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SURFACE IMPEDANCE OF SUPERCONDUCTING CADMIUM

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The apparatus described here can be used to measure the surface impedance of a metal at a wavelength of 3.2 cm in the "super-low" temperature region of $\sim 0.1°K$. The complex surface impedance of a cadmium single crystal has been measured in the temperature interval between 0.1 and 0.6°K. The results of the measurements are analyzed. The penetration depth of the electromagnetic field in the superconducting cadmium has been determined and found to be $\delta_0 = (13 \pm 1.4) \times 10^{-6}$ cm for $T \to 0$.

THE measurement of the surface impedance of superconductors is particularly interesting in the frequency range satisfying the condition $h\nu/kT_c \gtrless 1$, which can also be written as $\lambda T_c \lessgtr 1$ (λ = wavelength in cm, T_c critical temperature of the superconductor in °K). This condition can be satisfied by two different methods: by decreasing the wavelength λ of the applied electromagnetic radiation, and by investigating superconductors with lower critical temperatures T_c.

Until now most investigators have followed the first method[1-4] up to the present limits of radio techniques ($\lambda \sim 0.2$ cm). Only preliminary work has been carried out with the second method.[5,6] In this case, as in all work in this field, the temperature of the sample (Al) was reduced to 0.85°K by pumping off the liquid helium which cooled the whole of the apparatus under investigation.

This paper describes apparatus for the measurement of samples cooled to $\sim 0.1°K$ by the magnetic method (with ammonium iron alum used as the cooling agent). This makes possible a value $h\nu/kT_c = 0.9$ in investigations of cadmium at a wavelength of 3.2 cm, for example. This value is reached only at $\lambda = 0.46$ cm when working with tin. One of the features of the apparatus is that

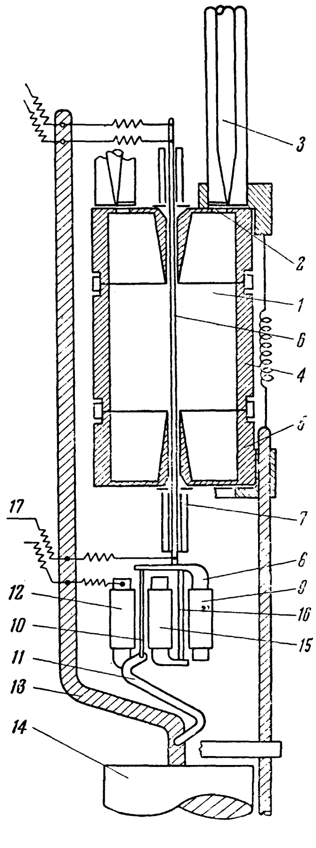

FIG. 1. Outline of the apparatus. On the right are shown some constructional details of the apparatus. The vacuum jacket is not shown.

only the sample, which serves as part of the measuring resonator, is cooled to the temperature of the alum. The remaining parts of the apparatus are at the temperature of the helium bath ($\sim 1.6°K$).

APPARATUS

The coaxial resonator (Fig. 1) with cavity 1 of diameter 1.6 cm and length 3.4 cm (somewhat larger than $\lambda = 3.2$ cm, owing to the effect of the conical ends of the internal conductor) has two symmetrical apertures 2, which are connected to the generator and to the detector by means of the coaxial lines 3. The power transfer efficiency (10^{-2} to 10^{-4} at resonance) may be regulated externally by means of a device which rotates the resonator about its axis of symmetry and displaces the two apertures relative to the coaxial lines. Cavity 1 of the resonator is formed by four conductors: cylinder 4 of length $\lambda/2$, two end flanges 5 of length $\lambda/4$, and the central part of the sample 6, which serves as the axial conductor. The electric contacts between the different parts of the resonator are not important, because all connections coincide with nodal lines of the resonance currents. The internal surfaces of the axial bores (diameter 1 mm) of flanges 5 are covered by layer of polystyrene of thickness 3 to 5μ. This prevents electrical contact between the specimen and the flanges. The cylinder and the flanges are made of oxygen-free copper and have been electropolished internally.

The cylindrical sample 6, with a diameter of 0.9 mm, fits closely into the bores of the end flanges of the resonator and extends in length right through the resonator. The losses in high-frequency power from the resonator via the sample are smaller by 1 to 1.5 orders of magnitude than the power transferred to the detector; this is brought about by the open-circuited quarter-wave lines, which are formed by the conical parts of the resonator flanges together with the sample, and also by the quarter-wave "reflectors" 7 placed at the ends of the sample. The lower part of the sample is soldered to copper strip 8 with cadmium — the same metal as the sample. Copper strip 8 serves to conduct the heat to carbon thermometer 9, which measures the temperature of the sample. A heat conductor 10, made of the same metal as the sample but having a thermal resistance an order of magnitude higher than the sample, is also soldered to strip 8. This arrangement ensures a homogeneous temperature in the sample when heated by high frequency currents throughout the temperature range over which the measurements are made. An additional thermometer measures the temperature at the upper end of the sample, to check that this condition was fulfilled.

Heat conductor 10 is soldered to copper strip 11 ($0.1 \times 1.5 \times 15$ mm^3), at the upper end of which is located heater 12. The latter enables one to raise the temperature of the sample above that of the salt by means of the heat flow produced in strip 11 and conducted away by heat conductor 13. This conductor is a copper wire, 2 mm in diameter, whose end has been flattened into a broad strip (2 cm) and pressed into the salt 14. Heater 15 is connected to strip 8 by means of heat conductor 16 whose thermal resistance is about half that of the sample. This heater is used to determine the power dissipated in the sample by the high-frequency current. For this an equivalent direct current is made to flow through the heater and adjusted so that there is no change in the reading of thermometer 9. The thermometers were made of resistor material in the shape of thin copper-plated platelets of a conducting substance ($0.5 \times 2 \times 5$ mm^3). The copper strips of the electric and thermal conductors were also electroplated to the thermometer; the completed thermometer was soldered with Wood's metal to a thin-walled copper screen. The resistance of the thermometer changes from $\sim 50\,\Omega$ at 4.2°K to $\sim 400\,\Omega$ at 0.1°K. The heaters are fabricated like the thermometers, but are made of a material whose resistance is practically constant in the temperature range of the experiment.

The thermometers were calibrated by the magnetic temperature of the ammonium iron alum used as cooling agent. The alum permeability was determined by measuring the mutual inductance of coils surrounding the salt with an R-56 ac potentiometer at 60 cycles. The alum was cooled by helium until the start of demagnetization; the helium was then removed from the cavity of the apparatus by means of a carbon adsorption pump. The pressure of the helium used as heat exchanger was estimated from the rise in the temperature of a special carbon thermometer over that of the helium bath. The thermometer was hung inside the vacuum jacket of the apparatus with thin caprone threads (measuring current $80\,\mu$a, resistance $3\,k\Omega$).

The electric leads to the sample for measuring its dc conductivity, to the thermometers, and to the heaters are made of constantan 30 microns in diameter. The center of each lead 17 (only a few of them are shown in Fig. 1) was brought into thermal contact with heat conductor 13.

By demagnetizing the salt from ~ 7 kilogauss at $\sim 1.6°K$ it was possible to obtain a temperature of $\sim 0.12°K$. The salt warmed up at a rate of

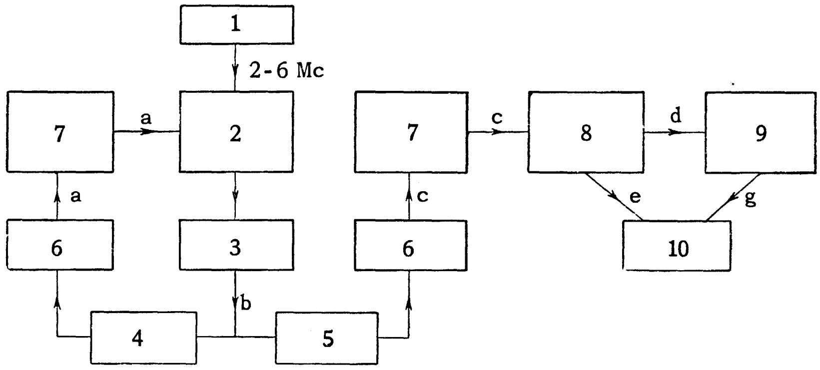

FIG. 2. Block-diagram of the electronic equipment for the continuous registration of the frequency characteristic and of the changes in the resonant frequency of the resonator. The letters represent the voltages and currents shown in Fig. 3. 1—Standard Signal Generator GSS-6; 2—Sweep Generator and Modulator of the Klystron; 3—Klystron K-20; 4—Standard resonator; 5—Resonator to be Measured; 6—Detectors; 7—Amplifiers; 8—First Differentiating Circuit; 9—Second Differentiating Circuit; 10—Six-Point Recording Potentiometer EPP-09.

~ 0.02°K/hr, corresponding to a heat influx of ~ 30 erg/sec. In the absence of an hf field (< 1 erg/sec), no direct heat flux to the sample could be detected (by thermal radiation and through points of contact with the resonator).

ELECTRONIC CIRCUIT

The frequency characteristics and changes in the resonant frequency of the resonator were measured with an electronic circuit which made it possible to register these values automatically (together with the temperature of the sample) on a six-point EPP-09 recording potentiometer.

The operating principle of the circuit is as follows (Figs. 2 and 3). The modulating voltage from standard signal generator GSS-6, with a frequency F = 2 to 6 Mcs, is applied to the reflector of the klystron K-20. This produces sidebands in the klystron-output spectrum, separated from the carrier frequency by an amount F. Furthermore, the carrier frequency of the klystron is modulated, at a frequency of 25 to 50 cycles, by a saw-tooth voltage applied to the reflector from the sweep generator. There is a linear shift in the frequency spectrum of the klystron during the first half cycle of the saw tooth voltage, as shown in Fig. 3b. During the second half cycle, there is a linear shift in the opposite direction.

The limits of deviation of the carrier frequency of the klystron are determined by the resonances of the standard resonator (Fig. 3a), which occur at the instant when the carrier frequency or one of the side bands of the spectrum of the klystron are equal to the resonant frequency f_{st} (Fig. 3b). A superconducting cylindrical lead resonator with a Q on the order of 10^6 (resonance mode H_{011}) served as standard resonator. The signal from the standard resonator triggers the sweep generator and, therefore, also the direction of the shift in the carrier frequency. In this way, the magnitude of the shift in the klystron frequency is equal to the modulation frequency F from the GSS-6 standard signal generator.

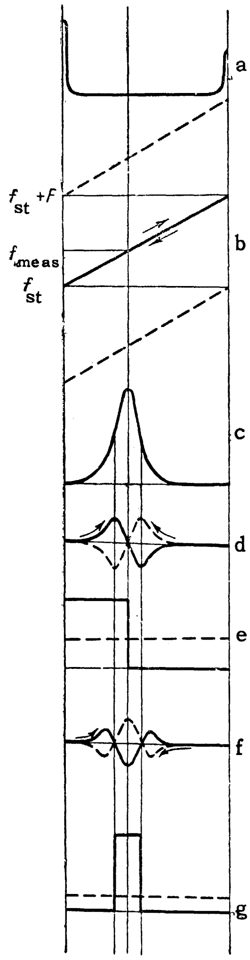

FIG. 3. Illustration of the operating principle of the electronic circuit. The time variations of the voltages and currents are shown at some points of the circuit in Fig. 2. During one half cycle of the sweep-voltage the changes take place from left to right, during the other half cycle – from right to left.

As the carrier frequency of the klystron covers the pass band of the tested resonator, a signal, which has the shape of the frequency characteristic of the resonator (Fig. 3c), reaches the detector. The first derivative of this signal (Fig. 3d) vanishes at the instant when the carrier frequency of the klystron coincides with the resonant frequency of the resonator. A pulse produced at this instant switches on or off one of the output currents of an electronic circuit (Fig. 3e) in such a way that the time average of this current (Fig. 3e, dotted line) is proportional to the difference in the frequencies of the measured and of the standard resonator

$f_{meas} - f_{st}$. This mean value of the current is recorded by the potentiometer EPP-09. In the same way, a second differentiation of the resonator signal (Fig. 3f) produces a current (Fig. 3g) whose time average is proportional to the width of the frequency band of the resonator. This current is also recorded by the potentiometer.

The electronic circuit contains about 60 vacuum tubes. A separate paper will be devoted to its description.*

THE SAMPLE

The sample investigated was an electropolished cylindrical single crystal of cadmium of purity 99.999% and a diameter of 0.9 mm. The hexagonal axis of the crystal was parallel to the axis of the cylinder to an accuracy of a few degrees. The hf currents travelled along the surface of the cylinder, so that the surface resistance of the cadmium was measured practically in the direction of its hexagonal axis. The dc conductivity of the sample at 4°K was $(22 \pm 5) \times 10^{20}$ esu, exceeding its conductivity at 300°K by a factor of $(1.5 \text{ to } 2) \times 10^4$.

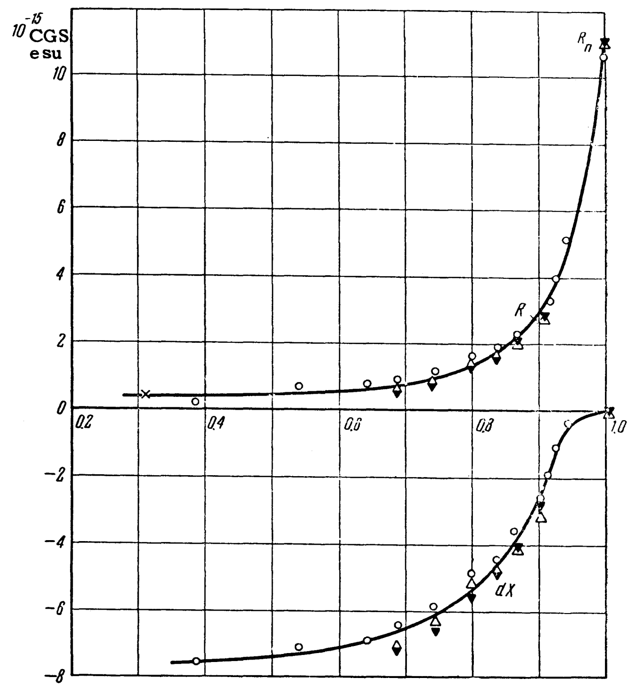

FIG. 4. The surface resistance R and the changes in the surface reactance dX of cadmium as a function of the relative temperature $t = T/T_c$ ($T_c = 0.56$°K). The three types of symbols represent three series of measurements.

THE MEASUREMENTS

The change Δf in the width of the frequency band of the resonator and the shift df in its resonant frequency were measured experimentally during the transition of the sample from the superconducting to the normal state (and back). These quantities determine the resistance R of the sample and the change dX in its reactance:

$$R = 2\pi K \Delta f; \quad dX = -4\pi K df.$$

The quantity $K = 4.7 \times 10^{-9}$ H was calculated from the dimensions of the resonator and from the type of oscillations excited in it; it was verified experimentally (to an accuracy of ~10%) by measuring the width of the frequency characteristic of the resonator at room temperature. At helium temperatures, the width of the frequency characteristic of the resonator with the cadmium sample in its normal condition was equal to 845 kc. Of these, 495 kc are due to losses in the copper walls and to losses by radiation in the coupling apertures, and 350 kc are caused by losses in the sample (accuracy ±5 kc). Figure 4 shows the results of measurements of the surface impedance of a sample of single crystal cadmium.

The "residual" resistance R_s of the superconducting sample at temperatures much below critical is quite small and its value, determined from the width of the frequency characteristic of the resonator, becomes quite inaccurate. For this reason R_s must be determined from the rf heating of the sample as mentioned above, by replacing the hf heating with the power dissipated in the heater 15. In practice, however, it is more convenient and accurate to measure the ratio of R_s to the resistance of the sample in the normal state R_n:

$$R_s / R_n = \Delta f_n W_s / \Delta f_s W_n,$$

where Δf_s and Δf_n are the widths of the resonator frequency characteristics with the superconducting and the normal sample, respectively, and W_s and W_n denote the power dissipated by the hf current in the superconducting and the normal sample. This ratio is equal to 2 ± 0.5% (Fig. 4, the cross on the R curve) for cadmium at ~0.15°K. During these measurements it is necessary to maintain constant the power from the generator (klystron) to the line that excites the resonator, and the power transfer efficiency, which should be small compared to unity.

The dc conductivity of the sample is determined during the high-frequency investigations by measurements with a tuned photoamplifier[7] at 4.64 cps. It is thus possible to check any differences in the critical temperature for transitions to the superconducting state under dc condictions and at high frequencies.[1,6] No such difference was found in the experiments described here (to an accuracy

*The circuit was developed together with I. I. Losev.

of $\sim 0.003°K$).

RESULTS

By measuring the surface resistance of a superconductor it is possible to determine the dielectric constant ϵ and the ratio of the conductivity of the normal electrons to their mean free path σ/l.[8-10] The two terms which appear in the expression

$$\varepsilon = \varepsilon_0 - c^2/\omega^2\delta^2, \quad (1)$$

can be separated if the results of independent measurements at different frequencies ω are known, or, in particular, if the depth of penetration δ of a weak static magnetic field into the superconductor is known. After measuring ϵ at one frequency ω, one can either calculate δ, assuming $\epsilon_0 \ll c^2/\omega^2\delta^2$, or one can try to separate the terms containing ϵ_0 and δ by making use of additional considerations, e.g., the law governing the variation of δ with the temperature T.

Figure 5 shows the variation of ϵ with the relative temperature $t = T/T_c$ ($T_c = 0.56°K$ is the critical temperature of cadmium), obtained from an analysis of the measurements of the complex surface impedance of cadmium in accordance with Abrikosov's equations.[9] The application of Ginzburg's[8] equations leads to practically the same results. The points lie a few percent higher but still within the possible error of the measurements.

As can be seen from Fig. 5, the variation $\epsilon(t)$ in the region $0.3 < t < 1$ can be expressed by the simple empirical law:

$$-\varepsilon = 14.4\,(1 - t^4)\cdot 10^8 \text{ CGS electrostatic units.} \quad (2)$$

The value of δ, calculated from the equation

$$\delta = c/\omega\sqrt{-\varepsilon} \quad (3)$$

assuming that $\epsilon_0 \ll c^2/\omega^2\delta^2$, leads to the expression

$$\delta = 13.4\,(1 - t^4)^{-1/2}\cdot 10^{-6} \text{ cm}, \quad (4)$$

which has the form of the usually[11,12] obtained relationship

$$\delta = \text{const}\,(1 - t^4)^{-1/2}. \quad (5)$$

The accuracy of determination of δ is about 10%.

By estimating the limiting value for δ at $T = 0$ as $\delta_0 = (13.4 \pm 1.4) \times 10^{-6}$ cm, we can calculate for cadmium the parameter κ of the theory of Ginzburg and Landau:[13]

$$\varkappa = 2.16\cdot 10^7 H_c \delta_0^2 = 0.11 \pm 0.02,$$

where $H_c = 28.8$ oersteds is the critical magnetic field for cadmium.[14] The value obtained for κ is

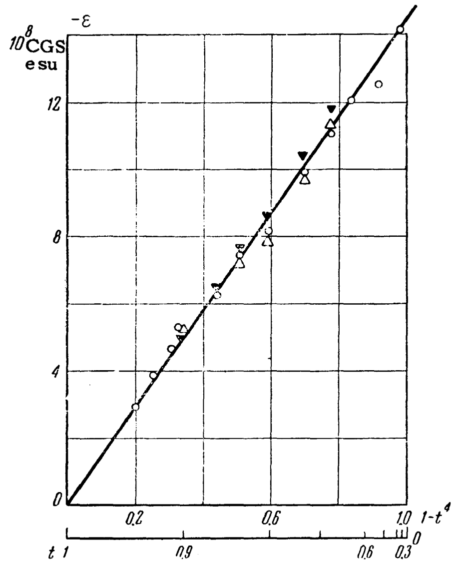

FIG. 5. Variation of the dielectric constant of superconducting cadmium with the relative temperature t. The three types of symbols represent three series of measurements.

in good agreement with theoretical considerations (for "soft" superconductors, $\kappa < 1/\sqrt{2}$).

By investigating the variation of the critical field of small cadmium spheres with their dimensions, Steele and Hein[15] obtained $\delta_0 = (9 \text{ to } 11) \times 10^{-4}$ cm. This value for δ_0 is far too high. This is particularly clear on calculating κ from it ($\sim 600!$). The result of Steele and Hein is apparently incorrect, though it is difficult to show the reason for their mistake.

The ratio of the conductivity produced by normal electrons to their mean free path σ/l may be found from the complex surface impedance of the superconductor $Z = R + iX$:

$$\sigma/l = \text{const}\,(1 + \eta)\,X^3/|Z|^6, \quad (6)$$

where the parameter $-1 \leq \eta \leq +3$ depends on the ratio R/X. (The surface reactance X of the superconductor is calculated from the equation $X = X_n - dX$, where dX is the experimentally-measured change in the surface reactance of the metal at the transition from the normal to the superconducting state, and the surface reactance X_n of the normal metal is determined from the relationship $X_n = R_n\sqrt{3}$, according to the theory of the anomalous skin effect.[16])

The results of the calculation of σ/l from Eq. 6 are shown in Fig. 6; the curve is drawn through the experimental points approximately. The large scatter of the points may be explained by the fact that the quantities X and R, measured to an accuracy of 3 to 5%, appear in Eq. 6 with high powers. This causes an almost tenfold increase in the relative error in the calculation of $(\sigma/l)_s$ for the superconductor.

The value of $(\sigma/l)_0$ for the normal metal is determined to an accuracy of about 10%. Knowing $(\sigma/l)_0$ and the conductivity σ, one can estimate the mean free path in the normal metal: $l \sim 0.1$ mm.

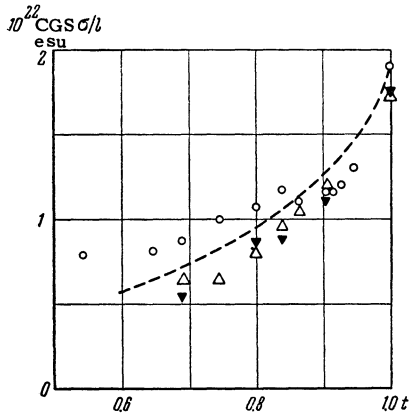

FIG. 6. Variation of the ratio of the conductivity produced by normal electrons to their mean free path with the relative temperature t. The curve has been drawn approximately through the experimental points. The three types of symbols represent three series of measurements.

The experimentally found variation of σ/l with temperature is not in disagreement (Fig. 7) with the law:

$$1 - [(\sigma/l)_s/(\sigma/l)_0] = C(1 - t^4). \tag{7}$$

A more positive verification cannot be given because of the above-mentioned insufficient accuracy of the results. For cadmium we obtain $C = 0.85 \pm 0.2$. Analogous experiments on tin give $C = 1.0 \pm 0.2$. A formula in the form of Eq. 7 can be obtained by making the following assumptions. We assume that the equation

$$n_0 = n_n + kn_s, \tag{8}$$

is correct, which establishes the relationship between the two electron densities: n_0 — normal electrons in the normal metal; n_n and n_s — normal and superconducting electrons in the superconductor. The coefficient k does not depend on temperature. Rewriting Eq. 8 with the use of the formulae

$$n_0 = \frac{mv_0}{e^2}(\sigma/l)_0; \quad n_s = \frac{m\omega^2}{4\pi e^2}\varepsilon,$$

and also assuming that the mean free path of normal electrons is the same in the normal metal and in the superconductor, we obtain

$$1 - \frac{(\sigma/l)_s}{(\sigma/l)_0} = \frac{kn_s}{n_0} = \frac{k\omega^2}{4\pi v_0}\frac{\varepsilon}{(\sigma/l)_0}, \tag{9}$$

where $v_0 = (1 \text{ to } 2) \times 10^8$ cm/sec is the Fermi velocity of the normal electrons (the effective masses of the normal and of the superconducting electrons are assumed to be equal).

Substituting the experimentally found values for ϵ and $(\sigma/l)_0$ in Eq. 9 and comparing with Eq. 7 we obtain k = 5 to 10 for cadmium. For tin we obtain k = 4 to 8, which is in agreement with the known data.[11] Thus for cadmium, as well as for tin and several other sufficiently investigated metals, it is not permissible to assume k = 1 when using Eq. 8.

The surface resistance of cadmium in the normal state is $R_n = (11 \pm 1) \times 10^{-15}$ esu. This is appreciably larger than the result of Chambers,[17] who obtained for polycrystalline cadmium $R_n = (7.7 \pm 0.3) \times 10^{-15}$ esu. Because of this, the corresponding values of $(\sigma/l)_0$ calculated from Eq. 6 differ by a factor of about 3. The discrepancy may be due to a strong anisotropy of the surface resistance of cadmium. Such an assumption, however, requires experimental verification.

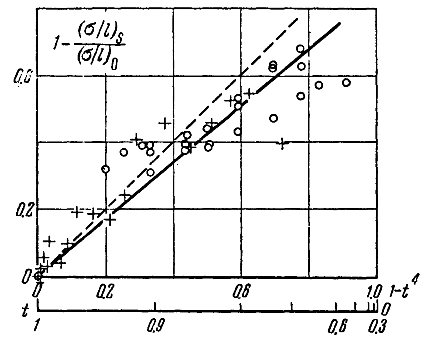

FIG. 7. The full line and the circles refer to measurements on cadmium, the dotted line and the crosses to measurements on tin.

Analyzing the measured surface resistances of different superconductors (Hg, Sn, Al, Zn, In) made at high frequencies (1200 to 36,000 Mc), Faber and Pippard[5] found that a plot of $\log(2 \times 10^5 G_1/T_c)$ and $\log G_2$ vs. $\log(\omega/T_c)$ is a straight line of slope $2/3$ (reference 5, Fig. 5). These quantities characterize the slope of the curve R(T) close to T_c and far away from T_c, respectively. Evaluation of the measurements of R(T) for cadmium (Fig. 4) leads to values of $G_1 = 11 \times 10^{-5}$ and $G_2 = 30$, which lie well on the extension of the straight line in Fig. 5 of the paper by Faber and Pippard.[5] Owing to the low critical temperature of cadmium, the points of this figure, observed by investigating cadmium at a frequency of 9400 Mc, lie at higher ω/T_c values than all other measurements made in that work.[5]

CONCLUSIONS

A method has been developed for measuring the complex surface impedance of superconductors in the temperature region down to $\sim 0.1°K$, obtainable by demagnetization of paramagnetic salts. By investigating superconductors with low critical temperatures it is possible to obtain values of the parameter $h\nu/kT_c$ several times larger than when working in the region of helium temperatures (1 to 4°K) at the same frequencies.

The complex impedance of a single crystal of superconducting cadmium has been measured. The temperature dependence has been obtained for (a) the depth of penetration of an electromagnetic field into superconducting cadmium, and (b) the ratio of the conductivity produced by normal electrons to their mean free path. In analyzing the second quantity, measurements of the surface resistance of tin have also been utilized.

The author takes the opportunity to thank P. L. Kapitza for his interest in the work, A. I. Shal'nikov for his continuous interest and valuable advice, and also G. S. Chernyshev and I. I. Losev for technical help.

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