times\left[\left(1 - 2\sin^2\frac{\theta}{2}\right)\delta_{\mu\alpha}\delta_{\nu\beta} - \frac{1}{2}\delta_{\mu\nu}\delta_{\alpha\beta}\right] \tag{10}$$
$$\times\, e^{\alpha}_{\sigma}e^{\beta}_{\tau}e^{\mu}_{\sigma'}e^{\nu}_{\tau'}\delta^{\omega\varepsilon,\sigma\tau}\delta^{\omega'\varepsilon',\sigma'\tau'};\quad \delta^{\omega\varepsilon,\sigma\tau} = \delta^{\omega\varepsilon}\delta^{\sigma\tau} - \delta^{\omega\sigma}\delta^{\varepsilon\tau} - \delta^{\omega\tau}\delta^{\varepsilon\sigma},$$

and from these, after the usual summation and averaging over polarizations of the particles involved, we obtain the differential cross sections

$$d\sigma_{\mathrm{sc}} = \frac{k^4M^2}{(16\pi)^2}\left(\frac{k_0^2 - m^2/2}{\mathbf{k}^2}\right)^2\frac{d\Omega}{\sin^4(\theta/2)};$$
$$d\sigma_{\mathrm{sp}} = \frac{k^4M^2}{(16\pi)^2}\left(\frac{k_0^2}{\mathbf{k}^2}\right)^2\frac{d\Omega}{\sin^4(\theta/2)}; \tag{11}$$
$$d\sigma_{\mathrm{em}} = \frac{k^4M^2}{(16\pi)^2}\cot^4(\theta/2)\,d\Omega;\quad d\sigma_{g} = \frac{k^4M^2}{(16\pi)^2}\,\frac{\cos^2\theta}{\sin^4(\theta/2)}\,d\Omega.$$

It is interesting that these cross sections all become the same at small angles for zero rest mass of the quanta, independent of the spin. For nonzero rest mass, however, as well as at large angles, the different dimensionalities of the tensors describing the fields lead to large differences in the cross sections. In general one can say that the mass causes the cross sections to increase.

Thus when the mass is included in the equations, it is found to have gravitational properties, from considerations of special relativity (energy considerations).

We note that one of the cross sections here obtained (that for photon scattering) was previously obtained by Pijr,[3] who clearly showed the analogy of this effect to the corresponding classical effect. Since, as we have seen, the scattering cross section of a particle with zero rest mass by a Schwarzschild field is

$$d\sigma_0 = (k^4M^2/8\pi)\,|\,d\theta/\theta^3\,|,$$

then, inserting the angle $\theta = k^2M/4\pi R$ (the angle through which light rays are bent according to the classical theory), we have $d\sigma_0 = 2\pi RdR$, which is the classical expression for the cross section for scattering by a sphere of radius R.

In conclusion I thank D. D. Ivanenko and M. M. Mirianashvili for their interest in the work.

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MOLECULAR AMPLIFIER AND GENERATOR FOR SUBMILLIMETER WAVES

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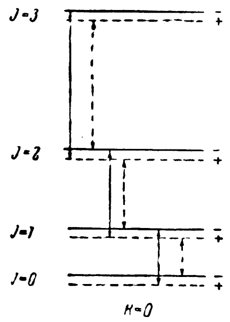
IN the present paper we consider the possibility of constructing a molecular amplifier and generator (MAG), for waves shorter than 1 mm, using ammonia molecules. The rotational transitions of the NH_3 molecules lie in the wavelength region below 1 mm. These transitions can be used to construct the MAG. The rotational transitions are sorted out at the same time as the inversion levels, viz.: molecules in the lower inversion level are sorted out by passing the molecular beam through a quadrupole condenser. The system of rotational-inversion levels after sorting is given in the figure for J = 3, 2, 1, and 0 and for K = 0. Levels which are not occupied by molecules are shown by dotted lines. The solid arrows show transitions increasing the energy of the incident radiation; dotted arrows show those absorbing energy.



The rotational spectrum of NH_3 for J = 3, 2, 1, and 0, and for K = 0.

An amplifier can be constructed using a device in which the radiation coming from one horn crosses a number of molecular beams and falls on a second horn. If the average density of the number of active molecules is equal to N, the coefficient of negative absorption is determined by the equation

$$\alpha = 8\pi^2\nu\,|\,\mu_{mn}\,|^2\,N/hc\Delta\nu, \tag{1}$$

where ν is the frequency of the transition, μ_{mn} the dipole-moment matrix element, $\Delta\nu$ the line width, h Planck's constant, and c the velocity of light.

If the power of the radiation leaving horn 1 is equal to P_0, the power after passing a path l and entering horn 2 rises to $P_k = P_0e^{\alpha l}$. Let $\nu = 6 \times 10^{11}$ cps ($\lambda = 0.5$ mm), $|\mu_{mn}|^2 = 2 \times 10^{-36}$, $\Delta\nu = 5 \times 10^3$ cps, and $N = 10^{10}\,cm^{-3}$. Then $\alpha = 1\ cm^{-1}$. If $l = 10$ cm, $P_k/P_0 = 2.2 \times 10^4$. The maximum power which such a beam can produce is about one microwatt. To construct a molecular generator one can use two plane-parallel mirrors as the resonator. If the distance between the mirrors is l, the reflection coefficient of the mirrors is k, and we assume that energy losses of the plane waves occur only upon reflection from the mirrors, the Q-factor of such a system is equal to

$$Q = (2\pi l/\lambda)/(1-k). \tag{2}$$

If $l = 1$ cm, $\lambda = 0.05$ cm, k = 0.95, then Q = 2400. However, energy losses occur also because the