$$C_v = C_{v0} - \frac{k\Theta a^{1/2}}{8\pi\alpha^{3/2}\sqrt{T-\Theta}} \iint u(\mathbf{r}_1 - \mathbf{r}_2)\,\rho_1(\mathbf{r}_1)\,\rho_1(\mathbf{r}_2)\,dV_1\,dV_2, \quad T > \Theta. \tag{15}$$

As has already been noted, Eqs. (8) and (9) are invalid for small **f** in the immediate neighborhood of the phase-transition point. This is true also for Eqs. (14) and (15). A necessary condition for their validity is $A\eta^2/2 \gg C\eta^4/4$, if $\eta^2 = (kT/4\pi\alpha d)\,e^{-nd}$. Here d is the distance between neighboring atoms, and $n = (a|T-\Theta|/\alpha)^{1/2}$

Noting that $a^2/2C$ is equal to the discontinuity in the specific heat per unit volume ΔC_p, and[2] that $\alpha \approx \Theta a d^2$, we obtain the following condition for the validity of Eqs. (14) and (15):

$$|T-\Theta|/\Theta \gg (k/16\pi d^3 \Delta C_p)\exp\{-(|T-\Theta|/\Theta)^{1/2}\}.$$

Using the value of Sykes and Wilkinson[3] for ΔC_p of β-brass, we obtain $|T-\Theta|/\Theta \gg 0.007$.

Within the limits of applicability of the expressions obtained, the inclusion of short-range order in the ordered phase leads to an insignificant decrease in the specific heat. In the disordered phase, the inclusion of short-range order adds the following correction term to the specific heat:

$$C_v = C_{v0} + \text{const}/\sqrt{T-\Theta}.$$

For β-brass, with $(T-\Theta)/\Theta = 3\times10^{-2}$, this additional correction term supplies about 5% of the discontinuity in the specific heat at the Curie point. This conclusion is in satisfactory agreement with the measurements performed by Sykes and Wilkinson.[3]

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CERTAIN SOURCES OF THE LOW-ENERGY ELECTRON-PHOTON COMPONENT OF COSMIC RAYS IN THE STRATOSPHERE

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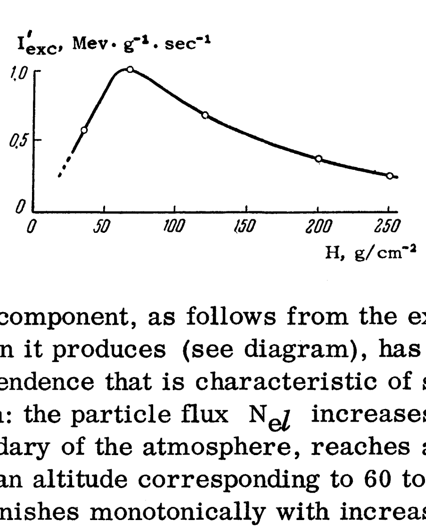
It is shown that part (20 to 30%) of the low-energy flux (range $R < 1.7$ g-cm^{-2} Al) registered in the cosmic radiation of the stratosphere, is genetically related to nuclear-disintegration products.

FROM the number of particles N ($\mathrm{cm^{-2}\,sec^{-1}}$) and the produced ionization I (pairs of ions per cm^3/sec), measured in the global intensity of cosmic rays,[1,2] it follows that the average ionizing ability of charged particles in the atmosphere, $K = I/N$, increases considerably with altitude, and that at an altitude of 10 to 17 km it exceeds the average ionizing ability of relativistic particles by a factor of 1.5 to 1.7 (Ref 3). This is evidence that there exists at these altitudes a considerable flux of secondary strongly-ionizing particles. The ionization due to these particles can be estimated from the value of the "excess" ionization, defined as the difference $I_{exc} = I - k_r N$ where k_r is the average ionizing ability of relativistic particles. The value of I_{exc} reaches one-third of the total ionization current.[4] Experiment has shown[4,5] that the variation of I_{exc} with altitude and its latitudinal effect are identical with the corresponding relations observed for stars in photoemulsions, and for ionization impacts observed in chambers. This suggests a possible genetic relationship between this variation and nuclear disintegrations.

Quantitative measurements of the ionization impacts, which experiments[5] have shown to be essentially due to strongly-ionizing protons and to heavier charged particles generated in nuclear disintegrations, account for a considerable portion[4] of the excess ionization — up to 57% at the threshold of registration of impacts of 5.5 relativistic particles,[6] corresponding under the experimental conditions to a proton energy range 1 Mev < E_p < 50 Mev. The ionizing contribution (~12% of I_{exc}) of the faster strongly-ionizing particles (particularly protons with energies E_p > 50 Mev) that were not recorded in the experiments of Refs. 4 and 6, were determined by measurement of the ionizing abilities of single particles with ranges R > 1.7 g-cm^{-2} in the experiments of Refs. 3 and 7. It was established[3] that, upon approaching the boundary of the atmosphere, most excess ionization is produced by multiply-charged particles from primary cosmic rays (primary α particles).

Comparison of the results of preceding experiments leads to the conclusion that approximately 0.3 I_{exc}, observed at an altitude of 10 to 19 km, cannot be attributed to a flux of strongly-ionizing protons and stronger particles generated in nuclear disintegrations. On the other hand, a study of the small ionization impacts observed at very low registration threshholds (~0.5 of the average ionization produced by relativistic particles) makes it possible to separate out the flux of particles that make up this portion of I_{exc} and to identify them as low-energy electrons that produce excess ionization I'_{exc} by multiple scattering.[3] The flux N_{el} of these short-range electrons (R < 1.7 g/cm^2), which travel in equilibrium with the photons that generate them (these photons have for the most part, low energies, ~10 Mev), makes up a considerable fraction (~20 percent) of the entire global flux of the charged particles at an altitude of 19 km (Ref. 3). (Further evidence of the considerable flux of low-energy photons in the stratosphere is found in the data of Ref. 8.)



This component, as follows from the excess ionization it produces (see diagram), has an altitude dependence that is characteristic of secondary radiation: the particle flux N_{el} increases from the boundary of the atmosphere, reaches a maximum at an altitude corresponding to 60 to 65 g/cm^2, and diminishes monotonically with increasing depth, following approximately an exponential law with an exponent $1/\mu = 130$ g/cm^2. The energy E_{el} scattered in the air by this flux can be determined from the equation

$$E_{el} = I'_{exc} + k_r N_{el} \approx 2 I'_{exc},$$

since the average ionization momentum produced by the short-range electrons in a chamber is approximately double the relativistic ionization.[3]

This naturally raises the question of the extent to which the value of I'_{exc}, in parallel with the other components of excess ionization, is genetically related to nuclear disintegrations. Since I'_{exc} is due to low-energy electrons, it is possible to take into consideration the γ and β activities of the residual nuclei, namely the products of disintegrations and of those reactions that can be excited in the air nuclei by the flux of secondary nucleons evaporated at low energies ($\lesssim$ 30 Mev) in the process of nuclear disintegration.

Along with radiation capture,[9] the neutron flux in the atmosphere can cause (nn), (np), or (nα) reactions with the air nuclei, causing production of the excited N^{14*}, C^{14*} and B^{11*} nuclei.[10–14] To estimate the average excitation energy $\epsilon_{ex}(E_n)$ produced in air (N^{14}) by a neutron of initial energy E_n we must know the relative probability σ_{ri} of the competing processes and the value of the average excitation energy ϵ_{exi} of the residual nuclei resulting from each reaction. The lack of literature data was made up, for the energy range 2 Mev < E_n < 30 Mev, by calculating (for reactions on nitrogen) the approximate values of σ_{ri} and ϵ_{exi}, in accordance with the "nonresonant" theory of nuclear reactions[15] and known data[16] on the energy levels of the produced nuclei. The resulting values of ϵ_{exi} are justified by an agreement, satisfactory for our purposes, with the individual experimental data available.[10,11] Also important is the fact that the quantity $\bar{\epsilon}_{ex}$ turns out to be little sensitive to a relative change in the cross sections of the various reactions. As a result, we obtained $\epsilon_{ex} \approx 0.5(E_n - 2.5)$ Mev.

Averaging this quantity over the spectrum of the neutron energies in the act of generation leads to a value $\bar{\epsilon}_{ex} = 3.5$ Mev, assuming a differential spectrum of the form $\sim E \exp\{-E/T\}$ (Ref. 17) and an average energy ~10 Mev at the instant of generation.[18] The excitation energy $\bar{\epsilon}_{ex}$, in view of the low probability of secondary emission of particles for the range of E_n under consideration[10,11,16] is radiated as γ quanta. Measured values of the γ-ray yield[12] confirm this fact. It is necessary to add to the quantity ϵ_{ex} the energy ϵ_γ of the γ radiation that occurs upon radiation capture of slow neutrons by the N^{14} nuclei. Comparing the various experimental data for absorption cross sections of slow neutrons[19] and for (np) and ($n\gamma$) reactions on nitrogen,[9,20] and taking into account the energy yield of the capture reaction ($Q = 10.8$ Mev),[16] we obtain as an average $\overline{\epsilon}_r = 1.6$ Mev per neutron absorbed in air.

The dominant reaction in the absorption of slow neutrons in the atmosphere is $N^{14}(np)C^{14}$ (Ref. 17), which leads to the formation of the β-active C^{14} nucleus. But the very long half life ($\sim 6{,}000$ years) makes this activity unimportant to the value of I'_{exc}.

The energies of the recoil nuclei, formed by elastic collision between the neutrons and the gas of the ionization chamber, are recorded as "excess" ionization, whose average value, for the equilibrium spectrum of neutrons in the atmosphere, turns out to be $\epsilon_0 = 1.1$ Mev/sec-g (referred to one neutron generated per gram of air per second).

The flux of secondary protons, the generation density of which can be assumed in the atmosphere to equal the generation density of the neutrons,[6] can produce various reactions with the N^{14} nucleus, resulting in excited and β-active residual nuclei. However, the ionization ranges of the protons turn out to be on the average considerably less than the average range for nuclear interactions, and consequently, the energy contributions of the latter to the production of electrons and photons does not exceed $\sim 3\%$ of the corresponding contribution of the reactions on neutrons. Even less significant here is the deuteron flux.

It is difficult to give an exact estimate of the excitation energy ϵ_d for the nuclei resulting from nuclear disintegration. Assuming this value to be bounded from above by the binding energy ϵ_b, we put as a rough approximation $\overline{\epsilon}_d \sim \epsilon_b/2$, which gives $\overline{\epsilon}_d \approx 1.8$ Mev for a single neutron (if the multiplicity of neutron generation in a light substance is assumed to be approximately 2, see Ref. 21). As to the β activity of the disintegration products, the data on observation of stars in photoemulsions exposed at high altitudes[22] apparently give no grounds for assuming this activity (with a short half-life) to be considerable.

A possible source of instrumental error in ionization measurements of cosmic rays[1-3] can be the occurrence of radioactivity in the material, namely the gas (argon) or the walls of the chamber (aluminum). The most significant here may be the radiation capture of slow neutrons, with production of β-active Al^{28} (half-life 2.3 minutes, maximum energy of radiated electrons 3 Mev).[23] The excess ionization due to this activity corresponds to $\overline{\epsilon}_\beta = 1.0$ Mev/sec-g (referred to one neutron absorbed in an equilibrium flux in one gram of air per second).

It is now possible to determine the total energy E_γ contributed by the above processes to the production of electrons and photons of low energy by making use of data on the generation density of neutrons in the atmosphere. For depths $H > 200$ g/cm^2, where the neutron flux can be assumed to be in equilibrium,[24] the necessary data are found in the absolute measurements of the absorption of slow neutrons in the atmosphere.[24,25] Extrapolation of these data, using an exponential law with exponent $1/\mu = 156$ g/cm^2 yields[25] the approximate values of the generation densities at lower depths (~ 100 g/cm^2). The calculated values of E_γ would be somewhat too high for low depths (~ 100 g/cm^2), owing to the shortage of slow neutrons in the observed spectrum compared with the equilibrium spectrum. The table gives the calculated values of E_γ for various depths, compared with the experimentally measured energy E_{el} (Ref. 3), after making suitable corrections in the latter for the values of $\overline{\epsilon}_0$ and $\overline{\epsilon}_\beta$.

Value of E_γ Compared with the Energy E_{el} at Various Depths in the Atmosphere (Geomagnetic latitude $\lambda = 52°$)

Depth, H, g/cm^2	65	120	200	250
E_{el}, Mev/g-sec	1.9	1.3	0.7	0.44
E_γ, Mev/g-sec	0.4	0.3	0.173	0.13
E_γ/E_{el}	0.21	0.23	0.25	0.3

It follows from these data that 20 to 30% (referred to the energy) of the short-range electrons ($R < 1.7$ g/cm^2), which produce approximately 10% of the total excess ionization, are genetically related to the products of nuclear disintegrations. The estimated value of E_γ can vary by $\pm 80°$, owing to the low accuracy of the initial data, principally the inaccuracy of the average neutron energy during the act of generation. The remaining portion of the flux of short-range electrons, which produces approximately 20 percent of the excess ionization, can be assumed due to the cascade process of the development of the electron-photon component in the atmosphere.

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THE CAUSALITY CONDITION AND SPECTRAL REPRESENTATIONS OF GREEN'S FUNCTIONS

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By means of the causality condition in the form of the requirement that field operators commute on a space-like surface, spectral representations are obtained for the vacuum expectation values of T-products of three Heisenberg operators. The analytic properties of these functions in the complex plane are discussed.

THE present paper presents a method for obtaining spectral representations for the vacuum expectation values of T-products of Heisenberg operators (Green's functions).

These representations [Eqs. (8), (9), (18), and (19)], being natural extensions of the Källén-Lehmann formulas[1,2] for the vacuum expectation values of T-products of two operators, provide a convenient means for investigating the analytic properties of these functions in the complex plane.

In the present paper, which is the first installment of this work, spectral representations are