THE QUESTION OF THE SYMMETRY OF THE MANY-ELECTRON SCHRÖDINGER WAVE FUNCTION

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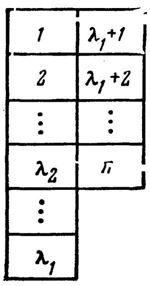
We consider the Schrödinger equation for a many-electron system. We assume that the Hamiltonian in this equation does not contain spin operators. Because of the equivalence of the electrons, the Hamiltonian is invariant under the symmetric group of interchanges of the spatial coordinates of the electrons. According to Wigner's theorem, the eigenfunctions belonging to each eigenvalue of the energy will then form the basis for one of the irreducible representations of the symmetric group. If we assume that the system is in a state with definite spin S, then the only allowed energy eigenvalues are those which correspond to the definite irreducible representation given by a Young scheme of two columns containing respectively λ_1 and λ_2 cells,[1] where

$$\lambda_1 \geqslant \lambda_2, \ \lambda_1 + \lambda_2 = n, \ \lambda_1 - \lambda_2 = 2S. \tag{1}$$

This gives the connection between the energy values and the value of the spin, although, as we mentioned, the Hamiltonian of our system does not contain spin operators.

The symmetry properties of the many-electron coordinate function necessary and sufficient to make this function an element of the subspace transforming according to a definite irreducible representation $\{\lambda_1\lambda_2\}$ are conveniently illustrated with the help of the corresponding Young scheme. We write into the cells of the Young scheme the position numbers of the arguments, e.g., in their natural order (cf. figure).



Then one can so choose the basis functions of the subspace transforming according to the irreducible representation $\{\lambda_1\lambda_2\}$, that each of them is antisymmetric with respect to the interchange of the arguments whose numbers lie in either of the columns of the Young scheme; one can also choose them such that each of the functions is symmetric with respect to the interchange of the arguments whose numbers lie in each of the rows of the Young scheme. These properties we shall call properties A.

There exists, however, another set of conditions, the three Fock conditions,[2] which are equivalent to the conditions A, as we shall show. The Fock conditions, which we shall call conditions B for brevity, consist in the requirement that the function be antisymmetric with respect to two groups of k and n − k arguments (two conditions), and in the requirement of cyclic symmetry. Moreover,

$$n - k \geqslant k, \ (n-k) - k = 2S. \tag{2}$$

Comparing (1) and (2), we see that $\lambda_1 = n - k$, $\lambda_2 = k$, and that, of course, the first two of the conditions B coincide with the first half of condition A. The requirement of cyclic symmetry may be interpreted as the impossibility of antisymmetrizing the function in more than n − k arguments.

We proceed to the systematic proof of the equivalence of conditions A and B. We shall make use of a group-theoretical method based on the material of chapters IV and V of Murnaghan's book.[3]

We consider the space of the functions subject to the first two of conditions B. We denote it by the symbol $(\lambda_1\lambda_2)$. This space is subdivided into several subspaces transforming according to the irreducible representations of the symmetric group. Among these will certainly be the irreducible representation $\{\lambda_1\lambda_2\}$, as well as some representations $\{\lambda_1'\lambda_2'\}$ with $\lambda_1' > \lambda_1$, but there will be no representation $\{\lambda_1''\lambda_2''\}$ with $\lambda_1'' < \lambda_1$.

We now impose the requirement of cyclic symmetry on the functions of space $(\lambda_1\lambda_2)$. Evidently, the functions of the subspace transforming according to $\{\lambda_1'\lambda_2'\}$ do not satisfy this requirement, since they can, according to property A, be antisymmetrized in more than λ_1 arguments. Of the whole space $(\lambda_1\lambda_2)$ there remains only one subspace, transforming according to $\{\lambda_1\lambda_2\}$, whose functions satisfy the requirement of spherical symmetry.[4] Indeed, the subspace transforming according to $\{\lambda_1\lambda_2\}$ is not contained in the spaces $(\lambda_1'\lambda_2')$, and hence cannot, according to the reciprocity theorem of Frobenius, be antisymmetric in more than λ_1 arguments.

Thus the three Fock conditions are necessary and sufficient for the functions satisfying these conditions to belong to the subspace transforming according to the irreducible representation of the symmetric group of interchanges of their arguments.

In closing I want to express my sincere gratitude to M. I. Petrashen' for a discussion of this paper.

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[2] V. A. Fock, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 961 (1940).

[3] F. Murnaghan, The Theory of Group Representations, Baltimore, 1938.

[4] Iu. N. Demkov, J. Exptl. Theoret. Phys. JETP **35** (1958), Soviet Phys. JETP **8** (1959) (in press).

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THE HEAVY NEUTRAL MESON: DECAY MODES AND METHOD OF OBSERVATION

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THE classification of the elementary particles admits the existence of a meson with strangeness 0 and isotopic spin 0 (see, e.g., the review by Okun'[1]). Such an hypothesis has been advanced repeatedly.[2,3] Evidently such a meson (let us call it ρ) must be neutral and interact strongly with nuclei; in particular, it may be produced singly in collisions of two nucleons.

We assume that the ρ meson differs from the neutral pion only in mass and isotopic spin, but that the space spin and parity of the ρ meson are the same as for the neutral pion (pseudoscalar). Let the ρ meson be equivalent to a nucleon-antinucleon pair in the state $0^S\,{}^1S_0$, whereas the neutral pion is equivalent to a pair in the state $1^S\,{}^1S_0$, $T_z = 0$ according to the classification of Bethe and Hamilton[4] (see also reference 5). These two pair states differ by the relative phase $\overline{PP}$ and $\overline{NN}$.

In this letter we consider the possible decay modes of the ρ meson and the method of observing it in an experiment. It is obvious that the mass of the ρ meson is greater than that of the neutral pion; otherwise the ρ meson would have been discovered in experiments on neutral pion production. The transformations $\rho \rightarrow 2\pi^0$ and $\rho \rightarrow \pi^+ + \pi^-$ are not possible as they would not conserve parity: two pions in a state with $L = 0$ are an even system, whereas the ρ meson is odd.

By applying the operator CT (C is charge conjugation, i.e., conversion of particles into antiparticles; T is charge symmetry, i.e., conversion of protons into neutrons). Bethe and Hamilton have shown that three-pion annihilation cannot occur in a 0^S state. Hence the decay modes $\rho \rightarrow 3\pi^0$ and $\rho \rightarrow \pi^+ + \pi^- + \pi^0$ are forbidden. This applies to any odd number of pions.

To treat the decay into four pions, we separate them into two pairs and denote the isotopic spin of the first pair by t_1, its orbital angular momentum by l_1, the corresponding quantities of the second pair by t_2 and l_2 and the angular momentum of the center of mass of the first pair relative to the other by L. From the assumption that the ρ meson is pseudoscalar and has $T = 0$, it follows that $t_1 = t_2$, $|L| = |l_1 + l_2|$, and that $L + l_1 + l_2$ is odd. If t_1 is even, then l_1 and l_2 are even; if t_1 is odd, then l_1 and l_2 are also odd.

If $l_1 = l_2$, then both pairs can be regarded as identical bosons, and the wave function must be symmetric with respect to their exchange.

The lowest values of the momenta that satisfy all these conditions are $l_1 = l_2 = 2$, $L = 1$, $t_1 = t_2 = 0$, or $t_1 = t_2 = 2$.

For $l_1 \neq l_2$, such a state is $l_1 = 1$, $l_2 = 3$, $L = 3$, and $t_1 = t_2 = 1$. The need for large orbital momenta can reduce substantially the probability of the $\rho \rightarrow 4\pi$ decay.

The decay $\rho \rightarrow \pi^0 + \gamma$ is forbidden, since radiative $0 - 0$ transitions are forbidden. The decay $\rho \rightarrow \pi^+ + \pi^- + \gamma$ is allowed, if the pion pair is in a state with $L = 1$. Also allowed is the decay $\rho \rightarrow 2\gamma$, which is analogous to the decay $\pi^0 \rightarrow 2\gamma$. If $m_\rho > 2m_\pi$, one can expect the single photon decay to be more probable.

The expected time of decay is 10^{-18} to 10^{-20} sec.

It will be extremely difficult to identify the decay $\rho \rightarrow \pi^+ + \pi^- + \gamma$ in the presence of photon background from the $\pi^0 \rightarrow 2\gamma$ decay.

We propose below a method for detecting events of single production of ρ mesons in interactions of charged particles by energy-momentum balance. Consider the reaction $p_1 + p_2 \rightarrow p_3 + p_4 + \rho$, where