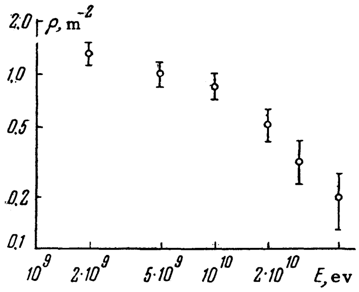

FIG. 2. Integral energy spectrum of nuclear-active particles in the energy region 2 to 50 Bev. The x axis represents the energy of nuclear-active particles in ev, and the y axis the absolute flux density of nuclear-active particles.

tances 0 to 9 m from the shower axis were constructed from 52 nuclear interaction events. Since the spectra obtained in the two variants of the experiments were identical, we averaged the results of both series of measurements.

The observed integral energy spectrum of nuclear-active particles in the 10 to 50 Bev energy region can be approximated by a power law E^{-k}, where $k = 0.95 \pm 0.25$ (cf. Fig. 2). This result is not surprising, since a spectrum of this shape can be expected from the integral energy spectrum of μ mesons in EAS, measured at the same altitude.[5] Furthermore, the same value of the spectrum exponent has been obtained for 10^{11} to 10^{12} ev nuclear-active particles.[1]

We estimated the fraction of nuclear-active particles of > 2 Bev, at a distance of 0 to 9 m from the axis, by comparing the observed number of nuclear-active particles with the electron flux density in showers detected by our array. We have obtained a value $(1.3 \pm 0.3)\%$ which, within the limits of experimental error, is in a good agreement with the value $(1 \pm 0.1)\%$ obtained earlier[6] by means of a hodoscope. It should be noted that the fraction of nuclear-active particles measured in the present experiment may be underestimated, in view of the difficulties in identifying nuclear-active particles when the number of electrons in the shower produced in the chamber is large.

*For details of the cloud chamber, cf. reference 2.

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Translated by H. Kasha
320

CALCULATION OF THE LIFETIMES OF EXCITED STATES OF Hf^{178} *AND* Hf^{180}

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THE nuclides Hf^{178} and Hf^{180} have excited states from which transitions occur to a level of the rotational band with total angular momentum $I = 8$, with emission of E1 radiation. The γ-ray energies are 88.8 and 56.6 kev, respectively. The lifetimes of these long-lived states are 3 seconds and 5.5 hours.[1,2] The standard theoretical estimates of the lifetimes of these excited states according to the independent particle model (examples of which can be found in reference 3) give values approximately 10^{16} times smaller. However, in highly deformed nuclei (such as Hf^{178} and Hf^{180}) a new quantum number K appears — the magnitude of the projection of the total angular momentum on the symmetry axis of the nucleus. For γ transitions in such nuclei there is therefore a new selection rule on K: $\Delta K \leq L$, where L is the angular momentum of the emitted radiation. The selection rule on K is not strict, since the rotational motion of the nucleus somewhat perturbs the nucleon configuration and distorts its shape. Transitions with $\Delta K > L$ are said to be K-forbidden, and their degree of forbiddenness is characterized by the number $\nu = \Delta K - L$. The occurrence of K-forbiddenness can explain the long lifetimes of the Hf nuclides.

The possibility of K-forbiddenness has been treated by A. Bohr in the uniform nuclear model which he proposed.[4] For a numerical estimate of the transition probability when K-forbiddenness occurs, we still have to make assumptions about the nature of the interaction of the nucleons in the nucleus. We shall use Nilsson's scheme,[5] which gives the correct order of levels for an unpaired nucleon. However, the nuclides Hf^{178} and Hf^{180} are even-even. We must therefore, in treating the nucleon, also take into account its interaction with its paired partner. We shall assume that the pairing energy is zero when the nucleons are in different single particle levels as given by the Nilsson scheme, while the pairing energy of nucleons which are in the same single particle level (so that they have equal and opposite projections of their total angular momenta on the symmetry axis) is equal to the difference between the excitation energy as calculated from the uniform model and the experimental value. The pairing energies for Hf^{178} and Hf^{180} are then 1100 and 1300 kev, respectively. According to the Nilsson scheme, in the ground state of the nucleus the nucleons successively fill all the single particle levels, and there can be no more than two nucleons in each of the levels. The long-lived states are the first excited states which involve a change in the nucleon configuration, since rotational excitations are associated with slow rotation of the nucleus, which does not affect its internal structure. It is therefore natural to assume that these long-lived states are formed as a result of the shift of a nucleon which is in the last filled level to the next higher level. According to the collective model, the lowest excitations with change of internal structure will be those in which $I = K = \Sigma\Omega_i$, where Ω_i is the projection of the total angular momentum of the i-th nucleon on the axis of symmetry. From the fact that an E1 transition was observed to a rotational level with $I = 8$, it follows that the total angular momentum I of the long-lived excited states is 7, 8, or 9. From the positions of the single particle levels in the Nilsson scheme, we see that, for the Hf^{178} and Hf^{180} nuclei, the only one of these three values which is possible is $\Sigma\Omega_i = 8$ (when a neutron is excited). However, there are experimental data which apparently favor $K = 9$ for the case of Hf^{180}.[2] This may occur, for example, as a result of a poor estimate of the pairing energy. If it happens that the difference between the pairing energies when the nucleons are in levels 49 and 40* (so that the projections of their angular momenta on the symmetry axis are $^9/_2$ and $^9/_2$) and when they are in levels 49 and 48 (with projections of their total angular momenta on the symmetry axis equal to $^9/_2$ and $^7/_2$) is less than or approximately equal to 200 kev, then the energy of the state with $K = 9$ will be lower than that of the state with $K = 8$. In this case it is difficult to say anything definite about the lifetimes of the excited states. However, if the pairing energy of nucleons which are in different levels is small, the lifetime of the excited state will be approximately one order of magnitude greater than that calculated here (because the degree of K-forbiddenness is increased by unity). The E1 transition to the rotational level with $K = 0$ is possible because K is not a strict quantum number.

We have treated the perturbations

$$u_1 = -(\hbar^2/J_1)I_1 j_1 - (\hbar^2/J_2)I_2 j_2,$$
$$u_2 = -\beta\sin\gamma M\omega_0^2 r^2 (Y_{22}(\vartheta,\varphi) + Y_{2-2}(\vartheta,\varphi))/\sqrt{2},$$
$$u_3 = (\hbar^2/4J_1 - \hbar^2/4J_2)(I_1^2 - I_2^2),$$

where I_κ, j_κ are the projections on the κ axis of the total angular momentum of the nucleus and the total angular momentum of the particular nucleon; r, ϑ, φ are the coordinates of the nucleon, and M its mass. ω_0 is determined by the number of nucleons in the nucleus, and $I_\kappa = 4B\beta^2 \sin^2(\gamma - 2\pi\kappa/3)$.

The matrix elements of u_2 and u_3 are different from zero only between states with different values of the quantum number n_γ which characterizes the nuclear surface oscillations. In the calculations we included corrections only from intermediate states with $n_\gamma = 1$, since the contribution from intermediate states with $n_\gamma > 1$ will be much smaller. The corrections from u_2 and u_3 are smaller, but are still significant in the cases we are considering, since they affect the order of magnitude of the lifetime.

It was assumed that the long-lived state of Hf^{180} results from the ground state through the shift of a neutron from level 49 ($\Omega = -^9/_2$) to level 48 (having $\Omega = ^7/_2$), so that $I = K = \Sigma\Omega_i = 8$. The single particle energies and nucleon wave functions were taken for $\beta = 0.2$. The lifetime of the long-lived excited state was found to be $\tau = 175$ hours (while the experimental value is $\tau = 5.5$ hours).

For Hf^{178} it was assumed that the long-lived state is formed by the transition of a neutron from level 41, with $\Omega = -^7/_2$, to level 49, with $\Omega = -^9/_2$, so that $I = K = \Sigma\Omega_i = 8$, and that $\beta = 0.25$. The calculated lifetime was $\tau = 130$ sec (while the experimental value is $\tau = 3$ sec).

The values of the deformation parameter β were chosen to be close to the experimental values (as determined from the nuclear quadrupole moment), and to give the smallest possible values of the lifetimes. We should remark that the result changes very slightly for small changes in the size of β.

The cases we are considering are of interest because the magnitudes of the lifetimes are determined by corrections in the 7-th order of perturbation theory, since the degree of K-forbiddenness is 7. Therefore any deficiencies of the scheme will strongly affect the result. The calculated values were found to be 30 to 40 times those observed in experiment. It should be remarked that, in addition to the perturbations considered here, there may also be perturbations due to the presence of the paired nucleon. Even though we cannot, because of possible deficiencies of the scheme, definitely assert that these perturbations give a contribution amounting to approximately $\frac{1}{4}$ of the contribution of the perturbations considered here (as would follow from our result), it is clear that these perturbations will be small. The main contribution will come from the perturbations which we have considered.

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*The levels are designated according to reference 5.

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321

PHONON INTERACTIONS OF ELECTRONS IN POLAR CRYSTALS

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THE interaction energy of two electrons in polar crystals can be split into parts. The first part is the direct Coulomb interaction of the electrons, including the polarization of the electron shells of the atoms. This polarization is evaluated by introducing the dielectric constant $\epsilon_\infty = n^2$, where n is the index of refraction of light. The second part is the interaction through the phonon field. An evaluation of the phonon interaction is also important for a system consisting of an electron and a hole, for instance, an exciton. This problem was considered by a number of authors: in the strong coupling approximation by Dykman and Pekar,[1] and in the intermediate-coupling approximation by Meyer,[2] Haken,[3] and others. The authors of the cited papers used different variational principles to find the energy spectrum of the exciton.

In the present communication the phonon interaction potential of the electrons is derived in the intermediate-coupling approximation, with allowance for their relative momentum.

The operator of the interaction energy of an electron with the phonon field can be written in the form[4,5] (we put henceforth $\hbar = 1$):

$$\sum_k V_k a_k e^{i\mathbf{kr}} + V_k^* a_k^+ e^{-i\mathbf{kr}}, \tag{1}$$

$$V_k = -(i\omega/k)(^1/_2\, m\omega)^{1/4}(4\pi\alpha/V)^{1/2},$$

$$\alpha = \frac{e^2}{2}\left(\frac{2m}{\omega}\right)^{1/2}\left(\frac{1}{n^2} - \frac{1}{\varepsilon}\right). \tag{2}$$

The quantity α plays the role of the coupling constant, m is the effective mass of the electron, ω the limiting frequency of the longitudinal optical vibrations, and the a_k are the second-quantization operators. In the center-of-mass system, the energy operator of the two-electron system under consideration is of the form

$$\hat{H} = -\frac{1}{2M}\nabla_R^2 - \frac{1}{2\mu}\nabla_r^2 + \frac{e^2}{n^2 r} + \sum_k \omega a_k^+ a_k$$
$$+ \sum_k 2\cos\frac{\mathbf{kr}}{2}(V_k a_k e^{i\mathbf{kR}} + \text{c. c.}), \tag{3}$$