I wish to thank Prof. A. A. Vlasov sincerely for his interest in the accomplishment of this work.

Translated by D. C. West
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LIBERATION OF GAS UPON CLEAVAGE OF CRYSTALLINE QUARTZ

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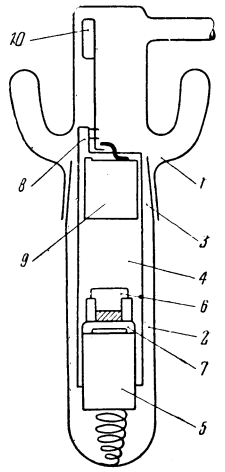
PREVIOUSLY reported experiments[1] disclosed electron emission upon cleavage of certain crystals in a vacuum of 10^{-4} to 10^{5} mm Hg.

To study and to explain further the nature of the emission, it was necessary to use higher vacuum, which in turn called for development of a new procedure. To attain high vacuum in the simplest possible manner, it was decided to make the equipment completely of glass, like the instruments used for the study of the kinetics of chemical reactions



(see diagram). The upper portion of the instrument consisted of a trap 1 for the lubricant vapor with a ground neck. The lower portion of the instrument consists of a sealed tube 2 which was inserted into ground section 3. Placed inside the instrument was a setup for cleaving solid specimens to incandescence, consisting of stainless steel tube 4 with windows, and of a brass cylinder with spring 5. Attached to this cylinder were guides for a knife 6 and a holder for x-ray film 7. Located in the upper portion of the tube was a trigger 8 for falling weight 9.

The instrument was sealed to a vacuum mercury pump and evacuated to a pressure of approximately 10^{-7} mm Hg (the vacuum was measured with an ionization manometer). After evacuating the instrument, lever 10 of the trigger was rotated with an electromagnet, and the weight fell and fractured a plate approximately 4 mm. The photographic film was exposed to the electrons emitted from the gap formed upon cleavage of the plate.

It was observed in the preliminary experiments that upon cleavage of glass and diffused quartz there is no noticeable change in the vacuum, and no electron emission was observed (like in the previous experiments).

Cleavage of crystalline quartz (like in the previous experiments) caused electron emission, and the pressure rose to 10^{-4} to 10^{-5} mm Hg (measured with an ionization manometer). The area of the fresh surface obtained upon cleavage of crystalline quartz was approximately 1 cm^2. The capacity of the vacuum system was about 1300 cubic cm. Liberation of gas was observed also upon splitting of mica and stripping of high-polymer films from glass.

The author is grateful to Professor B. V. Deriagin for valuable advice.

[1] N. A. Krotova and V. V. Karasev, Dokl. Akad. Nauk SSSR **92**, 607 (1953).

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ON ELECTRON CAPTURE IN BETATRONS

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LOGUNOV and Semenov[1] have pointed out the existence of statistical capture of electrons into betatron orbits and have estimated the efficiency of this mechanism. This calls for the following two essential remarks.

1. This mechanism can work only at not too large densities of the injected electrons. In par-

ticular, it will not work at the conditions of the example treated in Ref. 1. This can be seen from the following estimates.

The electrons are injected in a beam with an area $S = 1$ cm^2 and a density $N_0 = 4.22 \times 10^8$ electrons/cm^3. We shall assume a beam of round cross section. The electrostatic force repelling an electron situated on the edge of the beam is given by

$$F_{rep} = 2N_0 e^2 \sqrt{\pi S}. \quad (1)$$

On the other hand, the same electron will be attracted to the center of the beam by a force due to magnetic focusing:

$$F_{att} = -m(v/R)^2(1-n)\sqrt{S/\pi}, \quad (2)$$

where m is the electron mass, v the velocity of the injected electrons, R the radius of the equilibrium orbit, and n the magnetic field decrement index. From (1) and (2) we have

$$|F_{rep}/F_{att}| = 2\pi r_0 R^2 N_0/\beta^2(1-n), \quad (3)$$
$$r_0 = e^2/mc^2, \quad \beta = v/c.$$

For the conditions of the example of Ref. 1 this yields

$$|F_{rep}/F_{att}| \approx 22. \quad (4)$$

Thus the beam will begin to spread right after injection, and the electron dynamics will differ considerably from that assumed in Ref. 1. Electron-electron scattering of the kind on which the proposed capture mechanism was based will thus not occur for the majority of the electrons. Actually a large fraction of the electrons will be lost to the walls already within the first revolution and will thus not be able to experience even a single scattering event of the required kind. If one takes into account this and the loss of electrons in the subsequent revolutions, one finds that the estimate of the number of captured electrons obtained in Ref. 1 has to be decreased by a factor approximately equal to the ratio of the number of injected to captured electrons, i.e., by approximately two orders of magnitude. Mathematically this is expressed by a suitable decrease of N in the numerator of Eq. (4) in Ref. 1. Secondly, the capture efficiency will be decreased by another order of magnitude, owing to the decrease of the factor N_0 in the same formula. Thirdly, the following facts have to be considered. Besides radial oscillations the electrons oscillate also in the z direction and longitudinally. Only the transfer of energy from the radial oscillations into other oscillations was considered in Ref. 1. However, if these other oscillations are already excited, then energy will also be transferred into the radial oscillations. This actually will be the case in the example considered, and thus the efficiency of the proposed mechanism will be further decreased. So one finds that for large densities of the injected electrons, where the capture is most effective, this capture mechanism does not work. It therefore can not explain that part of the electron capture for which it was adduced in the first place. It plays only a subordinate role and only at sufficiently low densities, in the range between single electron capture and collective capture. We shall make a few remarks on this subject in the second comment.

2. The decisive capture mechanism is due to the Coulomb interaction and consists, in our example, of the following.

Right after the instant of injection, electrons begin to be lost to the walls close to the injector. The region of contact of the electron beam with the walls of the donut will increase at a rapid rate and after a time of one revolution electrons will hit the donut walls all around. The donut at that time will be filled with an electron cloud of decreasing density. At the time when the beam will stop expanding the electron density will be roughly

$$N_k \approx N_e/\ln(N_0/N_e), \quad N_e = \beta^2/4\pi r_0 R^2 = 0.3 \cdot 10^8. \quad (5)$$

From now on the electrons will start contracting toward the center of the beam, which itself will oscillate about the equilibrium orbit with a very small amplitude. This oscillation is due to the loss of electrons. Thus the survival of some electrons is due to the loss of others to the walls. The electrons which have survived the first revolution [their number is given by Eq. (5)] will now interact with the electrons which are being injected at this time and so a large number of electrons will be lost during the second revolution. Thus the mean life of electrons in the beam is approximately two revolutions or even a little less; for lower densities it would be a little more. During the second revolution the picture remains essentially the same except for the slight modification introduced by the presence of the electrons which have survived from the first revolution, etc. The number of electrons captured with the right momentum and being accelerated is given by

$$N_\gamma = N_k \, 2\pi R S_{eff} \approx 0.45 \cdot 10^{10}, \quad (6)$$

where S_{eff} denotes the area of the effective donut cross section. The obtained numerical value corresponds to the conditions discussed in Ref. 1. Equation (6) is valid for injection currents equal to or larger than the saturation current and has been derived for $N_0/N_e \gg S_{eff} > 1$. For smaller cur-

rents and densities the formula has to be modified.

This way one can give a mathematical description of the treated physical picture of the capture process. The theory then gives good agreement with experiment, both qualitatively and quantitatively.

[1] V. I. Logunov and S. S. Semenov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1513 (1957), Soviet Phys. JETP **6**, 1168 (1958).

Translated by M. Danos
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ANALYTICITY OF THE NONRELATIVISTIC SCATTERING AMPLITUDE AND THE POTENTIAL

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As is well known, the amplitude for scattering of a particle of given angular momentum l by a central field of force cannot be analytically continued into the upper half plane of the variable k. An interesting proof of this is connected with the inverse problem of scattering theory (Gelfand and Levitan,[1] Marchenko[2]).

For brevity we assume that there are no bound states and that scattering takes place in the s state; the generalization of the problem is obvious.

Following Marchenko,[2] the solution of the equation

$$d^2\psi(x,k)/dx^2 + (k^2 - V(x))\psi(x,k) = 0 \qquad (1)$$

can be given as an expansion in integrals over the system of functions

$$\varphi(y,k) = (2/\pi)^{1/2}\sin[ky + \delta(k)] \qquad (2)$$

[$\delta(k)$ is the scattering phase, known from experiment] in the following way

$$\psi(x,k) = \varphi(x,k) + \int_x^\infty A(x,y)\varphi(y,k)\,dy. \qquad (3)$$

In this equation A(x, y) is determined from the integral equation, the kernel and inhomogeneity of which are expressed through the Fourier component of the scattering amplitude M(k) = $\exp\{2\pi i\delta(k)\} - 1$:

$$m(z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} M(k)e^{ikz}dk, \quad \text{for } z > 0. \qquad (4)$$

If m(z) = 0 for z > 0, then it follows from the equation of Marchenko that A(x, y) = 0 and, according to Eq. (1), the wave function $\psi(x, k)$ coincides with the solution of the free equation everywhere except at the origin (contact interaction). On the other hand, m(z) going to zero for z > 0 means, according to Eq. (3), that the scattering amplitude does not have poles for Im k > 0 and grows with $k \to \infty$ (Im k > 0) no faster than as a power of k, since in this case the contour of integration can be closed around a half circle of large radius lying in the upper half plane.

From this it follows that, if the scattering is described by a potential, then either the amplitude has a pole (a so-called spurious pole, since we assumed that the system did not have a level), or it grows faster than a polynomial for $k \to \infty$ (Im k > 0).

If the potential is bounded in space [V(x) = 0 for x > a], then m(z) goes to zero for z > 2a. This follows from the relation

$$\int \varphi(y,k)\varphi(x,k)\,dk = \delta(y-x) - m(x+y). \qquad (5)$$

In fact, for (x and y) > a, $\varphi(x, k)$ and $\varphi(y, k)$ coincide with the solution of the Schrödinger equation and, therefore, should be orthogonal. But then it follows from Eq. (4) that the function

$$M_a(k) = M(k)e^{2ika} \qquad (6)$$

is analytic in the upper half plane. This result was obtained by Van Kampen[3] from other considerations.

If the scattering amplitude is known for all energies, then, as was shown most rigorously in the work of Khuri,[4] the function

$$g(E) = M(E,\tau) - V_\tau/4\pi \qquad (7)$$

(where M(E, τ) is the scattering amplitude, viewed as a function of energy E and given momentum τ, and V_τ is the Fourier component of the potential) can be analytically continued in the complex E plane (or upper half plane of k) and a dispersion relation can be given for it.*

From the dispersion relation for the function (7) obtained by Khuri, it can be seen that if the scattering amplitude is known, then the potential is determined by the amplitude without solution of the integral equation.

We emphasize that this assertion is valid if the scattering amplitude is known for all energies. Since even the Schrödinger equation is valid only in a limited region of energy, then the scattering am-